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Generalization of distance transform

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Suppose a binary matrix consisting of 0 and 1 is given. The distance transform is a problem of computing the distance to the nearest 1 element from each 0 element. This problem has been studied for various distances. Euclidean distance, Manhattan distance, and L_∞ distance are used as examples of the distances. Especially, the Euclidean distance is the most natural distance used for image processing. Therefore, the Euclidean distance transform for a binary image has been widely applied in the field of image processing such as computer vision, pattern recognition and so on.

Algorithms that solve the distance transform heavily depend on distances. It was relatively easy to design efficient algorithms for L_∞ and Manhattan distances. However, a quite efficient algorithm was not developed for the Euclidean distance. It is so complex and takes much time to solve the Euclidean distance transform. Therefore, the Euclidean distance transform was approximated by other distances to which an efficient algorithm had already been known.

The algorithms for computing the Euclidean distance transform in an efficient way were designed in 1995 and 1996. Kirkpatrick et al. proposed an algorithm using an idea of Voronoi diagram in 1995. A method which was proposed by Hirata in 1996 is to convert the problem of Euclidean distance transform into that of computing parabolic lower envelope. These two methods are algorithms that solve the Euclidean distance transform in time proportional to the size of a given matrix ,that is, in linear time.

Algorithms that solve the Euclidean distance transform for a binary image in linear time is known. However, no efficient algorithm like the distance transform has been proposed for a real valued matrix yet. Therefore, we consider the problem of generalizing the distance transform.

We define generalized distance transform as a problem that the concept of the distance transform was enhanced and for a matrix whose element is a real number. Then, we define a superior element for each element as the element with a value that is larger than its value. So, the generalized distance transform can be defined as a problem of computing the distance to the nearest superior element from each element. The generalized distance transform can be used for a given image to obtain a centerline in an object figure. For instance, we consider computing a ridge line of a topographical map. Rough figure of the ridge line can be guessed by computing the generalized distance transform when an altitude is given as the element of the matrix.

In this paper, we propose an efficient algorithm for generalized distance transform by assuming the size of an input matrix is $n \times n$. A simple algorithm is for each element to examine a superior element in order of distance. However, we have to examine $\mathrm{O}(n^2)$ elements for each element. Therefore, the entire computing time becomes $\mathrm{O}(n^4)$ time. Moreover, it is assumed that there are h different values of the elements included in the given matrix. Then, the matrix can be made a binary matrix by regarding a value of each element as a threshold. Repeat the algorithm in linear time for a binary matrix only h times, and the algorithm runs in time $\mathrm{O}(hn^2)$. However,since the number h of iteration may become $\mathrm{O}(n^2)$, it needs $\mathrm{O}(n^4)$ time to compute the Euclidean distance transform in the worst case. This corresponds to the quadratic time in the number of matrix elements.

In this paper, we improve this worst computational complexity. That is, we propose an efficient algorithm which computes generalized distance transform in subquadratic time in the matrix size. The proposed algorithm consists of two steps.

The first one is as follows. For each element we look for superior elements which exists in the distance $< k$. As the result, the nearest superior element can be found for large part of elements. However, there are some elements that can't find superior elements in their neighborhoods. Such elements are defined as the local maximum elements. It is necessary to look for the superior element separately for the local maximum elements.

The second step is as follows. we divide the matrix into $k \times k$ buckets. Then, a local maximum element appears as the maximum value in each bucket. Therefore, if the values of the maximum elements are compared between buckets, we can find the sets of buckets including the superior element for each local maximum element. So, we only have to search superior elements from this set. However, when it compares between arbitrary elements of the bucket, it might take too much time for computing. Then, each superior element in the set is converted into a straight line by applying the method of Hirata and using dual transformation. This straight line can convert a vertical distance between the straight line and an element into the distance between the corresponding elements and the elements. As a result, the problem of looking for the superior element can be converted to the problem of computing the nearest straight line , that is, that of finding the lower envelope of many piecewise linear curves. Thus, we can compute the nearest superior element for each element. Moreover, according to the change of division size k of the bucket, we can show the proposed algorithm solves generalized distance transform in $\mathrm{O}(n^2\sqrt[3]{n})$ time.