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# A study of topos semantics of formal theory based on intuitionistic logic

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The logic which was adjusted at the end of 1920's by Heyting is intuitionistic logic, descended from a constitutive way of thinking in the mathematical intuitionism that Brouwer advocated. The Curry-Howard isomorphism which makes correspondence of the typed  $\lambda$ -calculus of a kind of computation models to constructive proofs based on intuitionistic logic is known. This means that the analysis of the proof of a mathematical proposition leads the analysis of the property of the corresponding program, and also that a program can be extracted from a constructive proof. Therefore, it is important from a point of view of information science that a mathematical proposition has constructive proof. However, intuitionistic logic is weaker than the classic logic from definition and some propositions that were proved in classical logic are not proved in intuitionistic logic. Consequently, it has important meaning to clarify propositions that are proved in classical logic are not proved in intuitionistic logic.

Meanwhile, it is known that the sheaf that was devised to analyze a certain topological property in conventional mathematics is excellent with semantics of formal system based on intuitionistic logic. In addition, according to the viewpoint of the category theory, the thing which expanded the set making the root in current mathematics on topological space as a sheaf constitutes a topos. This topos is called spatial topos, and it is used in semantics of intuitionistic logic.

In this study, I use semantical technique and show that some mathematical propositions that are proved in classical logic are not proved in intuitionistic logic.

Generally we can show that a mathematical proposition is proved in the logic if we introduce a formal system of the logic and show the existence of the proof figure in the formal system. By contraries, a certain mathematical proposition tends to be shown it is not proved not using syntactical technique but semantical technique. Semantical technique is what we create meaning by structure to a formal system and use the property to be called soundness of the formal system, a mathematical proposition is valid if the one is proved in the formal system.

In this study, along the flow mentioned above, I introduce LJ, formal system of intuitionistic logic, and create meaning by structure by the sheaf over topological space as semantics of intuitionistic logic. Afterwards I show the existence of the proposition that are proved in classical logic are not proved in intuitionistic logic by constituting concrete structure by contraposition of the soundness of the formal system LJ, if a mathematical proposition is not right in a certain structure, it is not proved formal system LJ.

At first we think about the connected topological space  $\langle X, \mathcal{O} \rangle$  and think about a sheaf by the system of open sets  $\mathcal{O}$  over the topological space. In addition, we fix one open set  $V$  in the topological space other than  $X$  and  $\emptyset$  and define predicate  $P_V$  over the sheaf that maps all open sets to  $V$ . Then, by the structure over the topological space consisting of these, following fomulas  $P_V(U) \vee \neg P_V(U)$ ,  $\neg\neg P_V(U) \rightarrow P_V(U)$ ,  $(P_V(U_1) \rightarrow P_V(U_2)) \rightarrow (\neg P_V(U_1) \vee P_V(U_2))$  are not right in arbitrary  $U \in \mathcal{O}$ . Therefore, it is shown that following formulas  $\varphi \vee \neg\varphi$ ,  $\neg\neg\varphi \rightarrow \varphi$ ,  $(\varphi_1 \rightarrow \varphi_2) \rightarrow (\neg\varphi_1 \vee \varphi_2)$  are not proved in intuitionistic logic by this fact and soundness of LJ.

In this study, I was not able to mention it how to be interpreted formulas of intuitionistic high order predicate logic by topos. However, in intuitionistic first order predicate logic, I constituted structure over the topological space consisting of the sets of the sheaf where following formulas  $\varphi \vee \neg\varphi$ ,  $\neg\neg\varphi \rightarrow \varphi$ ,  $(\varphi_1 \rightarrow \varphi_2) \rightarrow (\neg\varphi_1 \vee \varphi_2)$  does not be right, and confirm these formulas are not proved in intuitionistic logic.