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Author(s)	Zankl, Harald; Hirokawa, Nao; Middeldorp, Aart
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KBO Orientability

Harald Zankl[‡] Nao Hirokawa[†] Aart Middeldorp[‡]

[†] Japan Advanced Institute of Science and Technology

[‡] University of Innsbruck

Term Rewriting

DEFINITION

- pair of terms $l \rightarrow r$ is rewrite rule if $l \notin \mathcal{V} \wedge \text{Var}(r) \subseteq \text{Var}(l)$
- **term rewrite system (TRS)** is set of rewrite rules
- (**rewrite relation**) $s \rightarrow_{\mathcal{R}} t$ if $\exists l \rightarrow r \in \mathcal{R}$, context C , substitution σ .
 $s = C[l\sigma] \wedge t = C[r\sigma]$

EXAMPLE

TRS \mathcal{R}

$$x + 0 \rightarrow x$$

$$x + s(y) \rightarrow s(x + y)$$

$$x \times 0 \rightarrow 0$$

$$x \times s(y) \rightarrow x \times y + x$$

rewriting

$$s(0) \times s(0) \rightarrow_{\mathcal{R}} s(0) \times 0 + s(0)$$

$$\rightarrow_{\mathcal{R}} 0 + s(0)$$

$$\rightarrow_{\mathcal{R}} s(0 + 0)$$

$$\rightarrow_{\mathcal{R}} s(0) \quad \text{terminated}$$

Termination

DEFINITION

TRS \mathcal{R} is **terminating** if there is no infinite rewrite sequence

$$t_1 \rightarrow_{\mathcal{R}} t_2 \rightarrow_{\mathcal{R}} \cdots$$

QUESTION

how to prove termination?

👉 **Knuth-Bendix order (KBO)**

- introduced by **Knuth and Bendix, 1970**
- **best studied** termination methods
- great success in **theorem provers** (Waldmeister, Vampire, ...)

Knuth-Bendix Orders

DEFINITION

- precedence $>$ is proper order on function symbols \mathcal{F}
- weight function (w, w_0) is pair in $\mathbb{R}_{\geq 0}^{\mathcal{F}} \times \mathbb{R}_{\geq 0}$
- weight of term t is

$$w(t) = \begin{cases} w_0 & \text{if } t \in \mathcal{V} \\ w(f) + w(t_1) + \cdots + w(t_n) & \text{if } t = f(t_1, \dots, t_n) \end{cases}$$

- weight function (w, w_0) is **admissible** for precedence $>$ if

$$w(f) > 0 \quad \text{or} \quad f \geq g$$

for all unary functions f and all functions g

DEFINITION

Knuth-Bendix order $>_{\text{kbo}}$ on terms $\mathcal{T}(\mathcal{F}, \mathcal{V})$:

$s >_{\text{kbo}} t$ if $|s|_x \geq |t|_x$ for all $x \in \mathcal{V}$ and either

- $w(s) > w(t)$, or
- $w(s) = w(t)$ and
 - $s = f^n(t)$ and $t \in \mathcal{V}$ for some unary f and $n \geq 1$; or
 - $s = f(s_1, \dots, s_{i-1}, s_i, \dots, s_n)$, $t = f(s_1, \dots, s_{i-1}, t_i, \dots, t_n)$, and $s_i >_{\text{kbo}} t_i$; or
 - $s = f(s_1, \dots, s_n)$, $t = g(t_1, \dots, t_m)$, and $f > g$

DEFINITION

let $X \subseteq \mathbb{R}_{\geq 0}$. TRS \mathcal{R} is KBO_X terminating if

- \exists precedence $>$
- \exists admissible weight function $(w, w_0) \in X^{\mathcal{F}} \times X$

such that $l >_{\text{kbo}} r$ for all $l \rightarrow r \in \mathcal{R}$

THEOREM Knuth and Bendix, 1970

TRS is terminating if it is $\text{KBO}_{\mathbb{N}}$ terminating

THEOREM Knuth and Bendix, 1970

TRS is terminating if it is $KBO_{\mathbb{N}}$ terminating

THEOREM Dershowitz, 1979

TRS is terminating if it is $KBO_{\mathbb{R}_{\geq 0}}$ terminating

THEOREM Dick, Kalmus, and Martin, 1990

*$KBO_{\mathbb{R}_{\geq 0}}$ termination is **decidable***

THEOREM Korovin and Voronkov, 2001, 2003

- *TRS is $KBO_{\mathbb{N}}$ terminating \iff it is $KBO_{\mathbb{R}_{\geq 0}}$ terminating*
- *$KBO_{\mathbb{R}_{\geq 0}}$ termination is decidable within **polynomial time***

THEOREM Zankl and Middeldorp, 2007

*$KBO_{\{0,1,\dots,B\}}$ termination ($B \in \mathbb{N}$) can reduce to **SAT** and **PBC***

Main Result

THEOREM

\mathcal{R} is $KBO_{\mathbb{N}}$ terminating $\iff \mathcal{R}$ is $KBO_{\{0,1,\dots,B\}}$ terminating
where, $B = n^{2^{n+1}}$

COROLLARY

Zankl and Middeldorp's SAT and PBC encodings are complete for this B

Summary

let \mathcal{R} be TRS of size $n = \sum_{l \rightarrow r \in \mathcal{R}} (|l| + |r|)$ and $B = n^{2^{n+1}}$

\mathcal{R} is $\text{KBO}_{\mathbb{R}_{\geq 0}}$ terminating

$\iff \mathcal{R}$ is $\text{KBO}_{\mathbb{N}}$ terminating

Korovin and Voronkov

$\iff \mathcal{R}$ is $\text{KBO}_{\{0,1,\dots,B\}}$ terminating

this talk

- theoretical interest of decidability issue is more or less closed

FUTURE WORK

find optimal B