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Parameterized Points-to Analysis for Java based on Weighted Pushdown Model Checking

Li Xin, Ogawa Mizuhito

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November 27, 2006

Points-to Analysis for Java

- Purpose
  - Approximate the set of heap objects pointed to by reference variables at runtime
- Why points-to analysis?
  - Essential to many other program analyses and compiler optimizations
  - Headachy issue in program verifications
- Precision and scalability is dominated by

Context-sensitivity calling contexts are distinguished
Flow-sensitivity execution orders are concerned
Field-sensitivity how instance fields are abstracted

A Running Example

```
1:    A x = new A(); ...o1
2:    B y = new B(); ...o2
3:    y.f = new Object(); ...o3
4:    x = y;
      if(...){
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      }

class A
m(B a):  { return a; }
class B inherits class A
m(B b):  { return b.f; }
```

Figure: An Example of Java Code Fragment

- Declared type strategy
- Virtual method invocation (dynamic binding) at line 5 and 7
- Call-by-value
- Abstract heap objects are associated with codes in blue

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class B *inherits class* A
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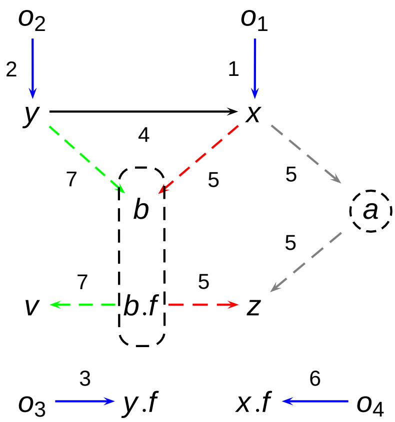

Figure: (a) Example Code Fragment (b) Pointer Assignment Graph of (a)

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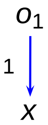

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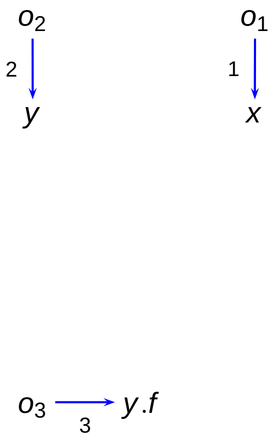

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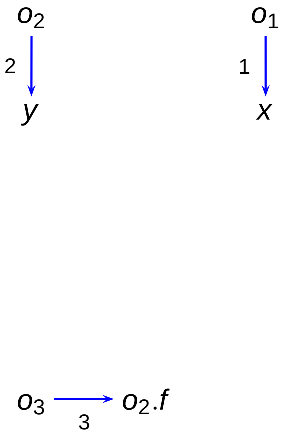

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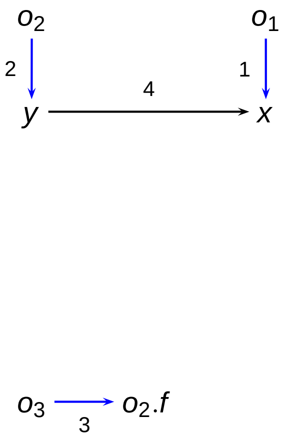

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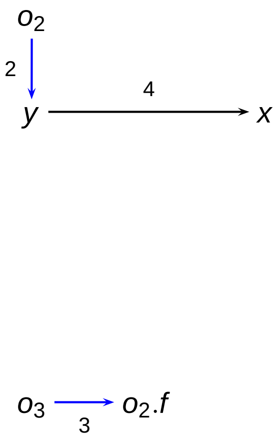

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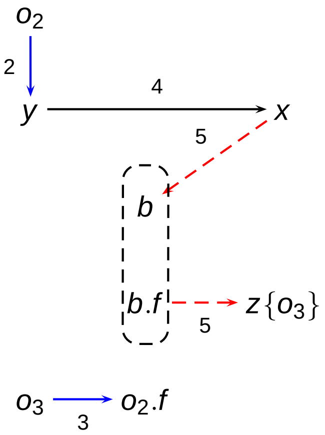

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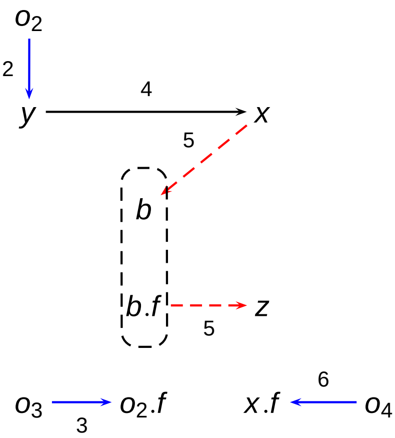

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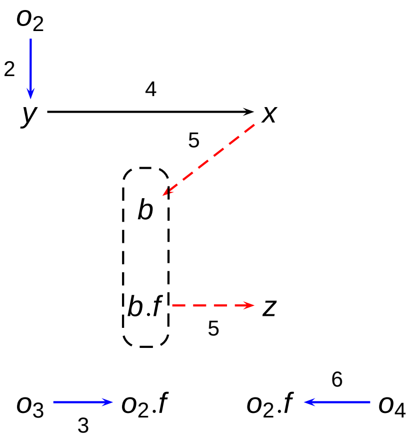

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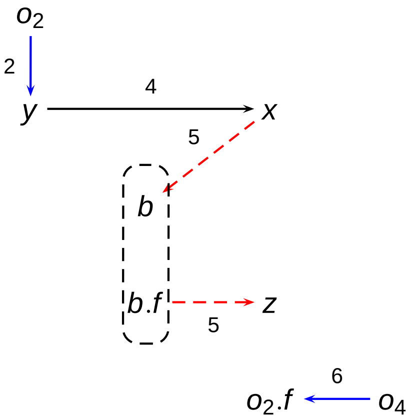

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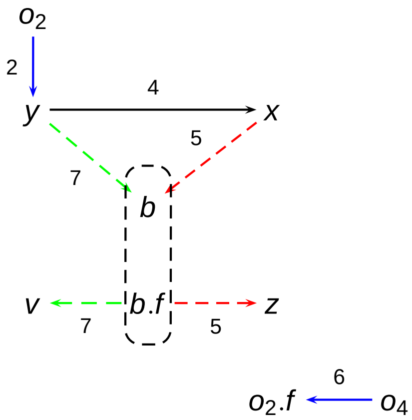

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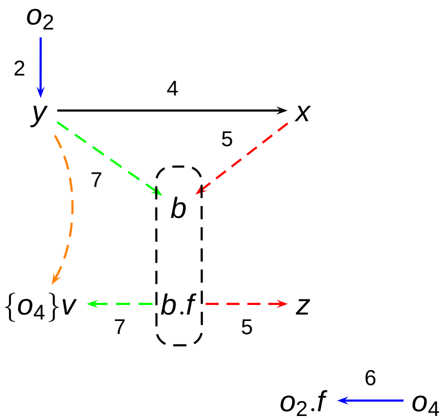


What does the example tell?

- Points-to analysis and call graph construction are mutually dependent
- Call graph construction
  - On-the-fly: constructed during points-to analysis
  - Ahead-of-time: a pre-computed approximated call graph is explored for points-to analysis
- Two occasions need points-to information:
  - Call graph construction
  - Instance field abstraction

Definition

Let $\mathcal{V}$ and $\mathcal{O}$ be a set of abstract reference variables and a set of abstract heap objects respectively. A transitive and reflexive points-to relation is defined as $\mapsto: \mathcal{V} \times \mathcal{H}$, where $\mathcal{H} = \mathcal{V} \cup \mathcal{O}$. Its inverse is defined as a flows-to relation $\rightsquigarrow$.

Definition

A pointer assignment graph is defined as $G_a = (N_a, E_a)$, where $N_a = \mathcal{V} \cup \mathcal{O}$ is a set of nodes, and $E_a = \rightsquigarrow$ is a set of edges.

Definition

Let $\mathcal{F}$ be a set of fields and $\mathcal{L}$ be a set of local variables. A field sensitive analysis abstracts an instance field $l.f(l \in \mathcal{L}, f \in \mathcal{F})$ as pairs of $\{(o, f) \mid f \mapsto o\}$.

Work Summary

- Program Analysis = Abstract Interpretation + Model Checking
- Context-sensitive points-to analysis algorithms based on weighted pushdown model checking
- Parameterized flow-sensitivity so that the abstraction design is easily tuned
- Variations of points-to analysis algorithms based on the following dimensions:
  - On-the-fly vs. Ahead-of-time call graph construction
  - Lightweight semiring operations vs. Smaller pushdown transitions in the abstraction design
- Evaluation within the SOOT framework

Pushdown Model Checking

- Model: Pushdown System (PDS)
- A PDS + (e.g. Simple) Valuation
  $\cong$ A Pushdown Automaton
  $\cong$ Context-free Language
- The intersection of context-free language and regular language is closed (context-free)
- The automata-theoretic approach works

$$\mathcal{M} \models \mathcal{S} \Leftrightarrow L(\mathcal{M}) \cap L(\mathcal{S})^{\mathcal{C}} = \emptyset$$

- Efficient algorithms are developed due to the fact that:
  "Regular sets of configurations are closed under forward and backward reachability"

Weighted Pushdown Model Checking

- Associate a weight from a bounded idempotent semiring to each pushdown transition rule
- Solve the Generalized Pushdown Reachability (GPR) problem: "Compute weights over paths in a pushdown graph leading from a pushdown configuration to a regular set of pushdown configurations"

Definition

A bounded idempotent semiring S is a semiring $(D, \oplus, \otimes, 0, 1)$, s.t.

- $\oplus$ is idempotent, i.e. $a \oplus a = a$.
- A partial order $\sqsubseteq$ is defined: $\forall a, b \in D, a \sqsubseteq b$ iff $a \oplus b = a$.

That is, no infinite descending chain on weight space is required.

Application of Pushdown Systems to Program Analyses

- Suitable for modeling interprocedural program analyses
  - Calls and returns are correctly paired (context-sensitivity)
  - No limitation on recursion steps (vs. K-CFA)
- Pushdown model checking
  - Model program's data domain
  - Demand finite domain abstraction (by automata-theoretic approach)
- Weighted pushdown model checking
  - Model program's flow function space
  - Demand infinite descending chains on the weight space, but infinite domain abstraction is possible
  - Regular pushdown configurations as an abstraction of calling contexts (context-sensitivity)

Intention Behind the Semiring Design

- Weight space $\Rightarrow$ Flow function space
- A weight intends a function to represent how a property is carried at each step of program execution.
- $1 \Rightarrow$ Properties keep unchanged by this transition step
- $0 \Rightarrow$ The program execution is interrupted by some error
- $f \otimes g \Rightarrow$ Function composition of $g \circ f$
- $f \oplus g \Rightarrow$ Conservative approximation over two control flows at their meet
- The optional commutativity of $\otimes$ facilitates modeling a flow-sensitive analysis

Abstraction of Heap Memory

Definition

Let $\mathscr{O}$ be a set of run-time objects allocated in the heap memory. Functions $\eta_{\mathcal{T}} : \mathscr{O} \rightarrow \mathcal{T}$ and $\eta_{\iota} : \mathscr{O} \rightarrow \mathcal{L}$ are defined respectively, where $\mathcal{T}$ is a set of types (class names) of heap objects, and $\mathcal{L}$ is a set of memory allocation sites in the program.

Definition

Let $\mathcal{O} \subseteq \mathcal{T} \times \mathcal{L} \cup \{\diamond\}$ be a set of abstract heap objects, where $\diamond$ represents null reference. An abstraction on $\mathscr{O}$ is defined as $\tilde{\alpha} : \mathscr{O} \rightarrow \mathcal{O}$, s.t. $\forall o \in \mathscr{O},\ \tilde{\alpha}(o) = (\tau, \iota)$, where $\tau = \eta_{\tau}(o) \in \mathcal{T}$, $\iota = \eta_{\iota}(o) \in \mathcal{L}$.

Remarks:

- $\forall (\tau_i, \iota_i), (\tau_j, \iota_j) \in \mathcal{O},\ \iota_i = \iota_j \Rightarrow \tau_i = \tau_j$
- $\forall o_i, o_j \in \mathscr{O}$, $\tilde{\alpha}(o_i) = \tilde{\alpha}(o_j)$ iff the allocation sites for them are the same.
- An array is approximated with a single element with its base type.

An Algorithm with Lightweight Semiring Operations

Approaches:

- Reachability analysis on the product of G_a and G_f.
- For efficiency, a variation of “exploded supergraph” is explored

Definition

A weighted pointer assignment graph is defined as $G_l = (N_l, E_l, L_l)$ from G_a, where $N_l = \{\Lambda\} \cup \mathcal{V}$ is a set of nodes, $E_l \subseteq N_l \times L_l \times N_l$ is a set of edges, and $L_l = \{\lambda x.x\} \cup \{\lambda x.o \mid o \in \mathcal{O}\}$ is a set of labels, such that

- $(v_1, \lambda x.x, v_2) \in E_l$ if $(v_1, v_2) \in E_a,\ v_1, v_2 \in \mathcal{V}$
- $(\Lambda, \lambda x.o, v) \in E_l$ if $(o, v) \in E_a,\ o \in \mathcal{O},\ v \in \mathcal{V}$

Remarks:

- Λ: an environment that allocates new heap objects
- Heap objects are labeled on the edges

The Underlined Model for Model Checking

Definition

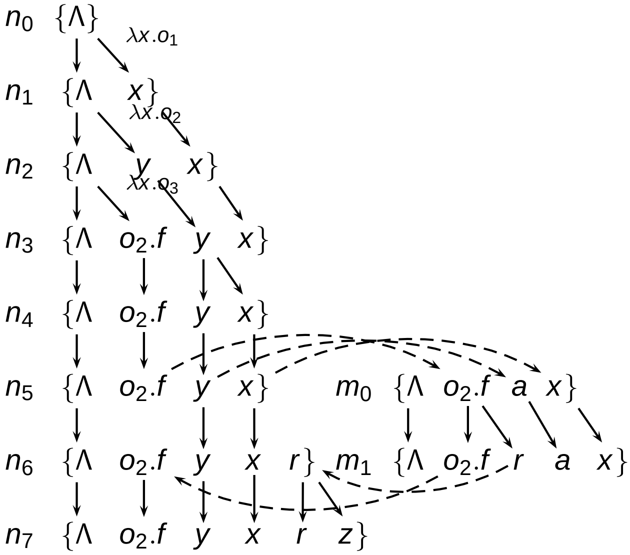
A weighted flows-to graph $G_p = (N_p, E_p, L_p)$ is the product of G_l and G_f, where $N_p = N_l \times N_f$ is a set of nodes, $E_p \subseteq N_p \times L_p \times N_p$ is a set of edges, and $L_p = L_l$ is a set of labels.

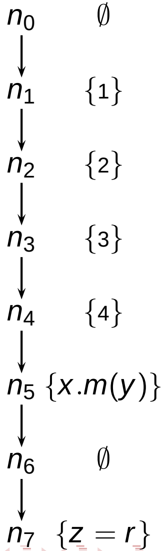
Algorithm

Let $\mathcal{A}[\![\cdot]\!] : S \to \mathcal{P}(\rightsquigarrow)$, *and* $N_l = \{\Lambda\} \cup V_g \cup V_l$ ($V_g \subseteq \mathcal{V}$ *represents global variables and* $V_l \subseteq \mathcal{V}$ *represents local variables), s.t.* $\forall e_f = (n_1, n_2) \in E_f$

$e_f \in E_i$	$\{((v,n_1),\lambda x.x,(v,n_2)) \mid v \in V\} \cup$
	$\{((v_1,n_1),\lambda x.x,(v_2,n_2)) \mid (v_1,\lambda x.x,v_2) \in E_l, (v_1,v_2) \in F, v_1 \in \mathcal{V}\} \cup$
	$\{((\Lambda,n_1),\lambda x.o,(v,n_2)) \mid (\Lambda,\lambda x.o,v) \in E_l, (c,v) \in F, o \in \mathcal{O}\} \subseteq E_p$
	where $F = \mathcal{A}[\![StmtOf(n_2)]\!], V = N_l - \{v \mid (h,v) \in F\}$
$e_f \in E_t$	$\{((v,n_1),\lambda x.x,(v,n_2)) \mid v \in V_l\} \subseteq E_p$
$e_f \in E_c$	$\{((v,n_1),\lambda x.x,(v,n_2)) \mid v \in V_g \cup \{\Lambda\}\} \cup$
	$\{((h,n_1),\lambda x.x,(v,n_2)) \mid (h,v) \in F\} \subseteq E_p$
	where $F = \mathcal{A}[\![StmtOf(n_1)]\!]$
$e_f \in E_r$	$\{((v,n_1),\lambda x.x,(v,n_2)) \mid v \in V_g \cup \{\Lambda\}\} \subseteq E_p$

Part of G_p for the Running Example





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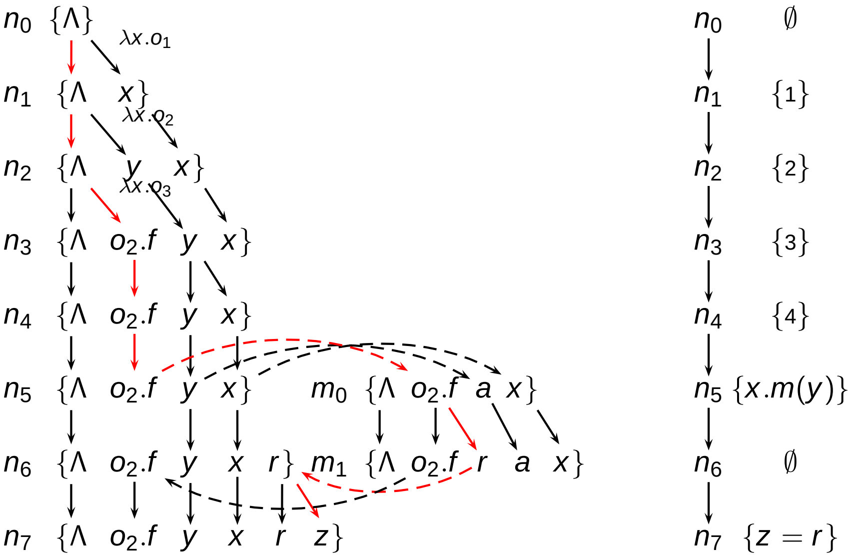

A Semiring Design

Let $\mathcal{S} = \mathcal{P}(\mathcal{O})$, $D_1 = \{\lambda x.s \mid s \in \mathcal{S}\}$ and $D_2 = \{\lambda x.x \cup s \mid s \in \mathcal{S}\}$

Definition

A bounded idempotent semiring $S = (D, \oplus, \otimes, 0, 1)$ is defined as

- The weight space $D = D_1 \cup D_2$
- **1** is defined as $\lambda x.x$ and **0** is defined as $\lambda x.\emptyset$
- The $\otimes$ operator is defined as

$$\forall d_i,\ d_j \in D \setminus \{\mathbf{0}, \mathbf{1}\},\ d_i \otimes d_j = d_j$$

- The $\oplus$ operator equals set union $\cup$, defined as

$$\forall d_i = \lambda x.s_i,\ d_j = \lambda x.s_j \in \tilde{D},\ d_i \oplus d_j = d_j \oplus d_i = \lambda x.s_i \cup s_j$$

$$\forall d_i = \lambda x.s_i \in \tilde{D},\ d_j = \lambda x.x \cup s_j \in \bar{D},\ d_i \oplus d_j = d_j \oplus d_i = \lambda x.x \cup s_i \cup s_j$$

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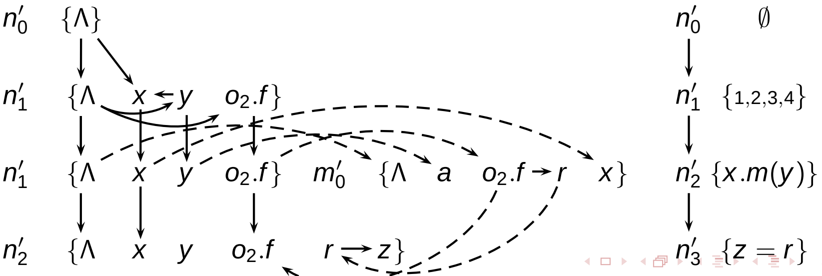
Distributivity of $\otimes$ over $\oplus$ is easily checked.

Parameterized Flow-sensitivity

- Problems: G_p will explode for large-scale programs
- Solutions: G_f is firstly shrunk by grouping nodes into blocks
  - One node possibly associated with a set of program statements
  - Each node has an unique entry after shrinking
- Parameterized flow-sensitivity by shrinking
  - Shrinking is NOT arbitrary to keep soundness (loops, branches)
  - An extreme shrinking collapses each method into a single node (flow-insensitive)



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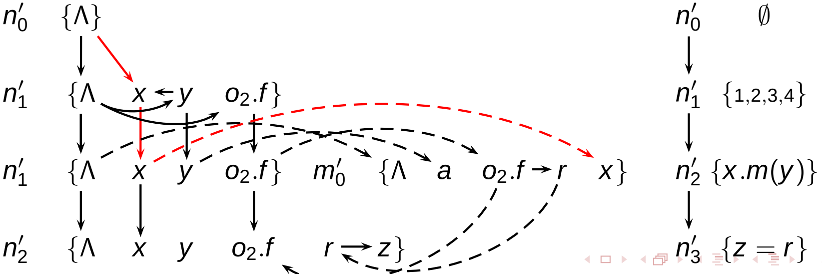

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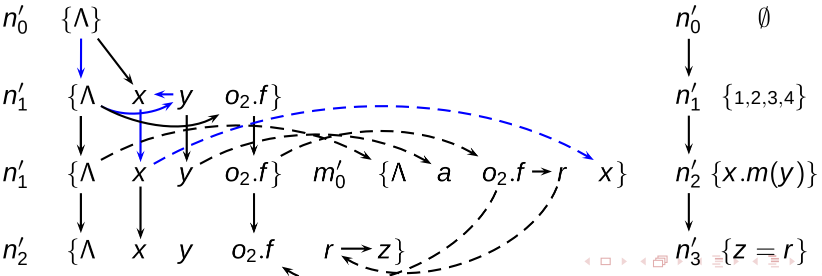
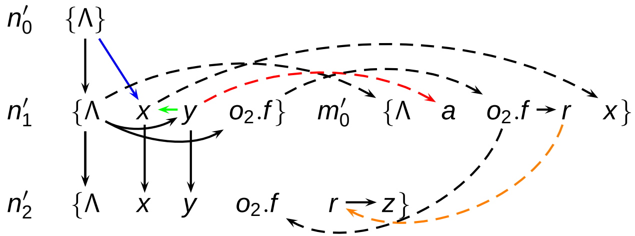


Encoding to Weighted PDS

Given a weighted flows-to graph $G_p = (N_p, E_p, L_p)$, with $N_p = \{\Lambda\} \cup \mathcal{V} \times N_f$

- $\{\Lambda\} \cup \mathcal{V} \Rightarrow$ control states
- $N_f \Rightarrow$ stack alphabets
- $E_p \Rightarrow$ pushdown transition rules



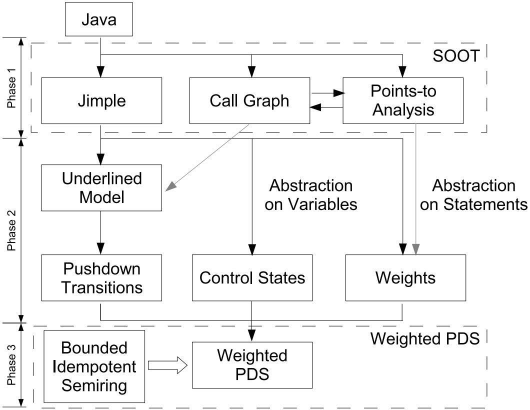
$\langle \Lambda, n'_0 \rangle \hookrightarrow \langle x, n'_1 \rangle$

$\langle y, n'_1 \rangle \hookrightarrow \langle x, n'_1 \rangle$

$\langle y, n'_1 \rangle \hookrightarrow \langle a, m'_0 n'_2 \rangle$

$\langle r, n'_1 \rangle \hookrightarrow \langle r, \varepsilon \rangle$

Evaluation within the SOOT Framework (On-the-fly + Lightweight Semiring Operation)



- Obstacle: restriction from the interaction of soot and weighted PDS library
- Bottleneck: weighted PDS constructed from scratch for each model checking request
- An incremental model construction is promising when possible

software	LOC	MCR	time (s)	TMCR(s)	TMC(s)
jetty	≈3000	1173	632.23	619.57 (97.99%)	468 (75.5%)

Points-to Analysis with Ahead-of-time Call Graph Construction

- Target: reduce frequent model checking demands
- Approaches
  - A pre-computed approximated call graph is explored
  - Invalid pathes are "removed" during model checking
  - Extra relations to model instance field accesses
- A semiring design with smaller pushdown transitions
  - $\hat{\mathcal{S}} \subseteq \mathcal{P}(\mathcal{V} \times \mathcal{H}) = \mathcal{P}(\mapsto)$, s.t. $\forall s \in \hat{\mathcal{S}}$

$$\forall (v_1, h_1), (v_2, h_2) \in s, h_1 = h_2 \text{ if } v_1 = v_2$$

  - "$x = y; y = z$" $\Rightarrow \{x \mapsto y, y \mapsto z\}$ instead of $\{x \mapsto z, y \mapsto z\}$
    $v_1 \mapsto v_2 \Longrightarrow v_1 \mapsto v_2'$ (flow-sensitive)
  - e.g. $\{x \mapsto y, y \mapsto o, z \mapsto x\} \Longrightarrow \{x \mapsto y', y \mapsto o, z \mapsto x'\}$ (i.e. A transitive closure on $s \in \hat{\mathcal{S}}$ does not make sense)

Definition

A bounded idempotent semiring $S = (D, \oplus, \otimes, 0, 1)$ is defined as

- The weight space $D = \mathcal{P}(\mathscr{D})$, where $\mathscr{D} = \hat{\mathcal{S}} \cup \{\textsc{id}\} \setminus \emptyset$
- $0 = \emptyset$ and $1 = \{\textsc{id}\}$
- $\forall w_1, w_2 \in D$, $w_1 \otimes w_2 = \{d_1 \odot d_2 \mid d_1 \in w_1, d_2 \in w_2\}$, where

$$d_1 \odot d_2 = \begin{cases} d_1 \text{ (resp. } d_2) & \text{if } d_2 = \textsc{id} \text{ (resp. } d_1 = \textsc{id}) \\ f_0(d_1, d_2) \cup f_1(d_1, d_2) & \text{o.w.} \end{cases}$$

$$f_0(d_1, d_2) = d_1 \setminus \{\ (v, h_1) \in d_1 \mid \exists h_2\ s.t.\ (v, h_2) \in d_2\ \}$$

$$f_1(d_1, d_2) = \{\ (v_2, h_2') \mid \forall (v_2, h_2) \in d_2, h_2' = \begin{cases} h_1 & \text{if } \exists h_1\ s.t.\ (v_1, h_1) \in d_1, v_1 = h_2 \\ h_2 & \text{o.w.} \end{cases}$$

- $\forall w_1, w_2 \in D$, $w_1 \oplus w_2 = w_1 \cup w_2$

Remarks on $w_1 \otimes w_2$: ① f_0: Relations in w_1 are changed by subsequent operations in w_2 (flow-sensitive); ② f_1: The second components of relations in w_2 are substituted w.r.t w_1.

Path Elimination

- $\mathscr{C} \subseteq \mathcal{P}(\mathcal{V} \times \mathcal{T})$: represent expected types of method receivers
- **type** : $\mathcal{O} \rightarrow \mathcal{T}$: get types of abstract heap objects
  loc : $\mathcal{O} \rightarrow \mathcal{L}$: get allocation sites of abstract heap objects
- $\propto$: $\mathscr{C} \times \mathscr{D} \rightarrow \{\textbf{TRUE}, \textbf{FALSE}\}$ is introduced as an judgement relation. That is, $\forall d \in \mathscr{D}, c \in \mathscr{C}$, $c \propto d$ iff $\exists (v, t) \in c$, and $(v, o) \in d$, such that $t' \ltimes t$, where $t' = \textbf{type}(o)$.
- $\ltimes : \mathcal{T} \times \mathcal{T} \rightarrow \{\textbf{TRUE}, \textbf{FALSE}\}$ defines a relation among classes. $\forall t, t' \in \mathcal{T}$, $t' \ltimes t$ iff

  *r*1. $t' \neq t$
  *r*2. *a*) t' does not inherit from t; or
  b) t' inherits from t, but t' redefines the method to be invoked.

- $\rtimes$ is defined as the reverse of $\ltimes$. That is,

$$\forall t, t' \in \mathcal{T},\ t' \rtimes t \text{ iff } t' \ltimes t = \textbf{FALSE}$$

A Semiring Design with Path Elimination

Definition

The previous semiring S is extended to be $S_e = (D_e, \oplus_e, \otimes_e, 0_e, 1_e)$, where

- $D_e = \mathcal{P}(\mathbb{D})$, where $\mathbb{D} = \{(d, c) \mid d \in \mathscr{D},\ c \in \mathscr{C}\}$
- $1_e = \{(\text{ID}, \emptyset)\}$ and $0_e = \emptyset$
- $\forall w_1, w_2 \in D_e$, $w_1 \otimes_e w_2 = \{\mathbb{d}_1 \odot_e \mathbb{d}_2 \mid \mathbb{d}_1 \in w_1, \mathbb{d}_2 \in w_2\}$, such that $\forall \mathbb{d}_1 = (d_1, c_1), \mathbb{d}_2 = (d_2, c_2) \in \mathbb{D}$,

$$\mathbb{d}_1 \odot_e \mathbb{d}_2 = \begin{cases} 0_e & \text{if } c_2 \propto d_1 \\ (d_1 \odot d_2, c_1 \uplus c_2) & \text{o.w.} \end{cases}$$

  where $c_1 \uplus c_2 = c_1 \cup f_8(c_2 \setminus c, d_1)$, and $c = f_7(c_2, d_1)$. $\forall c \in \mathscr{C}, d \in \mathscr{D}$,

$$f_7(c, d) = \{(v, t) \in c \mid \exists o \in \mathcal{C},\ \text{s.t. } (v, o) \in d,\ t' = type(o),\ t' \rtimes t\}$$

$$f_8(c, d) = \{(\tilde{v}, t) \mid \forall (v, t) \in c, \tilde{v} = \begin{cases} v' & \text{if } \exists (v, v') \in d, v' \in \mathcal{V} \\ v & \text{o.w.} \end{cases}\}$$

- $\forall w_1, w_2 \in D_e$, $w_1 \oplus_e w_2 = w_1 \cup w_2$

Remarks on Path Elimination

- $(v,t) \in c \Longrightarrow (v',t)$, where $c \in \mathscr{C}$
- $c_1 \uplus c_2$
  - f_7: remove constraints of c_2 satisfied by d_1
  - f_8: substitute variables of relations in c_2 w.r.t d_1
- Examples
  - $\{(x,o),\emptyset\} \odot_e \{\text{ID},(x,A)\} = 0_e$ if $(x,A) \propto (x,o)$
  - $\{(x,o)(y,x),\emptyset\} \odot_e \{\text{ID},(x,A)\} = \{(x,o)(y,x),\emptyset\}$ if $\textbf{type}(o) \rtimes A$
  - $\{(y,x),\emptyset\} \odot_e \{\text{ID},(y,A)\} = \{(y,x),(x,A)\}$
- Associativity of $\otimes_e(\odot_e)$ is not obvious but proved

Model Field Accesses

Definition

Let $\mathcal{L}$ be a set of local variables of reference type, and $\mathcal{F}$ be a set of field names. let $\hat{\mathcal{H}} = \mathcal{L} \cup \mathcal{O}$. A field read relation is defined as $\mathbb{R} : \hat{\mathcal{H}} \times \mathcal{F} \times \hat{\mathcal{H}}$. A field write relation is defined as $\mathbb{W} : \hat{\mathcal{H}} \times \mathcal{F} \times \hat{\mathcal{H}}$. The points-to relation is redefined as $\mathbb{P} : \mathcal{L} \times \hat{\mathcal{H}}$.

Remarks:

- $(h_1, f, h_2) \in \mathbb{R}$ models the field read access "$h_2 = h_1.f$" $(h_2 \rightarrowtail h_1.f)$
- $(h_1, f, h_2) \in \mathbb{W}$ models the field write access "$h_1.f = h_2$" $(h_1.f \rightarrowtail h_2)$
- $(h_1, f, h_2) \in \mathbb{R} \Longrightarrow (h_1', f, h_2)$
- $(h_1, f, h_2) \in \mathbb{W} \Longrightarrow (h_1', f, h_2')$
- A flow-sensitive analysis concerning field accesses seems intractable in this setting
  - $\{h_2 \rightarrowtail h_1.f\} \otimes \{h_3.f \rightarrowtail h_2\} \Rightarrow \{h_3.f \text{``} \rightarrowtail \text{''} h_1.f\}$
  - $\{h_2 \rightarrowtail h_1.f\} \otimes \{h_3 \rightarrowtail h_2.f\}$?

Conclusions

- Weighted pushdown model checking enables a fast design of interprocedural context-sensitive program analyses
- Pushdown systems provides us with handy context-sensitivity for program analyses
- Promising for developing a scalable analysis when the implementation allows
- Some future work
  - Evaluation on the ahead-of-time construction
  - Efficient data structures (like BDD) or other decision procedures could be explored

Thanks!

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