

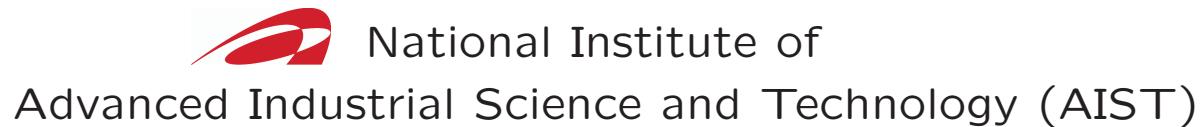
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# ACTAS

Automated Verification Based on Equational Tree Automata

Hitoshi Ohsaki

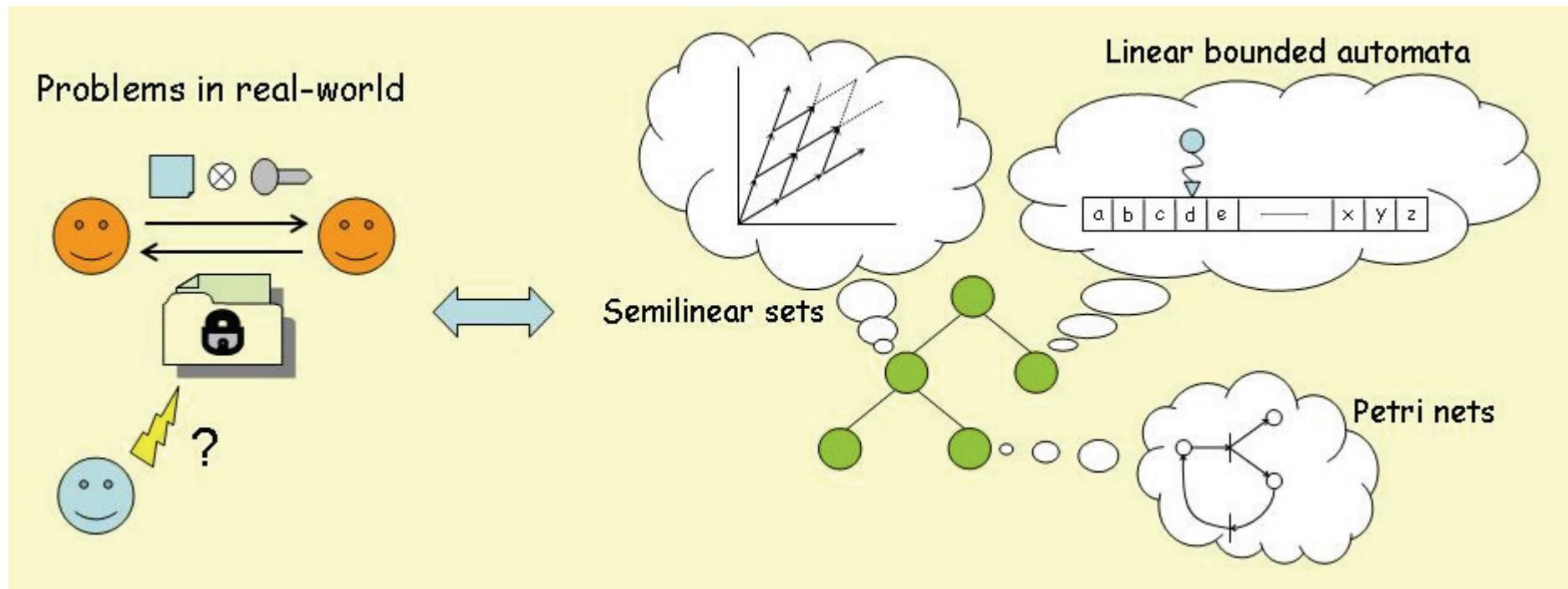


JAIST – AIST Workshop, Kanazawa

September 2005

## Automated reasoning

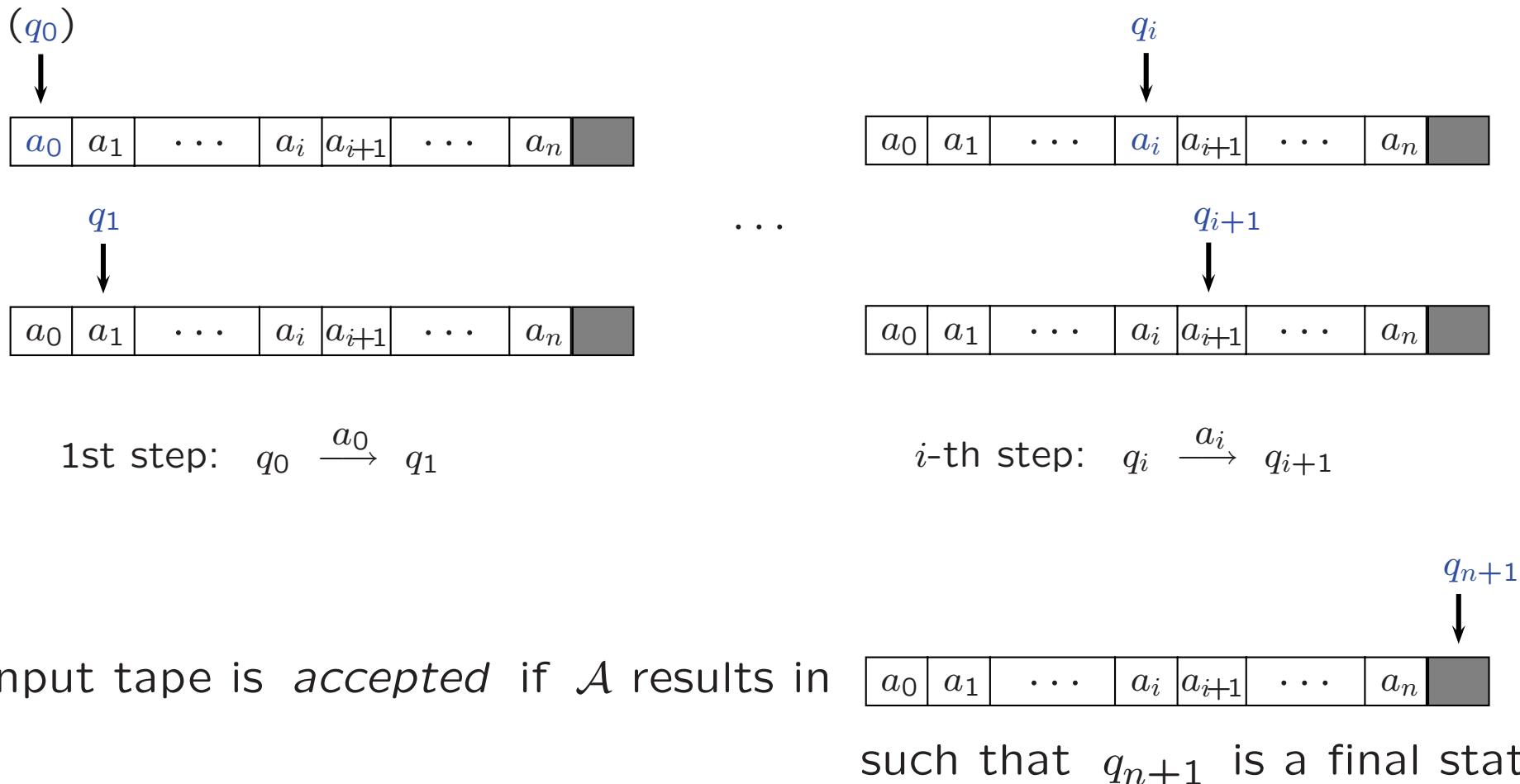
- problems with social needs
- decidable fragments of problems (in theory)
- heterogeneous data structures in **simple** framework



# I. Equational tree automata

## Automata for strings

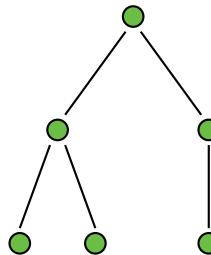
Given  $a_0 | a_1 | \dots | a_i | a_{i+1} | \dots | a_n$  and finite automaton  $\mathcal{A}$



## Tree automata vs. automata

input

tree automata



transition rules

$$f(\alpha_1, \dots, \alpha_n) \rightarrow \beta$$

$$\begin{aligned} f(\alpha_1, \dots, \alpha_n) &\rightarrow f(\beta_1, \dots, \beta_n) \\ \alpha &\rightarrow \beta \end{aligned}$$

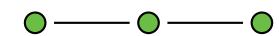
closure properties

$$\cup \quad \cap \quad ( )^c$$

decidability

$$\in \quad \subseteq \quad =\emptyset?$$

finite automata



$$\alpha \xrightarrow{a} \beta$$

$$\alpha \rightarrow \beta$$

$$\cup \quad \cap \quad ( )^c$$

$$\in \quad \subseteq \quad =\emptyset?$$

## Definition

$\mathcal{A}$ : tree automaton  $(\mathcal{F}, \mathcal{Q}, \mathcal{Q}_{fin}, \Delta)$

$\mathcal{F}$  signature, that is set of function symbols with fixed arity

$\mathcal{Q}$  set of state symbols (special constant symbols) such that

$$\mathcal{F} \cap \mathcal{Q} = \emptyset$$

$\mathcal{Q}_{fin}$  set of final state symbols such that  $\mathcal{Q}_{fin} \subseteq \mathcal{Q}$

$\Delta$  set of transition rules with the following forms:

$$f(p_1, \dots, p_n) \rightarrow q \quad (\text{Type 1})$$

$$f(p_1, \dots, p_n) \rightarrow f(q_1, \dots, q_n) \quad (\text{Type 2})$$

$$p \rightarrow q \quad (\text{Type 3})$$

## Transition move

- $\rightarrow_{\mathcal{A}}$  : move relation of tree automaton is defined below

$$s \rightarrow_{\mathcal{A}} t \text{ if } s = C[l] \text{ and } t = C[r]$$

for some  $l \rightarrow r$  in  $\Delta$  and context  $C$

E.g. Consider  $\mathcal{A}$  with transition rules  $\Delta$

$$a \rightarrow q_1 \quad b \rightarrow q_2 \quad f(q_1, q_2) \rightarrow q_3$$

then *tree language* accepted by  $\mathcal{A}$  is  $\{ f(a, b) \}$

- $\mathcal{L}(\mathcal{A})$  : set of trees  $t$  reaching by  $\mathcal{A}$  some final state, i.e.

$$t \rightarrow_{\mathcal{A}} t_1 \rightarrow_{\mathcal{A}} \cdots \rightarrow_{\mathcal{A}} t_n (\equiv q) \text{ for some } q \text{ in } Q_{fin}$$

- regular tree languages

## Basic properties (tree automata)

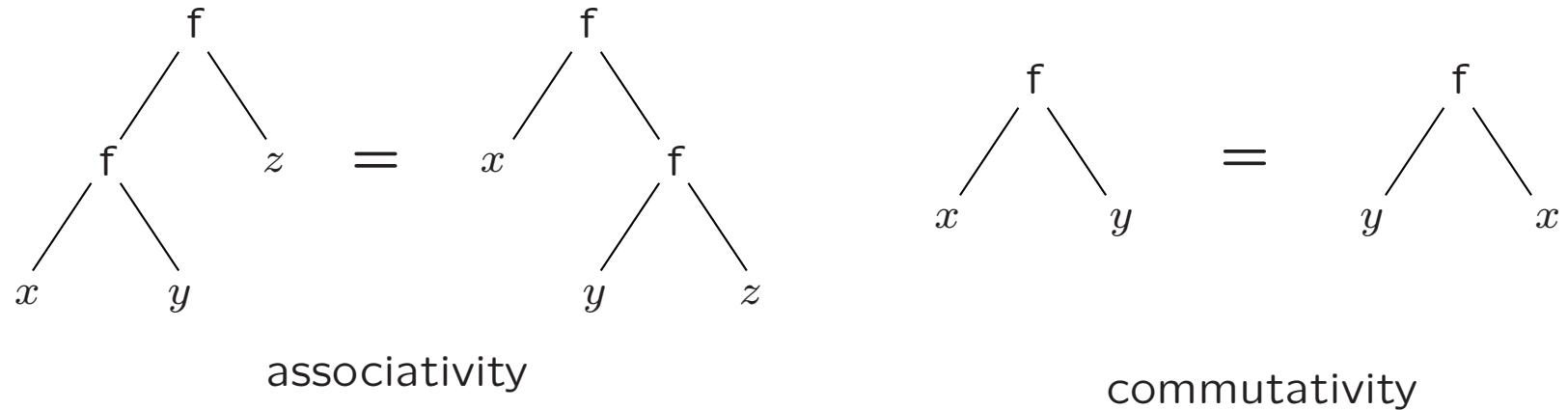
- Union
- Intersection
- Complementation:
  - Deterministic and complete tree automata
- Downsizing technique
  - Epsilon-free tree automata
- Emptiness problem:
  - Pumping lemma

## Associativity and commutativity axioms in tree automata

Given a signature  $\mathcal{F}$  with  $\mathcal{F}_{AC}$  some of the binary function symbols

AC-tree automaton  $\mathcal{A}/AC$ :

a pair of  $\mathcal{A}$  and the following **AC-axioms** for all symbols in  $\mathcal{F}_{AC}$



Likewise

**A**-tree automaton: tree automaton with associativity axioms of  $\mathcal{F}_A$

**C**-tree automaton: tree automaton with commutativity axioms of  $\mathcal{F}_C$

## Transition move with associativity and commutativity

- $\rightarrow_{\mathcal{A}/\text{AC}}$  : move relation for **AC**-tree automaton defined as below

$$s \rightarrow_{\mathcal{A}/\text{AC}} t \quad \text{if } s =_{\text{AC}} C[l] \text{ and } t =_{\text{AC}} C[r]$$

for some  $l \rightarrow r$  in  $\Delta$

E.g. Consider  $\mathcal{A}/\text{AC}$  with  $\mathcal{F}_{\text{AC}} = \{f\}$  and transition rules  $\Delta$

$$a \rightarrow q_1 \quad b \rightarrow q_2 \quad f(q_1, q_2) \rightarrow q_3$$

$$\text{then } f(b, a) \rightarrow_{\mathcal{A}/\text{AC}} f(q_2, a) \rightarrow_{\mathcal{A}/\text{AC}} f(q_2, q_1) \rightarrow_{\mathcal{A}/\text{AC}} q_3$$

- $\mathcal{L}(\mathcal{A}/\text{AC})$  : set of trees  $t$  reaching by  $\mathcal{A}/\text{AC}$  some final state, i.e.

$$t \rightarrow_{\mathcal{A}/\text{AC}} t_1 \rightarrow_{\mathcal{A}/\text{AC}} \cdots \rightarrow_{\mathcal{A}/\text{AC}} t_n (= q) \text{ for some } q \text{ in } \mathcal{Q}_{fin}$$

- AC-regular tree languages

## Basic properties (AC-tree automata)

- Union
- Intersection
- Complementation:
  - Translation by Parikh theorem
- regular AC-tree automata vs. non-regular AC-tree automata  
([6], Ohsaki *et al.* LPAR 2005)
- Emptiness problem:
  - Commutation lemma ([8], Ohsaki CSL 2001)

## Example

Consider regular AC-tree automaton with the transition rules

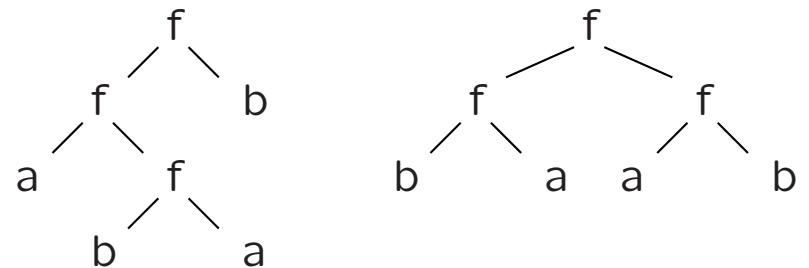
$$a \rightarrow q_a \quad b \rightarrow q_b \quad f(q_a, q_b) \rightarrow q \quad f(q, q) \rightarrow q$$

where  $\mathcal{F}_{AC} = \{f\}$  and  $q$  is final state

Then the accepted tree language  $L$  satisfies

$$|t|_a = |t|_b$$

for all  $t$  in  $L$



## Observations

- The above language  $L$  is **not** accepted by any tree automaton if  $\mathcal{F}_{AC} = \emptyset$
- In general, tree languages  $L$  represented by **linear** (in)equational constraints are accepted by **regular** AC-tree automata: e.g.

$$|t|_a > |t|_b$$

$$2|t|_a = |t|_b$$

$$|t|_a + 1 = |t|_b$$

## Closure under Boolean operations

[Ohsaki CSL'01, Ohsaki & Takai RTA'02  
 Ohsaki *et al.* RTA'03, Ohsaki *et al.* LPAR'05]

	regular TA	regular AC-TA	AC-TA
closed under $\cup$	✓	✓	✓
closed under $\cap$	✓	✓	✓
closed under $( )^c$	✓	✓	✗

regular TA  $<$  regular AC-TA  $<$  AC-TA

	regular TA	regular A-TA	A-TA
closed under $\cup$	✓	✓	✓
closed under $\cap$	✓	✗	✓
closed under $( )^c$	✓	✗	✓

regular TA  $<$  regular A-TA  $<$  A-TA

## Proof of not closed under complement of AC-TA (outline)

Given a signature  $\mathcal{F} = \{f\} \cup \{a_1, \dots, a_n\}$

$P$ : arithmetic constraint over the natural numbers s.t.

$$\begin{aligned} P &:= C \\ | \quad &P \vee P \\ | \quad &P \wedge P \\ | \quad &\neg(P) \end{aligned}$$

$$\begin{aligned} C &:= \#a_i \geq c \quad (c \in \mathbb{N}) \\ | \quad &\#a_i + \#a_j \geq \#a_k \\ | \quad &\#a_i \times \#a_j \geq \#a_k \end{aligned}$$

$L_P$ : tree language over  $\mathcal{F}$  s.t. each tree in  $L_P$  satisfies  $P$ , meaning that

e.g.  $L_{\#a=\#b}$  is the tree language whose element  $t$  satisfies  $|t|_a = |t|_b$

Suppose tree languages recognizable with AC-tree automata are closed under  $( )^C$

- Given  $P \equiv c_1 x_1^{k_1} + \dots + c_n x_n^{k_n} = c$ ,  $L_P$  is recognizable with AC-tree automata
- $L_P \neq \emptyset$  iff  $\exists (x_1, \dots, x_n)$  in  $\mathbb{N}^n - \mathbf{0}$ :  $c_1 x_1^{k_1} + \dots + c_n x_n^{k_n} = c$

It contradicts to the fact that (weak variant of) Hilbert's 10th problem is undecidable

## Decidability results

	regular TA	regular AC-TA	AC-TA
$t \in \mathcal{L}(\mathcal{A}/\text{AC})?$	✓	✓	✓
$\mathcal{L}(\mathcal{A}/\text{AC}) = \emptyset?$	✓	✓	✓
$\mathcal{L}(\mathcal{A}/\text{AC}) \subseteq \mathcal{L}(\mathcal{B}/\text{AC})?$	✓	✓	✗
$\mathcal{L}(\mathcal{A}/\text{AC}) = \mathcal{T}(\mathcal{F})?$	✓	✓	?

	regular TA	regular A-TA	A-TA
$t \in \mathcal{L}(\mathcal{A}/\text{A})?$	✓	✓	✓
$\mathcal{L}(\mathcal{A}/\text{A}) = \emptyset?$	✓	✓	✗
$\mathcal{L}(\mathcal{A}/\text{A}) \subseteq \mathcal{L}(\mathcal{B}/\text{A})?$	✓	✗	✗
$\mathcal{L}(\mathcal{A}/\text{A}) = \mathcal{T}(\mathcal{F})?$	✓	✗	✗

Note 1: One question remains open since CSL 2001

Note 2: The question is registered in the list of RTA Open Questions  
[\(<http://www.lsv.ens-cachan.fr/rtaloop/problems/101.html>\)](http://www.lsv.ens-cachan.fr/rtaloop/problems/101.html)

## Yet further tree language hierarchy

monotone A-TA = monotone A-PTA  $\gtrsim$  monotone AC-PTA

$\cup\downarrow$

$\cup\downarrow$

regular A-PTA

$\not\sqsubset$

monotone AC-TA

$\cup\downarrow$

$\chi\triangleright$

$\cup\downarrow$

regular A-TA

$\not\sqsubset$

regular AC-TA

$\parallel$

$\supseteq$  : tree language inclusion

regular AC-PTA

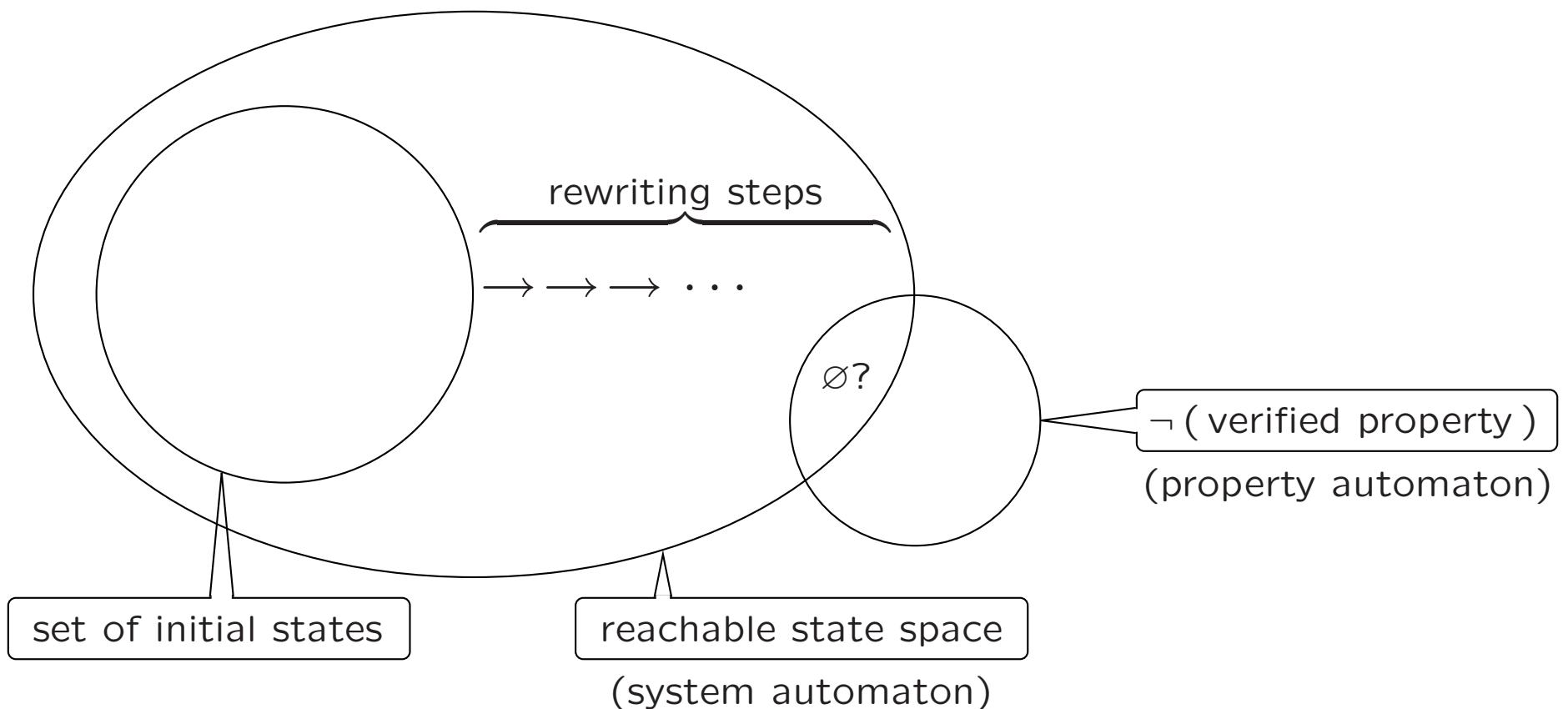
$\gtrsim$  : representability relation

## II. Towards system verification

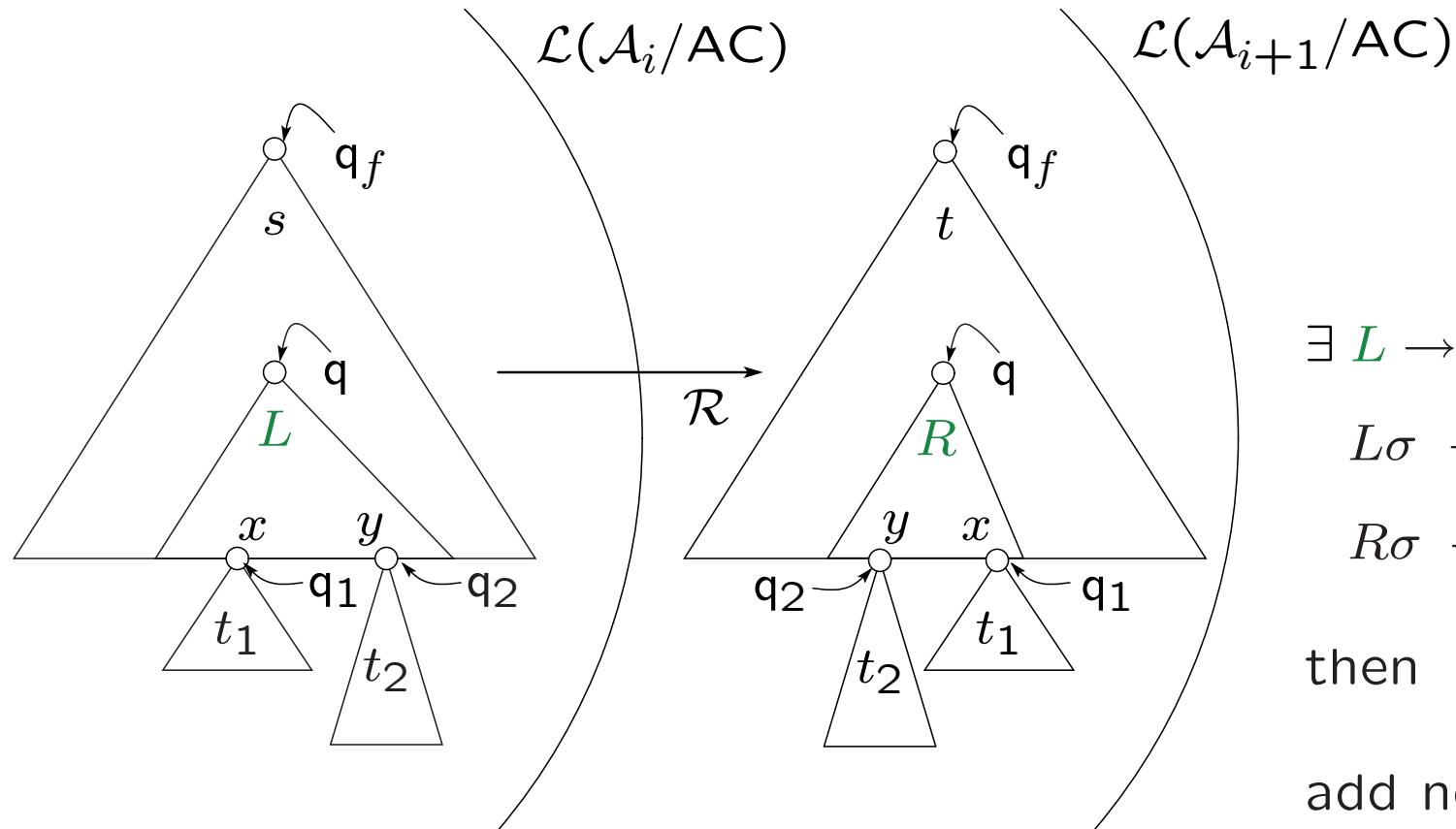
## Model checking based on term rewriting and tree automata

Observation1 Undecidable fragments exist in the fixpoint computation

Observation2 Intersection-emptiness problem is EXPTIME-complete  
for regular TA



## One step of the procedure



$\exists \textcolor{green}{L} \rightarrow \textcolor{green}{R}$  in  $\mathcal{R}$  such that

$L\sigma \xrightarrow{*_{\mathcal{A}_i/\text{AC}}} q$  but  
 $R\sigma \not\xrightarrow{*_{\mathcal{A}_i/\text{AC}}} q$

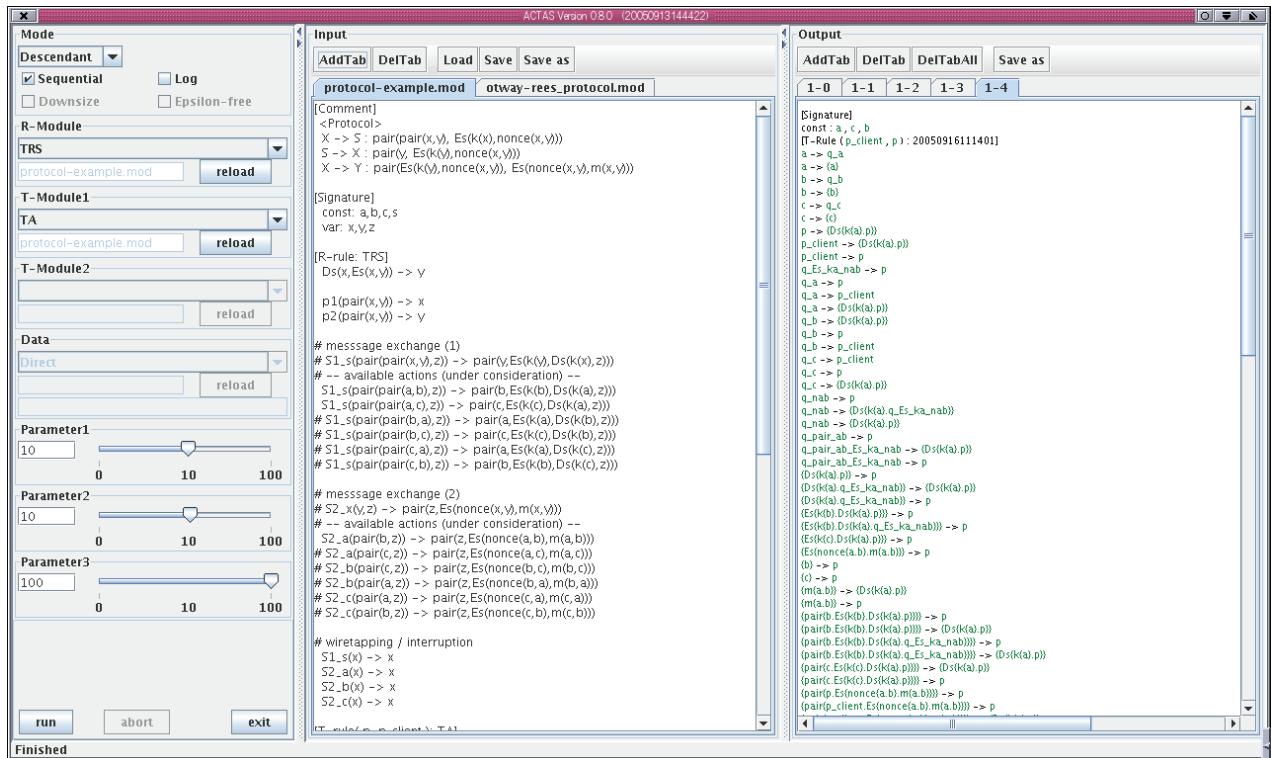
then

add new transition rules  
to  $\mathcal{A}_i/\text{AC}$  to satisfy

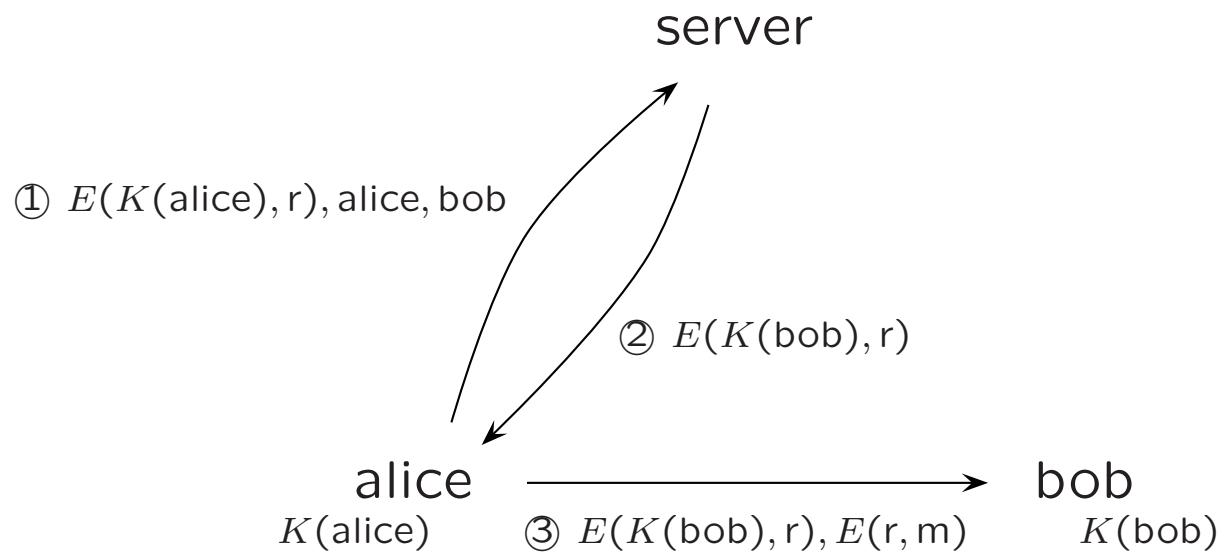
$R\sigma \xrightarrow{*_{\mathcal{A}_{i+1}/\text{AC}}} q$

## Specs on ACTAS & user-interface

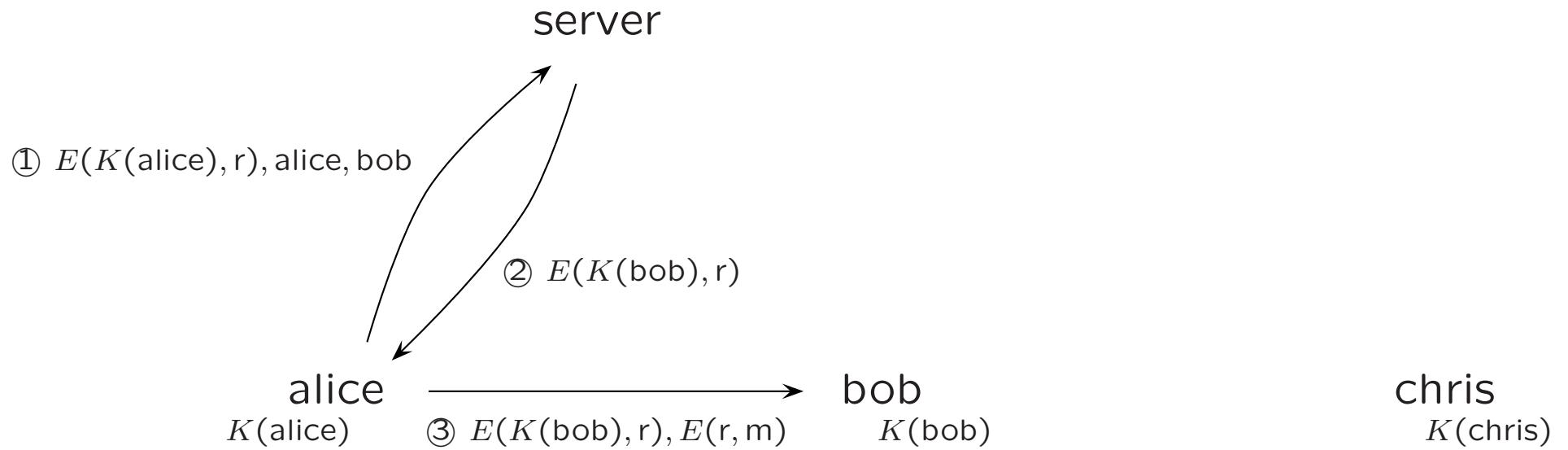
- Platform OS:  
Linux, Solaris
- Software requirement:  
Java  
ant (for rebuild)
- Memory:  
~ 512M byte  
(standard model)  
~ 2G byte  
(advanced model)
- Version:  
0.8.20050912



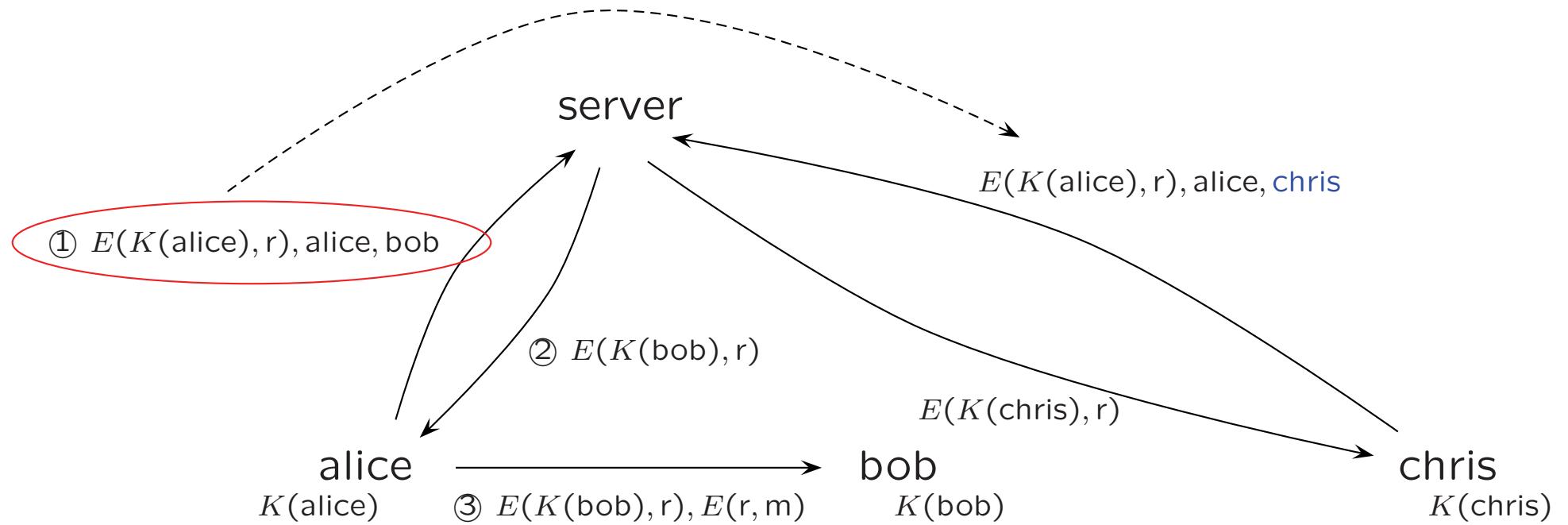
## Security flaw in a network protocol (1)



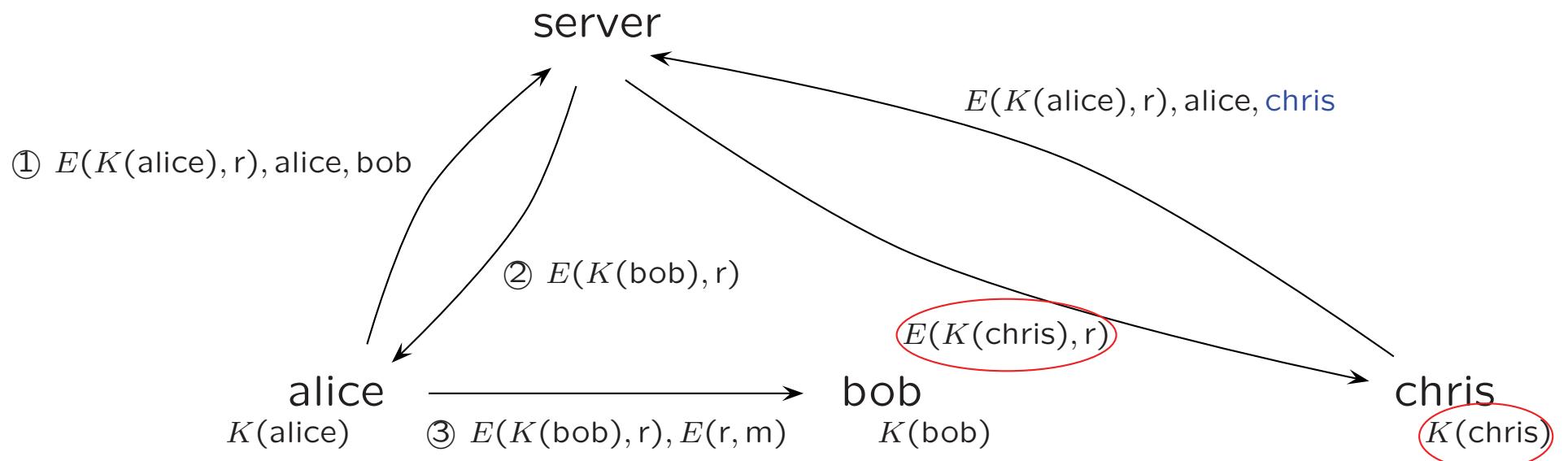
## Security flaw in a network protocol (2)



### Security flaw in a network protocol (3)



## Security flaw in a network protocol (4)

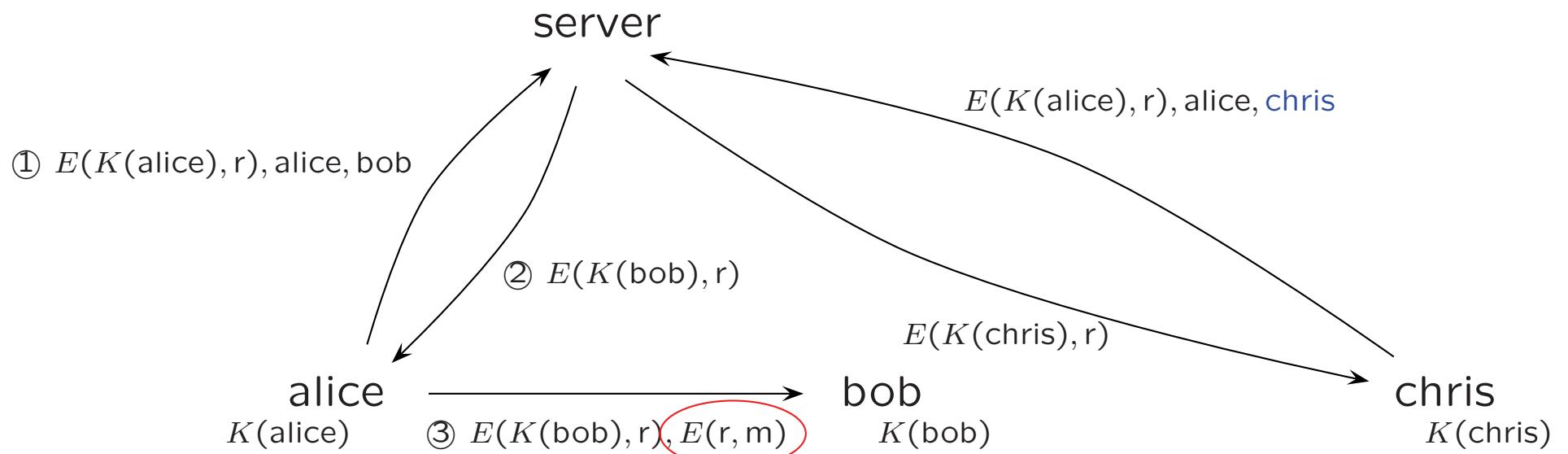


Axiom

$$D( x, E( x, y ) ) \rightarrow y$$

$$D( K(\text{chris}) , E(K(\text{chris}), r) ) \rightarrow r$$

## Security flaw in a network protocol (5)



Axiom

$$D( x, E( x, y ) ) \rightarrow y$$

$$D( K(chris) , E(K(chris), r) ) \rightarrow r$$

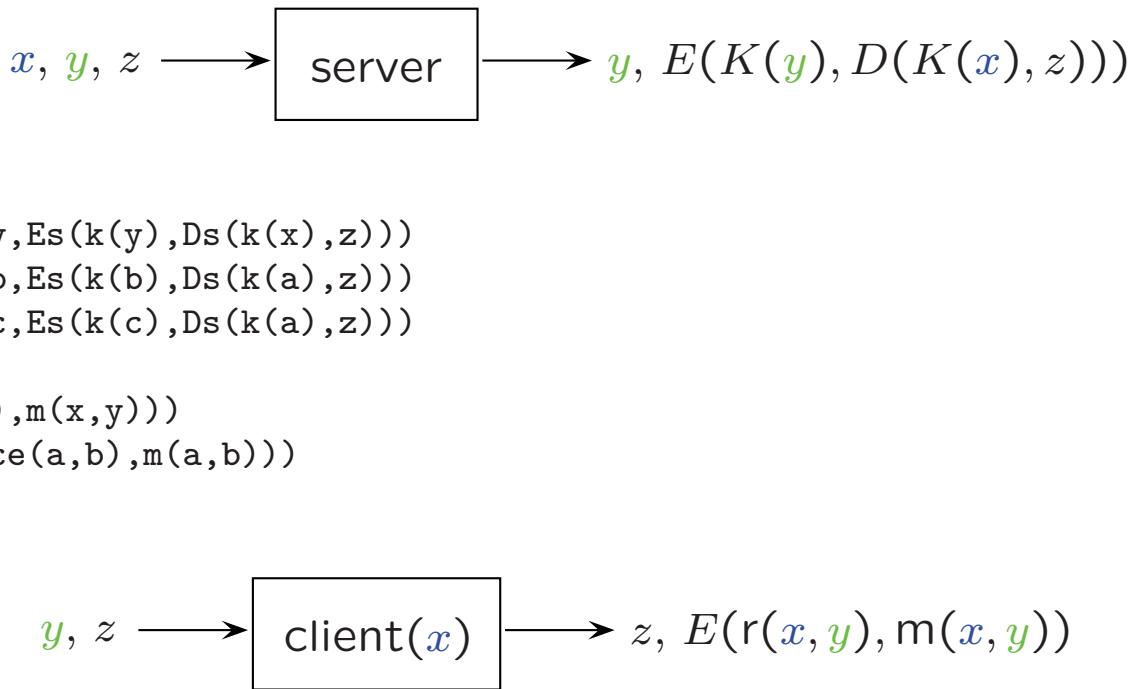
$$D( r , E(r, m) ) \rightarrow m \text{ (secret message)}$$

## ACTAS specification (Lines 1–25)

```

1: [Signature]
2: const: a,b,c,s
3: var: x,y,z
4:
5: [R-rule: TRS]
6:   Ds(x,Es(x,y)) -> y
7:   p1(pair(x,y)) -> x
8:   p2(pair(x,y)) -> y
9:
10:
11: # S1_s(pair(pair(x,y),z)) -> pair(y,Es(k(y),Ds(k(x),z)))
12:   S1_s(pair(pair(a,b),z)) -> pair(b,Es(k(b),Ds(k(a),z)))
13:   S1_s(pair(pair(a,c),z)) -> pair(c,Es(k(c),Ds(k(a),z)))
14:
15: # S2_x(y,z) -> pair(z,Es(nonce(x,y),m(x,y)))
16:   S2_a(pair(b,z)) -> pair(z,Es(nonce(a,b),m(a,b)))
17:
18:   S1_s(x) -> x
19:   S2_a(x) -> x
20:
21: [T-rule( p, p_client ): TA]
22:   Es(p,p) -> p
23:   Ds(p,p) -> p
24:   p1(p) -> p
25:   p2(p) -> p

```

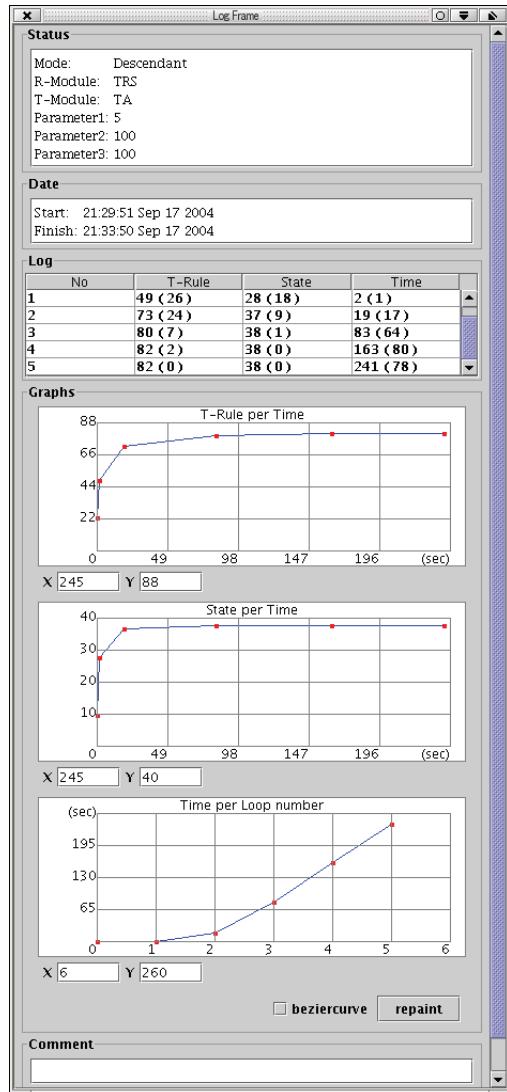


## ACTAS specification (Lines 26 – )

```
26: pair(p,p) -> p
27: pair(p_client,p_client) -> p
28: q_a -> p_client
29: q_b -> p_client
30: q_c -> p_client
31:
32: S1_s(p) -> p
33: S2_a(p) -> p
34:
35: # C's initial knowledge
36: k(q_c) -> p
37:
38: # initial messsage transfer:
39: # S1_s(pair(pair(a,b),Es(k(a),nonce(a,b)))) -> p
40:
41: # --- subterm decomposition ---
42: S1_s(q_p_ab_Es_ka_nab) -> p
43: pair(q_p_ab,q_Es_ka_nab) -> q_p_ab_Es_ka_nab
44: pair(q_a,q_b) -> q_p_ab
45: Es(q_ka,q_nab) -> q_Es_ka_nab
46: k(q_a) -> q_ka
47: nonce(q_a,q_b) -> q_nab
48: a -> q_a
49: b -> q_b
50: c -> q_c
```

1. If chris knows  $x$  and  $E(x,y)$ , then chris also knows  $y$
2. If chris knows  $x$  and  $y$ , chris can construct  $E(x,y)$  and  $D(x,y)$
3. chris knows its own secret key  $K(\text{chris})$  and all principals names: alice, bob, chris
4. chris knows message going through the network (**wiretapping**)
5. chris decomposes sequences of data (**modification**)
6. chris pretends to be other principals (**impersonation**)

## Descendant computation for reachability analysis



Loop number	$\sharp(T\text{-rules})$	$\sharp(\text{states})$	time (sec)
0	23	13	3
1	56	34	4
2	102	46	6
3	109	46	18
4	109	46	23

Note 1.  $\forall i: \mathcal{L}(\mathcal{A}_i/\text{AC}) \subseteq \mathcal{L}(\mathcal{A}_{i+1}/\text{AC})$

Note 2.  $\exists i: \mathcal{L}(\mathcal{A}_i/\text{AC}) = \mathcal{L}(\mathcal{A}_{i+1}/\text{AC})$

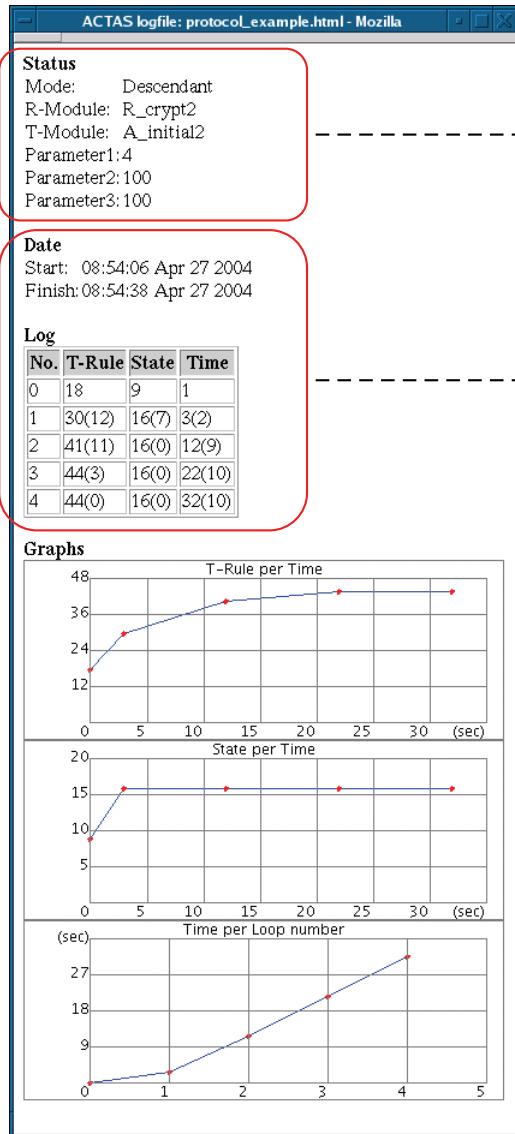
$\Rightarrow \exists i: \mathcal{L}(\mathcal{A}_j/\text{AC}) = \mathcal{L}(\mathcal{A}_{j+1}/\text{AC}) \text{ for all } j \geq i$

( $\Rightarrow \exists i: \mathcal{L}(\mathcal{A}_i/\text{AC}) = \mathcal{L}(\mathcal{A}_\infty/\text{AC})$ )

Note 3.  $\exists i: m(a, b) \in \mathcal{L}(\mathcal{A}_i/\text{AC})$

$\Rightarrow$  secret message  $m$  is retrieved by chris

## Tool support for state space analysis



Computation mode

Module names (i.e. selected R-rule and T-rule names)

Parameters 1–3 ( $0 \leq i \leq 100$ )

Execution time

Number of transition rules

Number of state symbols for each loop computation

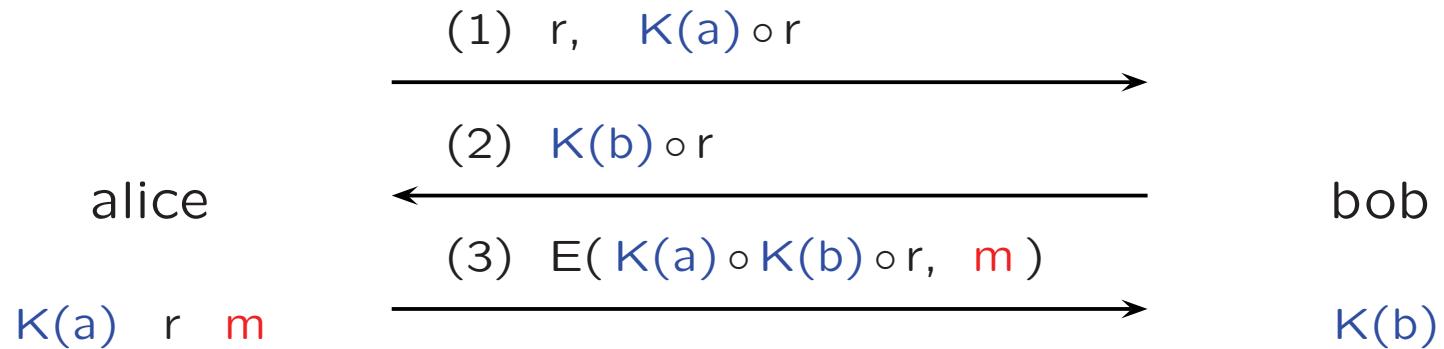
Graph1: number of transition rules  $\times$  time(sec)

Graph2: number of state symbols  $\times$  time(sec)

Graph3: time(sec)  $\times$  loop number

(in HTML file format)

## AC-axioms in encryption scheme



$\mathsf{K}(a)$   $\mathsf{K}(b)$  : secret keys

$r$  : random number

$\mathsf{m}$  : secret message

$E$  : encryption function

$\circ$  : AC symbol (infix operator)

Claim: secret message  $\mathsf{m}$  is not retrieved by wiretapping only

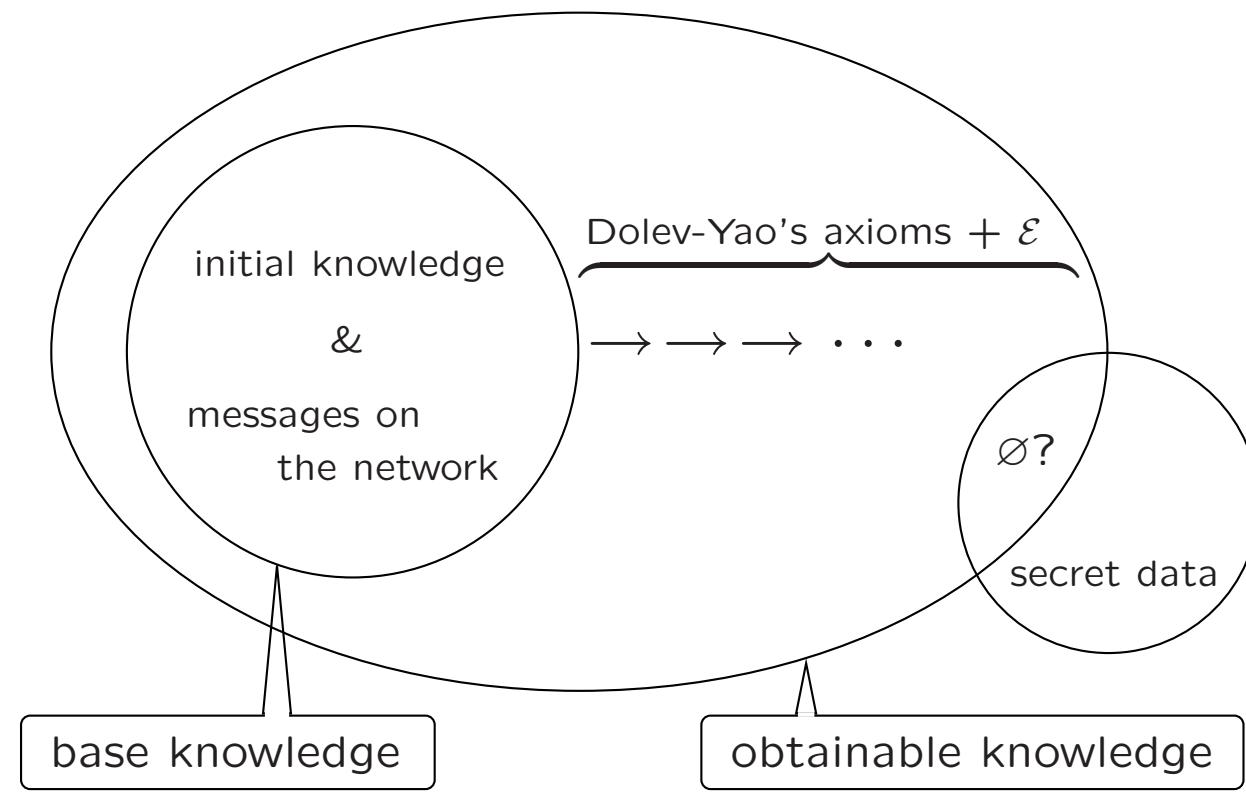
(Cf. “Easy Intruder Deductions” by Comon-Lundh & Treinen 2003)

## AC-function symbols in ACTAS specification

```

1: [Signature]
2: AC: f
3: const: a,b,c,m,r
4: var: x,y
5:
6: [R-rule: TRS2]
7: Ds(x,Es(x,y)) -> y
8:
9: [T-rule( p ): TA2]
10: Ds(p,p) -> p
11: Es(p,p) -> p
12: f(p,p) -> p
13:
14: f(q_ka,q_r) -> p
15: r -> q_r
16:
17: r -> p
18:
19: f(q_kb,q_r) -> p
20: k(q_b) -> q_kb
21: b -> q_b
22:
23: e(q_f_kba_r,q_m) -> p
24: f(q_kab,q_r) -> q_f_kba_r
25: f(q_kb,q_ka) -> q_kba
26: k(q_a) -> q_ka
27: a -> q_a
28: m -> q_m
29: # C's initial knowledge
30: a -> p
31: b -> p
32: c -> p
33: k(q_c) -> p
34: c -> q_c

```



## Intruder deduction problem (general version)

Given two sets  $L, M$  (of messages) and equational rewrite system  $\mathcal{R}/\mathcal{E}$ :

Is the intersection of  $[\rightarrow_{\mathcal{R}/\mathcal{E}}^*](L)$  and  $M$  the empty or not ?

Note 1. In the previous setting

$L$  : initial knowledge + messages on the network

$M$  : secret data

$\mathcal{R}/\mathcal{E}$  : Dolev-Yao's axioms and  $AC(\{f\})$

Note 2. Tree languages recognized by AC-TA, called *AC-recognizable tree languages* are closed under  $\cap$  and

*AC-regular tree languages* are also closed under  $\cap$

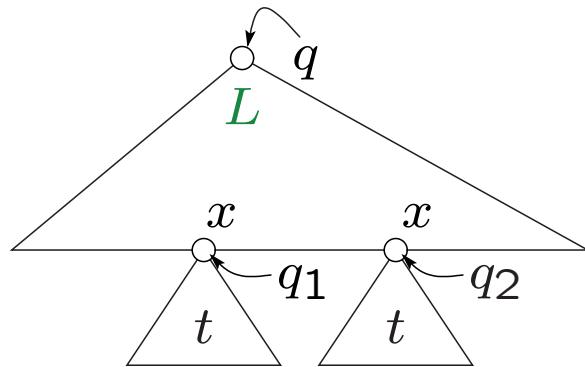
Note 3. The emptiness problems for AC-TA and regular AC-TA are decidable

## Non-left-linear case

$\exists L \rightarrow R$  in  $\mathcal{R}$  such that  $L = C[x, x]$  e.g.  $D(x, E(x, y)) \rightarrow y$

- Check  $\mathcal{L}(\mathcal{A}_i/\text{AC}, q_1) \cap \mathcal{L}(\mathcal{A}_i/\text{AC}, q_2) \neq \emptyset$

If the above condition holds for  $q_1 \neq q_2$ , we take the following assignment:

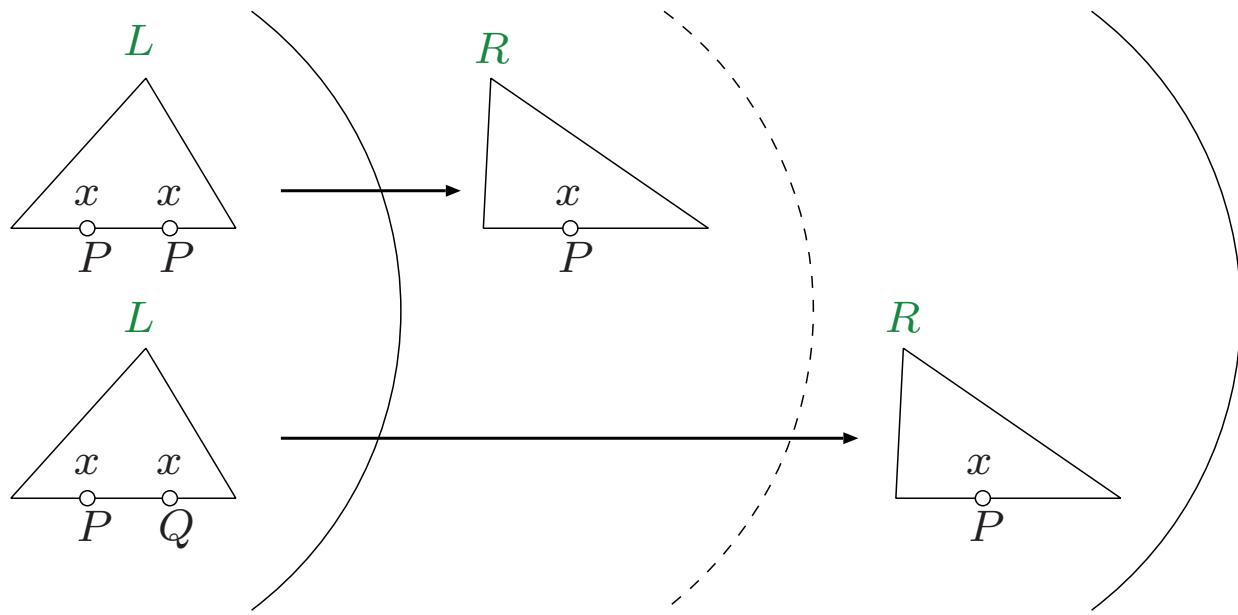


- In fully automated verification, under- or over-approximation is needed to be applied

The intersection-emptiness for AC-TA is decidable but the complexity is extremely high!

- In fact, we use in ACTAS under- (or over-)approximation algorithms when solving emptiness problems in AC-case

## Bounded computation for emptiness test



$$\#(\text{T-rule}) = 21$$

$$\#(\text{state symbols}) = 16$$

search depth for emptiness test is controlled by  
Parameters 2, 3

P2 = 10		
P3	state space for emptiness test	time (sec)
1	0	1
2	256	2
3	4096	2
4	65536	3
5	1048576	9
6	16777216	—

## References

## Research collaboration

### [1] Sophie Tison & Jean-Marc Talbot

Université des Sciences et Technologies de Lille, France

- Invited position, June 2002
- Invited position, June 2005



### [2] José Meseguer & Joe Hendrix

University of Illinois at Urbana-Champaign, IL, USA

- Invited position, January – March 2004
- Invitation (Hendrix), July – August 2005



### [3] Ralf Treinen

École Normale Supérieure de Cachan, France

- Invited position, August – September 2004
- Invitation (Treinen), December 2001 & December 2005

## Publications I: software & applications

- [4] ACTAS: A System Design for Associative and Commutative Tree Automata Theory

Hitoshi Ohsaki & Toshinori Takai

5th International Workshop on Rule-Based Programming (RULE 2004)

Aachen (Germany), June 2004

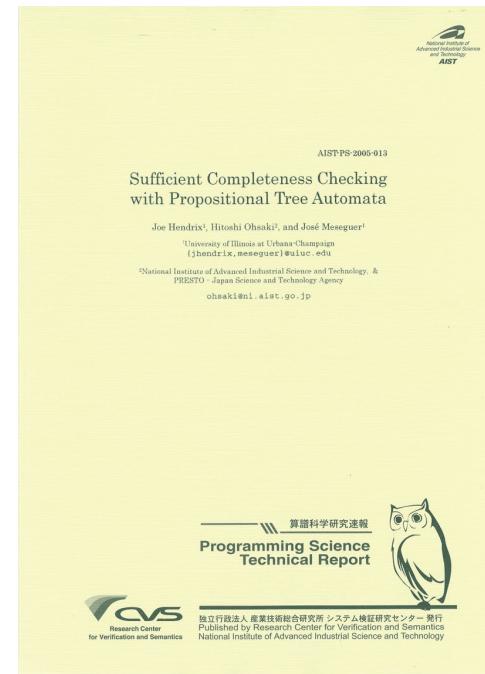
ENTCS 124, pp. 97–111

- [5] Sufficient Completeness Checking with Propositional Tree Automata

Joe Hendrix & Hitoshi Ohsaki & José Meseguer

Technical Report AIST-PS-2005-013

AIST/CSV, 2005 (new !)



## Publications II: equational tree automata

### [6] AC-Monotone Tree Languages

Hitoshi Ohsaki & Jean-Marc Talbot & Sophie Tison & Yves Roos

12th International Conference on Logic for Programming

Artificial Intelligence and Reasoning ([LPAR 2005](#))

Montego Bay (Jamaica), December 2005

To appear in [LNCS](#)

### [7] Recognizing Boolean Closed A-Tree Languages with Membership Conditional Rewriting Mechanism

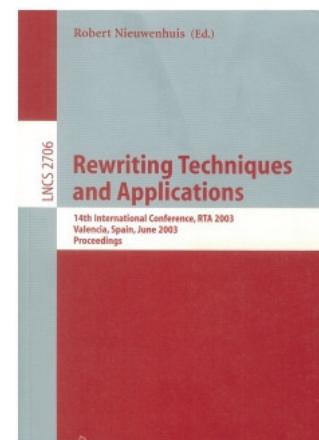
Hitoshi Ohsaki & Hiroyuki Seki & Toshinori Takai

14th International Conference on Rewriting

Techniques and Applications ([RTA 2003](#))

Valencia (Spain), June 2003

[LNCS 2706](#), pp. 483–498



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## Publications II (cont'd)

- [7] Decidability and Closure Properties of Equational Tree Languages  
Hitoshi Ohsaki & Toshinori Takai  
13th International Conference on Rewriting Techniques and Applications  
([RTA 2002](#))  
Copenhagen (Denmark), July 2002  
[LNCS 2378](#), pp. 114–128
- [8] Beyond Regularity: Equational Tree Automata for Associative and Commutative Theories  
Hitoshi Ohsaki  
15th International Conference of the European Association for Computer Science Logic ([CSL 2001](#))  
Paris (France), September 2001  
[LNCS 2142](#), pp. 539–553

## Tool demonstration

### [9] ACTAS: Associative and Commutative Tree Automata Simulator

(presented by Toshinori Takai)

4th International Conference on Application of Concurrency to System Design (**ACSD 2004**), Hamilton (Canada), June 2004

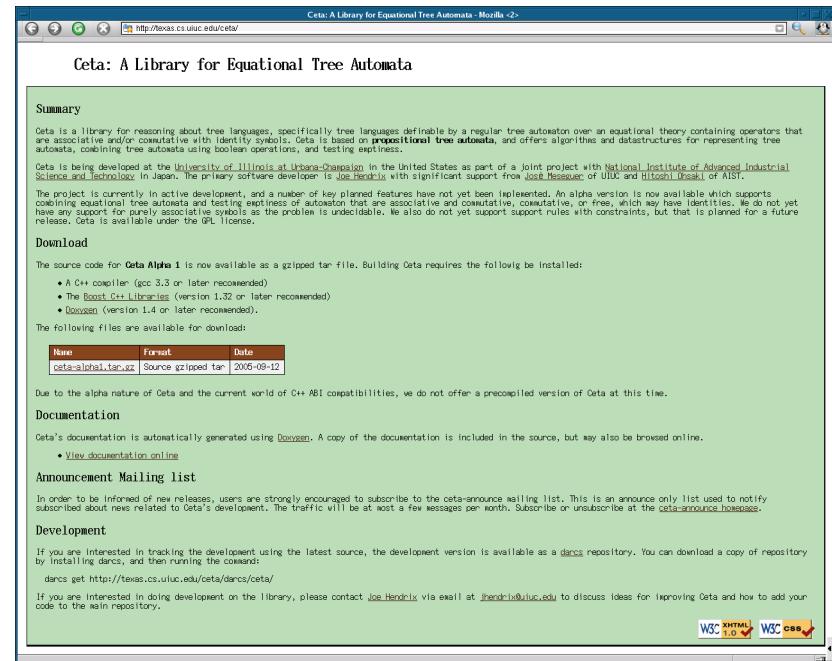
## Software products

### [10] CETA: Library for Equational Tree Automata

Joe Hendrix

<http://texas.cs.uiuc.edu/ceta/>

### [11] ACTAS Hitoshi Ohsaki (to be announced)



CETA homepage

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