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# Self-stabilizing Bipedal Locomotion Employing Neural Oscillators

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***Abstract*— For attaining a stable and robust dynamic bipedal locomotion, we address an efficient and powerful alternative based on biologically inspired control framework employing neural oscillators. Neural oscillators can be used to generate sustained rhythmic signals, and show superior features for stabilizing bipedal locomotion particularly when coupled with virtual impedance components. By building a network of neural oscillators, we can enable humanoid robots to walk stably and exhibit robustness against unexpected disturbances. Specifically, in order to maintain stability, the neural oscillator plays an important role by controlling the trajectory of the COM in phase with the ZMP input. The effectiveness of the proposed control scheme is verified through simulations.**

## I. Introduction

Human and animal locomotion exhibit inherent stable rhythmic movements adapted to the natural frequency of their body dynamics in spite of differences in their sensors and actuators. The neural circuits of oscillators on the spinal cord [1] known as Central Pattern Generators (CPGs) contribute to such efficient motor movement and novel stability properties in biological models. Moreover, neural oscillators can entrain to the sensory feedback, which plays a key role to adapt locomotion in a changing environment. Hence, biologically inspired motion control based on the neural oscillator has been gaining increasing attention as an attempt to implement natural motions in humanoid locomotion.

Mathematical descriptions of a neural oscillator were addressed in Matsuoka's works [2], [3], where neurons were proven to generate the rhythmic patterned output. His work provided necessary and sufficient conditions on the parameters to sustain self-oscillations. Employing Matsuoka's neural oscillator model, Taga *et al.* investigated mutual entrainment of neural oscillators performed by a musculo-skeletal system, which created stable locomotion in a certain environment. Specifically, sensory signals from the joints of a biped robot were fed back to entrain the oscillators [4], [5]. As a result, the robot became robust against perturbation and was able to climb an upward slope [6]. These attributes were later applied to a 3D locomotion in simulation by Miyakoshi *et al.* in [7]. In addition to these prior researches, neural oscillators were successfully implemented in a dynamic quadrupedal walking by Fukuoka *et al.* [8], and in the control of rhythmic robot arm movement by Williamson [9]. On the other hand, a new intuitive control scheme based on virtual model control for legged locomotion was proposed by Pratt [10]. Even though this algorithm is extended for rough terrain walking, the approach is only applied to 2-D locomotion.

In practice, oscillator based approaches were applied later to various locomotion systems [11], [12], showing that neural oscillators made the robot adaptive to uneven terrains through the entrainment property. Even though they embody adaptive bipedal locomotion on 3-D environment incorporating a real humanoid robot, those works mainly dealt with the global stability on a periodic locomotion employing the phase locking characteristic of the neural oscillator. Hence, there are still some considerable issues on how to balance during stance phase, to precisely cope with locomotion planning, etc..

From a practical point of view, it would be advantageous if humanoid robots can maintain its stability without using sophisticated controllers. This work involves a new application of the interaction between neural oscillators and virtual components to humanoid locomotion. This allows humanoid robots to adapt their motions through entrainment responding to unknown disturbances. The motion of the inverted pendulum is often regarded as a supporting leg in stance phase of humanoid locomotion [11]. We therefore develop a new control method for sustaining and enlarging the stability of inverted pendulum. Based on this, we focus on implementation of periodic swing motions according to a desired pattern by incorporating the sensory signals that detect changes in the Center of Mass (COM) of humanoid robots. This also requires a converged swing motion regardless of external disturbances. It is verified through simulations that the proposed approach yields a robust yet efficient control of rhythmic locomotion.

## II. Rhythmic Movement Generation

### A. Matsuoka's Neural Oscillator

Our work is motivated by studies and facts of biologically inspired locomotion control employing oscillators. Especially, the basic motor pattern generated by the Central Pattern Generator (CPG) of inner body of human or animal is usually modified by sensory signals from motor information to deal with environmental disturbances. Similar to the sensory system of human or animal, the neural oscillators are entrained with external stimuli at a sustained frequency. They show stability against perturbations through global entrainment between the neuro-musculo-skeletal systems and the ground [4]. Thus, neural oscillators have been applied to the CPG of humanoid robots with rhythmic motions [5], [6].


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The oscillators provide robust performance in a wide variety of rhythmic tasks, when they are implemented to such a system as a robotic arm. The reason is that the oscillators use sensory signals about the joint state to adapt the frequency and phase of the joint motion regardless of the references corresponding to change of the environment.

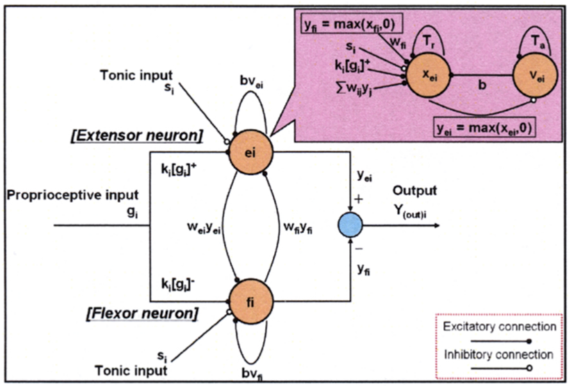



Fig. 1 Schematic diagram of the NEURAL OSCILLATOR

Matsuoka's neural oscillator consists of two simulated neurons arranged in mutual inhibition as shown in Fig. 1 [1], [2]. If gains are properly tuned, the system exhibits limit cycle behaviors. The trajectory of a stable limit cycle can be derived analytically, describing the firing rate of a neuron with self-inhibition. The neural oscillator is represented by a set of nonlinear coupled differential equations given as

$$
\begin{aligned}
T_r \dot{x}_{ei} + x_{ei} &= -w_{fi} y_{fi} - \sum_{j=1}^{n} w_{ij} y_j - b v_{ei} - \sum k_i [g_i]^+ + s_i \\
T_a \dot{v}_{ei} + v_{ei} &= y_{ei} \\
y_{ei} &= [x_{ei}]^+ = \max(x_{ei},\, 0) \\
T_r \dot{x}_{fi} + x_{fi} &= -w_{ei} y_{ei} - \sum_{j=1}^{n} w_{ij} y_j - b v_{fi} - \sum k_i [g_i]^- + s_i \\
T_a \dot{v}_{fi} + v_{fi} &= y_{fi} \\
y_{fi} &= [x_{fi}]^+ = \max(x_{fi},\, 0), \quad (i = 1,\, 2,\, \cdots,\, n)
\end{aligned}
\tag{1}
$$

where $x_{e(f)i}$ is the inner state of the $i$-th neuron which represents the firing rate; $v_{e(f)i}$ is a variable which represents the degree of the adaptation, modulated by the adaptation constant $b$, or self-inhibition effect of the $i$-th neuron; the output of each neuron $y_{e(f)i}$ is taken as the positive part of $x_i$, and the output of the whole oscillator as $Y_{(out)i}$; $w_{ij}y_i$ represents the total input from the neurons inside a neural network; the input is arranged to excite one neuron and inhibit the other, by applying the positive part to one neuron and the negative part to the other; the inputs are scaled by the gains $k_i$; $T_r$ and $T_a$ are time constants of the inner state and the adaptation effect of the $i$-th neuron respectively; $b$ is a coefficient of the adaptation effect; $w_{ij}$ is a connecting weight from the $j$-th neuron to the $i$-th neuron; $s_i$ is an external input with a constant rate. Especially, $w_{ij}$ (0 for $i \neq j$ and 1 for $i=j$) is a weight of inhibitory synaptic connection from the $j$-th neuron to the $i$-th, and $w_{ei}$, $w_{fi}$ are also a weight from extensor neuron to flexor neuron, respectively.

### B. *Coupling Neural Oscillator to Mechanical Systems*

This subsection addresses a new control method exploiting the natural dynamics of the oscillator coupled to the dynamic system that closely interacts with environments. This method enables a robot to adapt to changing conditions. For simplicity, we employed a general 2$^{nd}$ order mechanical system connected to the neural oscillator as seen in lower system of Fig. 2. The desired torque input at the $i$-th joint can be given by [9]

$$\tau_i = k_i(\theta_{vi} - \theta_i) - b_i \dot{\theta}_i, \tag{2}$$

where $k_i$ is the stiffness of the joint, $b_i$ the damping coefficient, $\theta_i$ the joint angle, and $\theta_{vi}$ the desired joint position which is the output of the neural oscillator. The output of the neural oscillator drives the mechanical system corresponding to the sensory signal input (feedback) from the actuator (displacement or torque). The oscillator entrains the input signal so that the mechanical system can exhibit adaptive behavior even under the unknown environment condition.

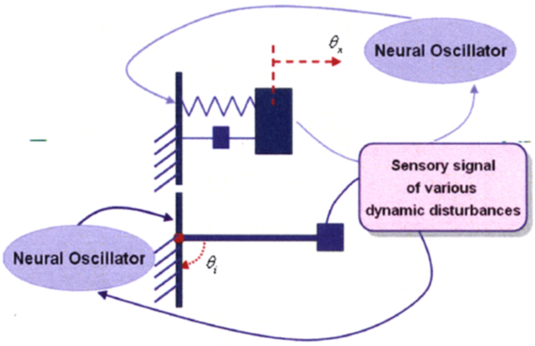



Fig. 2 Mechanical system model coupled to the neural oscillator

## III. The Control Model for Bipedal Stability

In humanoid locomotion, the pitching motion should be performed under the stable single support phase of the rolling motion. Now we explain how to attain the stable single support phase corresponding to the periodic bipedal locomotion. For the stable rolling motion in the frontal plane, we consider an inverted pendulum model coupled to such a virtual mechanical component as a spring and damper and the neural oscillator, as seen in Fig. 3 (a) for generating an appropriate rolling motion. The coupled model enables the inverted pendulum to stably move in a frontal plane according to a desired ZMP (Zero Moment Point) trajectory sustaining the stability.

Assuming that $\theta$, the angle between the vertical axis and the pendulum in Fig. 3 (a), is small enough and linearized near 0, the dynamic equation of the coupled inverted pendulum is given by

$$\ddot{y} = \frac{G}{l}(y - u) + F_y \tag{3}$$

where $y$ is the displacement of the pendulum in the rolling direction, $l$ is the length of the pendulum, and $u$ is the position of the massless cart of the pendulum. $G$ is the gravitational constant and $F_y$ indicates the force that should be applied to the COM of the pendulum in the rolling direction.

If the desired ZMP trajectory, $u$, is given in Eq. (3), a stable periodic motion of the COM of the pendulum is generated in terms of the coupled neural oscillator with state feedback [11]. If a mechanical system is coupled to the neural oscillator, the dynamic stability is improved [12]. Hence, a stable limit cycle behavior is induced and the periodic COM motion is achieved by the impedance controller of the virtual components, as illustrated in the block diagram in Fig. 4. Accordingly, $F_y$ in Eq. (3) is given by

$$F_y = \frac{1}{ml}\left\{(2k_s(h\theta_v - k_p y) - k_v \dot{y}) + (i_p(y_d - y) - i_v \dot{y})\right\} \tag{4}$$

where $k_s$ is the stiffness coefficient and $h$ is the output gain of the neural oscillator. $k_p$ and $k_v$ are the gains of state feedback, and $i_p$ and $i_v$ are the gains of the impedance controller. In the proposed controller, $\theta_v$ and $y_d$ denote the output of the neural oscillator and a desired ZMP input, respectively. The current COM position and velocity of the humanoid robot are obtained again by Eq. (3). For a stable rolling motion corresponding to the ZMP input, $F_y$ in Eq. (4) is transformed into joint torque using the Jacobian that needs to be applied to each joint of both legs for the rolling motion in Fig. 3 (b). As illustrated in Fig. 3 (b) and (c), the humanoid robot exhibits stable rolling motion satisfying the desired ZMP.

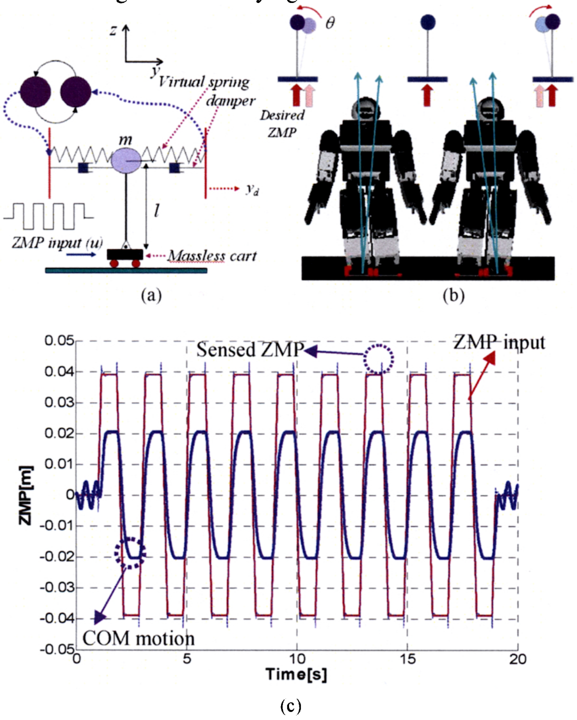



Fig. 3 (a) The proposed control model for stable inverted pendulum motion (b) Simulation result with respect to rolling motion of humanoid robot (c) The rolling COM motion of humanoid robot

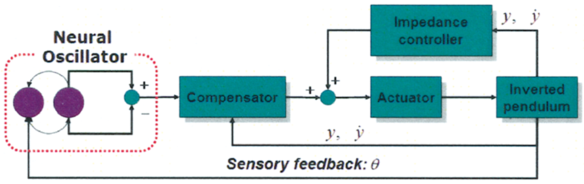



Fig. 4 Control block diagram of the coupled inverted pendulum

## IV. Implementation and Basic Tests of The Coupled Model

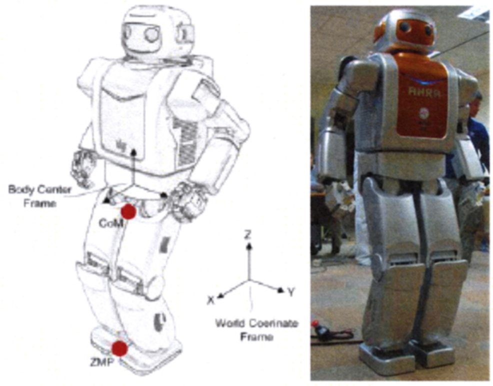



Fig. 5 KIST Humanoid robot

### *A. Implementation*

This subsection describes how to embody the bipedal locomotion of the humanoid robot using the proposed control scheme to guarantee the stability on the single and double support phases. The humanoid robot in Fig. 5, developed by KIST, is used to test and evaluate the proposed controller. It has 35 degrees of freedom (DOF): 6 in each arm, 6 in each leg, 4 in each hand, 2 in neck, and 1 in waist. It's 150cm high and weighs about 67kg. The robot is equipped with a stereo camera, a microphone, four force/torque sensors, and a pose sensor

We propose the control method that uses several virtual mechanical components such as springs and dampers, and couple them with neural oscillators as seen in Fig. 6. A virtual spring-damper is defined from the hip to the ankle for each leg and could generate the virtual vertical forces $F_{z,r}$ and $F_{z,l}$ at each hip joint of the right leg and left leg, respectively. It is used to control the vertical stable position of the hip corresponding to the reaction force of the foot. $F_z$ is the summation of the virtual vertical forces ($F_{z,r}$ and $F_{z,l}$). These forces bring about significant positive effects on the landing stability of swing legs. Also the virtual springs-dampers are attached to the hip along the two horizontal directions to generate the virtual horizontal forces $F_x$ and $F_y$ at the hip. Here $F_x$ and $F_y$ include the virtual forces exerted by virtual mechanical components and neural oscillator. Using those forces, the humanoid robot should perform the desired motion and satisfy the stability simultaneously under unknown environments or disturbances.

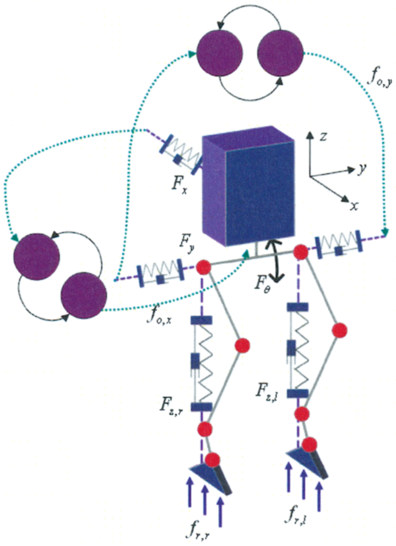



Fig. 6 The coupled model with virtual springs and dampers

Separating the right and left legs, the dynamic equation of the three-link is given in the following form

$$M(\theta)\ddot{\theta} + V(\theta,\dot{\theta}) + G(\theta) = \tau \tag{5}$$

where $M$ is the 3×3 inertia matrix, $V$ is the 3×3 Coriolis /centripetal vector, and $G$ 3×1 is the gravity vector. Also a rotational spring- damper is attached at the hip to generate a virtual rotational force $F_\theta$ about the y-axis in Fig. 6. This is used to maintain the upright posture of the hip. The virtual forces and moment can be transformed into equivalent torques as follows:

$$\begin{pmatrix} \tau_a \\ \tau_k \\ \tau_w \end{pmatrix} = \begin{pmatrix} J_{12} & J_{22} & -1 \\ J_{13} & J_{23} & -1 \\ J_{14} & J_{24} & -1 \end{pmatrix} \begin{pmatrix} F_x \\ F_z \\ F_\theta \end{pmatrix}, \tag{6}$$

where

$$F_x = k_x(x_d - x) + c_x(\dot{x}_d - \dot{x}) + f_{o,x}$$
$$F_{z,r} = f_{r,r} + k_z(z_d - z) + c_z(\dot{z}_d - \dot{z})$$
$$F_{z,l} = f_{r,l} + k_z(z_d - z) + c_z(\dot{z}_d - \dot{z})$$
$$F_\theta = k_\theta(\theta_d - \theta) + c_\theta(\dot{\theta}_d - \dot{\theta})$$

$J$ is the Jacobian matrix, which relates the virtual velocity with regard to the Cartesian coordinate between frames of the ankle joint and the COM position to the joint velocities. $k_i$ and $c_i$ are the spring stiffness and damping coefficient, respectively for the virtual components in $i$ (=$x$, $y$, $z$ or $\theta$) coordinate. $f_{r,r}$ and $f_{r,l}$ indicate the reaction forces of the right and left leg as seen in Fig. 6. Therefore, the right leg is controlled in terms of $F_{z,r}$ and $F_{z,l}$ is employed to drive the left leg. Note that $f_{o,x}$ is the driving force induced by the individual neural oscillators of the sagittal plane. In the frontal plane, since one-link dynamics is considered similar to the inverted pendulum, $F_y$ can be described as Eq. (4) including $f_{o,y}$. $x$, $y$, $z$ *and* $\theta$ denote the current position of the hip and $x_d$, $y_d$, $z_d$ and $\theta_d$ are the desired position. Finally, substituting the equations of the virtual forces into Eq. (6), the hip, $\tau_w$, knee, $\tau_k$, and ankle joint torques, $\tau_a$, can be obtained to drive the each leg joint of the humanoid robot.

*B. Basic Control Tests of the Coupled Model*

We performed numerical simulations to verify the proposed control method. In this simulation, we use the humanoid robot simulator, named *Simstudio*, developed in *SimLab* co. Prior to bipedal locomotion simulation, we test a few motions during the double support and single support phases, since the analysis of those supporting phases is essential for bipedal walking or other movements. Also, we investigate whether the output of the neural oscillator is adapted to the dynamic motion of the humanoid robot.

*1) Motion verification in the double support phase:*

After implementing the control algorithm to the *Simstudio*, we acquired the simulation results of the hip positions during rolling and pitching as seen in Figs. 7 and 8. The hip positions are generated appropriately by an arbitrarily defined ZMP input seen in Figs. 7 and 8. This implies that the proposed control method in Sec. III works properly. The dashed line in the figures indicates the input pattern of the desired ZMP. The solid lines of Figs. 7 and 8 show the COM of the humanoid robot with regard to the rolling and pitching motions, respectively. Remarkably, the dotted line in Fig.7 and the dash-dotted line in Fig. 8 are the outputs of the neural oscillators, when the humanoid robot periodically moves in the lateral and sagittal planes. The COM motion of the humanoid robot fed again is considered as the sensory signal of the neural oscillator. Then the outputs of neural oscillator entrain that of the humanoid robot and drive the humanoid robot corresponding to the sensory input appropriately as a torque input. From these results, it can be observed that the neural oscillator causes the self-adapting motion to follow the sensory input. Consequently, we note that this leads the adaptive motion of humanoid robots to maintain the bipedal stability even under an unknown disturbance.

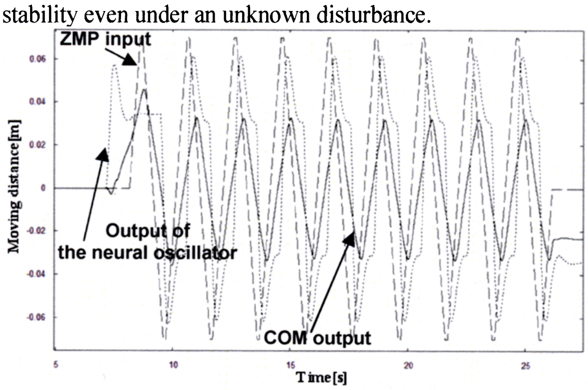



Fig. 7 Input ZMP profile (dashed line), the output of the COM position (solid line) and the output of the neural oscillator in the rolling motion (dotted line)

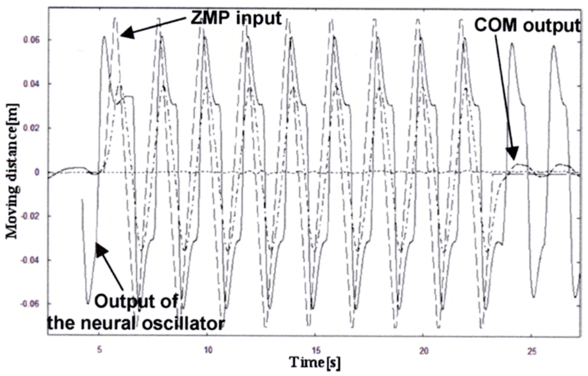



Fig. 8 Input ZMP profile (dashed line), the output of the COM position (solid line) and the output of the neural oscillator in the pitching motion (dash-dotted line)

*2) Motion verification in the double and single support phase:* There is the difficulty on how to or when to switch the double support phase and the single support phase under the bipedal locomotion or various humanoid behaviors. To solve this problem, we propose the proper switching rule based on the COM position and the ZMP position. Basically the balancing motion is controlled considering the only COM position. If the projected COM position in the rolling direction moves within the size of the left foot, this indicates that the left leg only supports the whole body of the humanoid robot. On the contrary, in the right leg, the single support phase becomes diametrically opposed to that. In consequence, there is the double support phase when the projected COM position is placed at inner empty space of the size of both foots. Then both legs control the whole body of the humanoid robot.

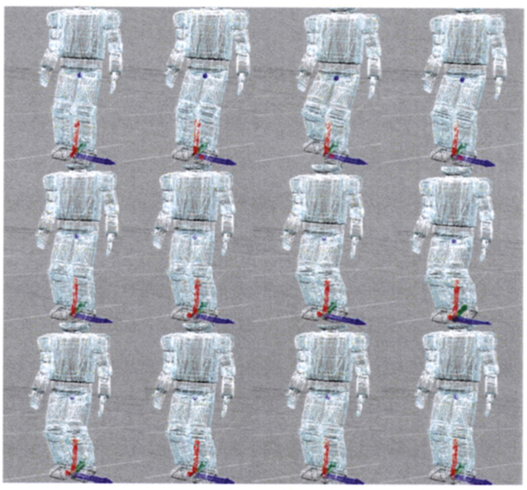


Fig. 9 Simulation result on the lifting leg motion with the transition velocity of the ZMP, 0.4cm/s.

The lifting motion of the leg is dominated in terms of the calculated ZMP position. If the ZMP position exists within the ZMP criterion of a foot, the corresponding leg can be used as the supporting leg in order to maintain the lifting motion of another leg. For instance, when the calculated ZMP position is inside the ZMP criterion of the right leg, the lifting motion of the left leg becomes possible. However, this criterion is changeable according to the moving velocity of the current ZMP position between the ZMP criterion of a foot and that of another foot. Here we should establish the condition to evaluate the performance of the bipedal walking control based on the neural oscillator coupled virtual model. The motion for keeping balance of the humanoid robot can be yielded properly in terms of the coupled model. In the simulation seen in Fig. 9, we verified the smooth and natural lifting motion regardless of the double or single support phase.

## V. Adaptive Motion under unknown Disturbance

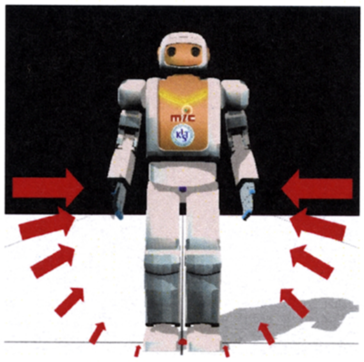


Fig. 10 Schematic figure on an unknown harmonic disturbance applied to the humanoid robot

In section V, in order to evaluate the inherent adaptive feature of neural oscillators, we investigate the proposed balance controller based on the neural oscillator coupled to the virtual components under two states of the humanoid robot with an unknown disturbance. Fig. 10 illustrates how to apply an external acceleration as a disturbance to the MAHRU robot. The size and direction of the red arrow indicate the magnitude and direction of the external acceleration as unknown disturbances, respectively as seen in Fig. 10. The unknown external acceleration is induced into the COM position of the MAHRU robot periodically as time.

We perform four cases in simulation as following cases I through IV. In figures 11, 12 15 and 16, it can be observed that the humanoid robot not coupled to the neural oscillator and virtual components tips over if there is an unknown disturbance. In the graphs of figures 12, 14, 16 and 18, the red thin line, the blue dashed line and the gray thick line indicate the calculated ZMP, the input of an external acceleration as an unknown disturbance and the COM position, respectively. Also, note that the black dotted line is the output of the neural oscillator and the red dash-dotted line is the desired ZMP input. The COM and the ZMP plots of figures 12 and 16 represent that the humanoid robot become unstable. Here the positive and negative signs of the moving distance imply the left and right direction of humanoid robot. Comparing these results with cases II and IV, it can be verified from figures 14 and 18 that the COM follows well the external disturbance regardless of the motion of the humanoid robot. This is caused by the neural oscillator coupled to the COM of the humanoid robot since the neural oscillator entrains the sensory signal fed from the COM motion of the humanoid robot.

*A. Case I: Under standing state on the spot without the proposed controller*

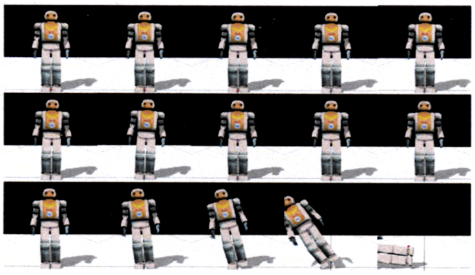


Fig. 11 Snap shots on the motion of the MAHRU robot under an unknown sinusoidal disturbance of 1.9G; G is set with 9.8 m/s$^2$

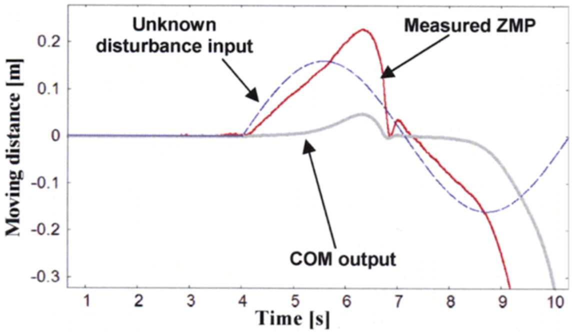



Fig. 12 The outputs of the COM position and the ZMP of the humanoid robot; The input of the external acceleration

*B. Case II: Under standing state on the spot with the proposed controller*

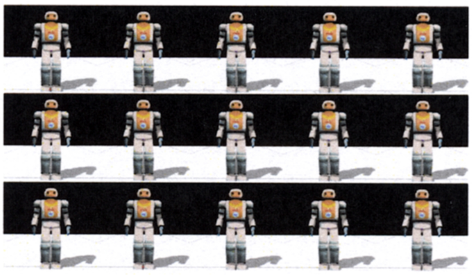


Fig. 13 Snap shots on the motion of the MAHRU robot under an unknown sinusoidal disturbance of 1.9G. G is set with 9.8 m/s$^2$

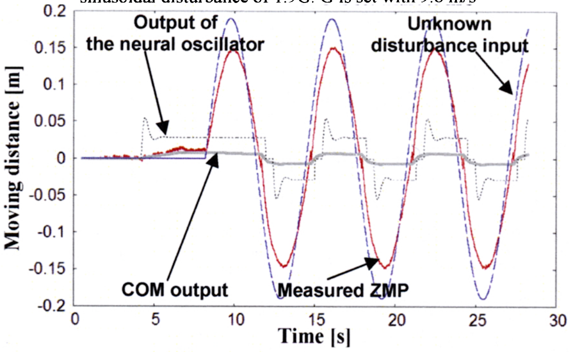



Fig. 14 The outputs of the COM position and the ZMP of the humanoid robot; The input of the external acceleration; The output of the neural oscillator

*C. Case III: Under rolling motion without the proposed controller*

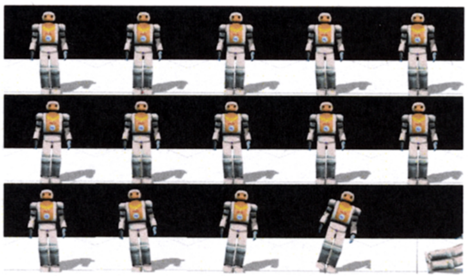


Fig. 15 Snap shots on the motion of the MAHRU robot under an unknown sinusoidal disturbance of 1.3G. G is set with 9.8 m/s$^2$

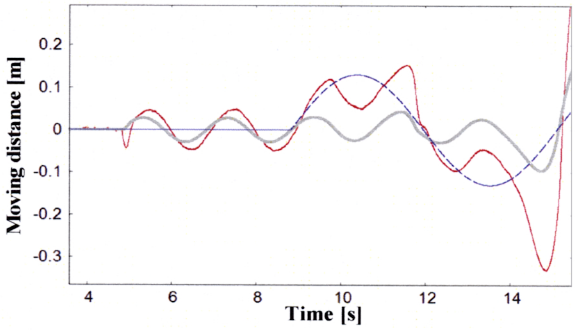



Fig. 16 The outputs of the COM position and the ZMP of the humanoid robot; The input of the external acceleration

*D. Case IV: Under rolling motion with with the proposed controller*

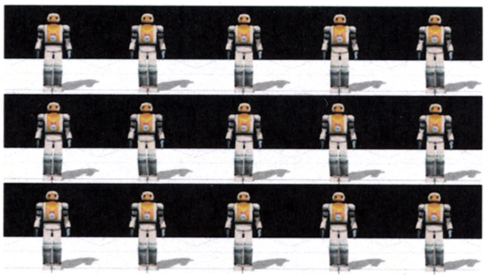


Fig. 17 Snap shots on the motion of the MAHRU robot under an unknown sinusoidal disturbance of 1.3G. G is set with 9.8 m/s$^2$

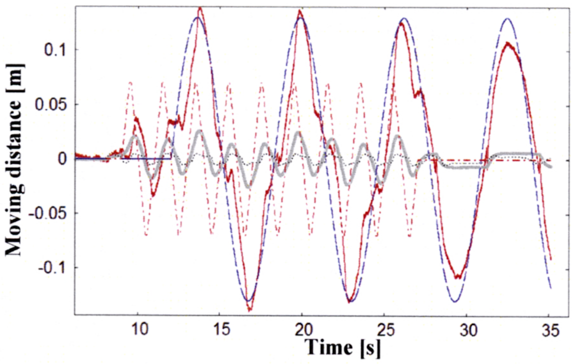



Fig. 18 The outputs of the COM position and the ZMP of the humanoid robot; The input of the external acceleration; The output of the neural oscillator

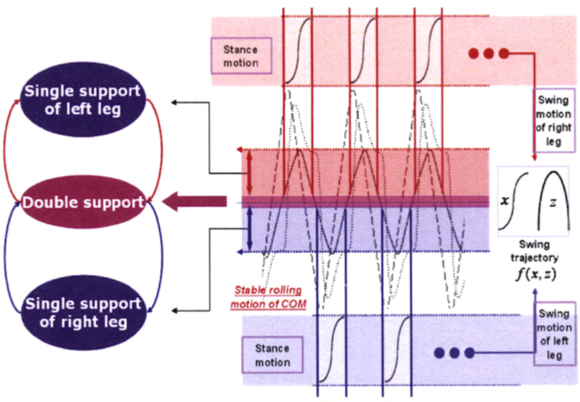
*E. Case V: Experimental verification*

Finally we implemented the proposed control method to the real humanoid robot. During a dancing motion under the double stance phase, unknown disturbances composed of two dumbbells of weigh 10kg are applied in the frontal and sagittal sides, respectively, as seen in Fig. 19. It is observed from the experiment that the humanoid robot is able to maintain the bipedal stability. Hence even though the external disturbance of a sinusoidal form effects to the humanoid robot, the humanoid robot coupled to the neural oscillator and virtual components can stably exhibit a novel adaptive motion corresponding to an unknown disturbance.

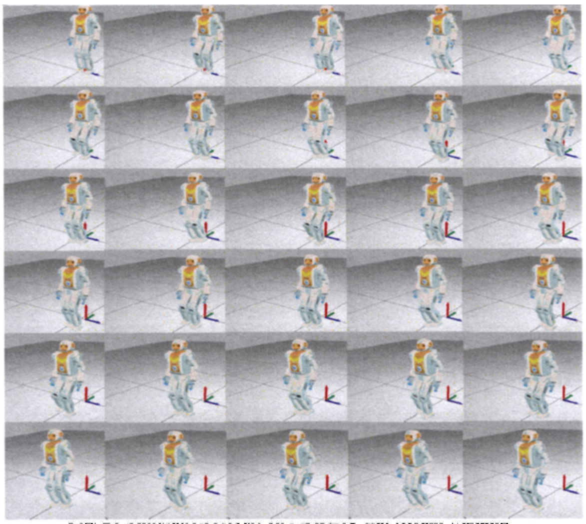
disturbances, Dotted circles indicate the dumbbells employed as unknown disturbances

## VI. Bipedal Locomotion Strategy

We finally propose the appropriate bipedal walking strategy of humanoid robots for our control model. Our objective is to embody the simple and integrated algorithm for various bipedal motions of humanoid robots. We also desire to have robust and adaptive properties under an unknown disturbance even in an unpredictable terrain. The proposed algorithm is well illustrated in Fig. 20.




The output of the COM motion in the rolling direction dominates the entire bipedal walking motion such as walking period, velocity, stride distance, etc.. The output of the COM motion causes the COM motion in the pitching direction. By this, the rolling motion and pitching motion of the COM are harmonized periodically. Also the output of the hip motion in the lateral plane determines the trajectory of the swing motion. Therefore, if the COM is inhibited by an unknown disturbance such as pushing or pulling forces, the neural oscillator properly generates the adapting motion. And then the swing and stance motions can be autonomously changed according to the altered COM motion. Ultimately the COM of a humanoid robot will be appropriately controlled sustaining the bipedal stability according to an arbitrary ZMP input. In addition, we exploit the cycloid function for the trajectory generation of the swing leg due to decreasing the acceleration in the pitching direction. Fig. 21 shows the stable walking of MAHRU robot on the flat terrain controlled in terms of the proposed walking algorithm.



## VII. Conclusions

To attain a stable periodic locomotion of biped humanoid robots, we proposed a new control architecture consisting of neural oscillators and virtual components. Specifically, the COM position was controlled to follow the time-varying desired ZMP input sustaining the bipedal stability. For this, we simply let the virtual components and the neural oscillator generate the Cartesian forces. Then, we could determine the required joint torques incorporating the Jacobian of the three link inverted pendulum. It is also noted that the appropriate swing and stance motion were generated according to the rolling and pitching motion. Since the stable rolling motion of the COM were induced by the ZMP reference input, which also properly generated the pitching motion of the COM in accordance with the rolling motion, the stable walking could be achieved. Extensive simulations were carried out to verify the effectiveness of the proposed method. Experiments are currently under way with a real robot for further evaluation of the proposed control method.

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