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Description	

# Self-configurable Mobile Robot Swarms with Hole Repair Capability

Geunho Lee and Nak Young Chong

**Abstract**—We address the problem of deploying a swarm of autonomous mobile robots toward building an *ad hoc* network of robotic sensors with spatial uniform density. For the purpose, each of the robots configures themselves into an area with geographical constraints through local interactions with two adjacent neighboring robots. The basic idea underlying this work is that robots can be thought of as liquid particles that change their positions conforming to the shape of the container they occupy. The main challenge is how to cope with the accuracy limitations of sensors and possible holes in the configuration. Considering such realistic conditions, the convergence of the proposed method is proved using Lyapunov's theorem. The proposed method is verified to be effective through the simulation for the secure deployments of robotic sensor network.

## I. INTRODUCTION

Recently, there has been increasing attention paid to swarm robotics, because it is possible to have a swarm of robots cover an area of interest for such applications as environmental or habitat monitoring, search-and-rescue, and exploration [1]. These applications require that robots adaptively configure themselves into the area controlling the individual robot's motion in a decentralized way [2], and be dispersed in a uniform spatial density without holes. Toward the end, we propose a geometric approach to self-configuration that enables a swarm of robots to adapt its shape to the geographically constrained plane with equilateral triangle lattices. Based on a partially connected mesh topology [14], the proposed approach can take advantage of the redundancy provided by a fully connected topology without the expense and complexity of networking processes.

What is important from the practical point of view is that we consider the problem of limitations in sensor technology and holes in the configuration process and/or node failure. Our main contribution lies in providing an effective approach against the measurement errors employing the Kalman filter [16], and the self-repair capability. By increasing the number of neighboring robots positioned at the uniform distance, the swarm repairs the holes and improves the network connectivity. Regarding the convergence of the proposed algorithms, Lyapunov's theorem is utilized, leading to asymptotic stability of the desired configuration from an arbitrary distribution. Both individual behavior of robots and overall shape of the swarm can be coordinated with scalability in this work.

Decentralized control for robot swarms can be broadly classified into global and local strategies according to whether sensors have range limits. Global strategies [3]

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may provide fast, accurate, and efficient deployment, but are technically non-feasible and lack scalability as the number of robots increases. On the other hand, local strategies are based on interactions between individual robots inspired by colonies of ants or schools of fish, or physical phenomena such as crystallization. Local strategies can further be divided into biological emergence [4][5], behavior-based [6], and virtual physics-based [7]-[13] approaches. Many of the behavior-based and virtual physics-based approaches used such physical phenomena as electric charges [7], gravitational forces [8], spring forces [9][12][13], potential fields [10], van der Waals forces [11], and other virtual models.

Robot swarm configurations achieved by the above-mentioned local interactions may result in lattice-type networks. These configurations offer high level coverage and multiple redundant connections ensuring maximum reliability and flexibility from the standpoint of topology. Depending on whether there are interactions among all robots, the network can be classified into fully and partially connected topologies [14]. The fully connected topologies have each robot interact with all of other robots simultaneously within a certain range. Thus, those approaches might over-constrain individual robots and frequently lead to deadlocks. On the contrary, using the partially connected topology, robots interact selectively with other robots, but are connected to all robots in the formation. For example, robots may choose to exert forces in a certain direction [12], where this selective interaction helps prevent them from being too tightly constrained. Due to similar reason, robots are enabled to achieve faster formation without deadlocks [13]. Using the partially connected topology, our work is to enable robot swarms to construct uniformly spaced equilateral triangles with a minimum number of interacting robots in an area with geographical constraints.

## II. PROBLEM STATEMENT

### A. Robot Model

We consider a swarm of mobile robots denoted as  $r_1, \dots, r_n$ . It is assumed that an initial distribution of all robots is arbitrary and distinct. Each robot autonomously moves on a 2-D plane. They have no leader and no identifiers, and do not share any common coordinate system, and do not retain any memory of past actions [3]. Due to limited sensing range, they can detect the position of other robots only within a certain range. In addition, each robot does not communicate explicitly with other robots. Let the position  $p_i$  of a robot  $r_i$  be denoted as a state vector

$$p_i = [p_{i,x} \ p_{i,y}]^T. \quad (1)$$

We can also define  $r_i$ 's kinematics by  $\dot{p}_{i,x} = u_i \cos \theta_i$ ,  $\dot{p}_{i,y} = u_i \sin \theta_i$ , where  $u_i$  and  $\theta_i$  are the translational and angular velocity of  $r_i$ , respectively. In addition, to provide an optimal estimation of noisy sensor measurements, a Kalman filter is employed [16]. Let's consider the system state vector  $p_{i,k}$ ,  $[p_{i,1,k} \ \cdots \ p_{i,n,k}]^T$ , defined as the positions of other robots at time  $k$ . The Kalman filter model describes the state transition from  $k$  to  $k+1$  as follows:

$$p_{i,k+1} = A_{i,k} p_{i,k} + \omega_{i,k} \quad (2)$$

where  $A_{i,k}$  is the state transition matrix given by an identity matrix and  $\omega_{i,k}$  means the system random noise  $\omega_{i,k} \sim N(0, W_{i,k})$ . The predicted estimate covariance can be obtained as follows:

$$P_{i,k+1|k} = A_{i,k} P_{i,k|k} A_{i,k}^T + W_{i,k} \quad (3)$$

where  $W_{i,k} = w_{i,k} I$ . (Do not confuse the notation for the  $r_i$ 's position  $p_i$  in (1) with that for the error covariance  $P_i$ .) Then, when  $r_i$  detects the position of  $r_j$  with respect to  $r_i$ 's local coordinate system, the measurement  $z_{i,j,k}$  is given by

$$z_{i,j,k} = p_{i,j,k} - p_{i,i,k} + v_{i,j,k} \quad (4)$$

where  $v_{i,j,k}$  is zero-mean white Gaussian random process. The estimated measurement would be  $\hat{z}_{i,j,k} = \hat{p}_{i,j,k} - \hat{p}_{i,i,k}$ . Since  $p_{i,i,k}$  and  $\hat{p}_{i,i,k}$  becomes zero, the error for the measurement  $\tilde{z}_{i,j,k}$  is given by

$$\tilde{z}_{i,j,k} = z_{i,j,k} - \hat{z}_{i,j,k} = \tilde{p}_{i,j,k} + v_{i,j,k} \quad (5)$$

where  $\tilde{p}_{i,j,k} = p_{i,j,k} - \hat{p}_{i,j,k}$ . Therefore, if  $r_i$  measures the position of other robots, the measurement error equations can be written as

$$\tilde{z}_{i,k} = H_{i,k} \tilde{p}_{i,k} + v_{i,k} \quad (6)$$

where  $v_{i,k}$  is the measurement random noise  $v_{i,k} \sim N(0, M_{i,k})$  and  $H_{i,k}$  is the measurement relation function matrix given by an identity matrix. The innovation covariance by the measurement sensor error is given by

$$S_{i,k} = H_{i,k} P_{i,k|k-1} H_{i,k}^T + M_{i,k} \quad (7)$$

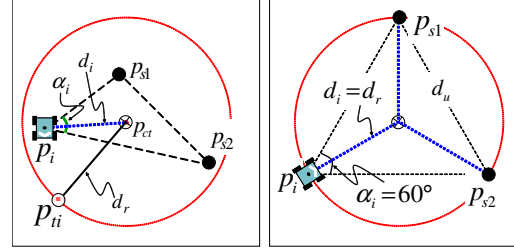
where  $M_{i,k}$  is the covariance of the relative state measurements expressed as  $m_{i,k} I$ . Thus, the filtered state estimate and the error covariance are obtained as follows:

$$\begin{aligned} K_{i,k+1} &= P_{i,k+1|k} H_{i,k+1}^T S_{i,k+1}^{-1} \\ \hat{p}_{i,k+1|k+1} &= \hat{p}_{i,k+1|k} + K_{i,k+1} z_{i,k+1} \\ P_{i,k+1|k+1} &= P_{i,k+1|k} - K_{i,k+1} S_{i,k+1}^{-1} K_{i,k+1}^T \end{aligned} \quad (8)$$

where  $K_{i,k+1}$ ,  $\hat{p}_{i,k+1|k+1}$ , and  $P_{i,k+1|k+1}$  are an optimal Kalman gain, an updated state estimate, and an updated estimate covariance, respectively.

### B. Notations and Problem Definitions

The distance between the robot  $r_i$ 's position  $p_i$  and the robot  $r_j$ 's position  $p_j$  is denoted as  $dist(p_i, p_j)$ . We define a uniform interval  $d_u$ , the desired distance between each robot in the configuration.  $r_i$  detects the position  $\{p_1, p_2, \dots\}$  of other robots located within its sensing boundary  $SB$ ,



(a) range and bearing (b) equilateral triangle  
Fig. 1. Two control parameters in local interaction

yielding a set of the positions  $O_i$  with respect to its local coordinates. Next,  $r_i$  can select two robots  $r_{s1}$  and  $r_{s2}$  within  $r_i$ 's  $SB$  that we call the neighbors of  $r_i$  and denote their positions,  $\{p_{s1}, p_{s2}\}$ , as  $N_i$ . Given  $p_i$  and  $N_i$ , the *Triangular Configuration*, denoted by  $\mathbb{T}_i$ , is defined as a set of three distinct positions  $\{p_i, p_{s1}, p_{s2}\} = \mathbb{T}_i$ , where the internal angle  $\angle p_{s1} p_i p_{s2}$  of  $r_i$  is denoted by  $\alpha_i$ . Now we define the *Equilateral Configuration*, denote by  $\mathbb{E}_i$ , as a configuration that all the distance permutations of  $\mathbb{T}_i$  are equal to  $d_u$ . We need a measure indicating to which degree  $\mathbb{T}_i$  is configured into  $\mathbb{E}_i$ . Given  $\mathbb{T}_i$ , we can express the distance permutations with respect to  $r_i$  as the following matrix  $\mathbf{D}_i$ .

$$\mathbf{D}_i = \begin{cases} (dist(p_m, p_n) - d_u)^2 & \text{if } m \neq n \\ 0 & \text{otherwise} \end{cases} \quad (9)$$

where  $\{\{p_m, p_n\} | p_m, p_n \in \mathbb{T}_i = \{p_i, p_{s1}, p_{s2}\}\}$ . We will denote  $(dist(p_m, p_n) - d_u)^2$  for simplicity as  $(d_k - d_u)^2$ . Using  $\mathbb{T}_i$  and  $\mathbb{E}_i$ , we can formally define the local interaction as follows: Given  $\mathbb{T}_i$ , *Local Interaction* is to have  $r_i$  maintain  $d_u$  with  $N_i$  at each time instant toward forming  $\mathbb{E}_i$ . Based on the local interaction, we formally address the ADAPTIVE SELF-CONFIGURATION PROBLEM.

*Given a swarm of robots with arbitrarily distinct positions in a 2-D plane, how to enable the robots to configure themselves into  $\mathbb{E}_i$  while conforming to the plane border.*

### III. LOCAL INTERACTION

The local interaction algorithm enables three neighboring robots to generate an equilateral triangle of side length  $d_u$ , consisting of the function  $\varphi_{interaction}$  whose arguments are  $p_i$  and  $N_i$  at each time. Consider  $r_i$  and its two neighbors  $r_{s1}$  and  $r_{s2}$  located within  $r_i$ 's  $SB$ . As shown in Fig. 1-(a), three robots are configured into  $\mathbb{T}_i$  whose vertices are  $p_i$ ,  $p_{s1}$ , and  $p_{s2}$ , respectively. First,  $r_i$  finds the centroid  $p_{ct}$  of the triangle  $\triangle p_i p_{s1} p_{s2}$  with respect to its local coordinates, and measures the angle  $\phi$  between the line connecting two neighbors and  $r_i$ 's horizontal axis. Using  $p_{ct}$  and  $\phi$ ,  $r_i$  calculates the target point  $p_{ti} = (p_{ti,x}, p_{ti,y})$  by the following equations

$$(p_{ct,x} + d_u \cos(\phi + \pi/2)/\sqrt{3}, p_{ct,x} + d_u \sin(\phi + \pi/2)/\sqrt{3}).$$

$r_i$  attempts to form an isosceles triangle with its two neighbors at each time. By repeatedly doing this, three robots configure into  $\mathbb{E}_i$  as illustrated in Fig. 1-(b).

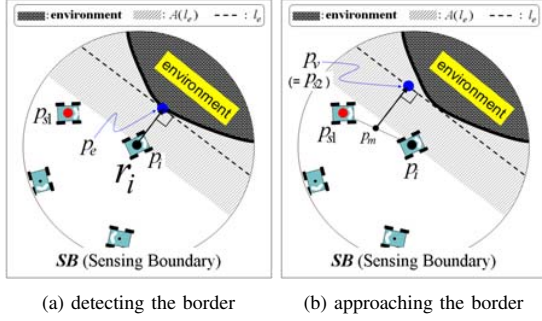


Fig. 2. Robots on the boundary of the swarm interact with the border

Now let's consider the circumscribed circle of an equilateral triangle whose center is  $p_{ct}$  of  $\triangle p_i p_{s1} p_{s2}$  and radius is  $d_r$ . The position of robots can be modeled by the distance from  $p_{ct}$  and the internal angle (see Fig. 1-(b)). The distance is controlled by the following equation

$$\dot{d}_i(t) = -a(d_i(t) - d_r), \quad (10)$$

where  $a$  is a positive constant and  $d_r$  represents  $d_u/\sqrt{3}$ . Indeed, the solution of (10) is  $d_i(t) = |d_i(0)|e^{-at} + d_r$  that converges exponentially to  $d_r$  as  $t$  approaches infinity. Next, the internal angle is controlled by the following equation

$$\dot{\alpha}_i(t) = k(\beta_i(t) + \gamma_i(t) - 2\alpha_i(t)), \quad (11)$$

where  $k$  is a positive constant. Because the total internal angle of a triangle is  $180^\circ$ , (11) can be re-written as

$$\dot{\alpha}_i(t) = k'(60^\circ - \alpha_i(t)), \quad (12)$$

where  $k'$  is  $3k$ . The solution of (12) is  $\alpha_i(t) = |\alpha_i(0)|e^{-k't} + 60^\circ$  that converges exponentially to  $60^\circ$  as  $t$  approaches infinity.

Note that (10) and (12) imply that three robots eventually form an equilateral triangle of side length  $d_u$ . In order to show the convergence, we will take advantage of Lyapunov stability theory [15]. Consider the following scalar function

$$f_{l,i} = \frac{1}{2}(d_i - d_r)^2 + \frac{1}{2}(60^\circ - \alpha_i)^2 \quad (13)$$

that is always positive definite except  $d_i \neq d_r$  and  $\alpha_i \neq 60$ . The derivative of the scalar function is given by

$$\dot{f}_{l,i} = -(d_i - d_r)^2 - (60^\circ - \alpha_i)^2, \quad (14)$$

which is obtained using (10) and (12). Eq. (14) is negative definite. The scalar function  $f_{l,i}$  is radially unbounded since it tends to infinity as  $\|\mathbf{x}\| \rightarrow \infty$ . Therefore, the equilibrium state is asymptotically stable, implying that  $r_i$  reaches a vertex of  $\mathbb{E}_i$ .

#### IV. ADAPTIVE SELF-CONFIGURATION

Adaptive self-configuration is decomposed into self-configuration and uniform conformation, each of which is solved based on the local interaction. If detecting the plane border within its  $SB$  as illustrated in Fig. 2-(a),  $r_i$  defines a point  $p_e$  projected from  $p_i$  onto the surface with the minimum distance  $d_e$  and then computes the tangent  $e'(t)$  to the surface

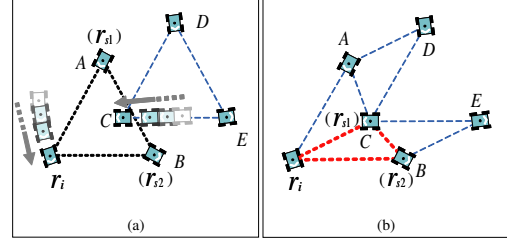


Fig. 3. Dynamically changing neighbors (a)  $r_i$  moves toward  $p_{ti}$ , (b)  $r_i$  selects new neighbors according to the minimum perimeter condition)

at  $p_e$ . (For convenience,  $l_e$  will be used instead of  $e'(t)$ .) It is obvious that  $l_e$  is perpendicular to the vector  $\overline{p_i p_e}$ , termed the surface direction. Let  $A(l_e)$  denote the area between the border and the line passing through  $p_i$  and parallel to  $l_e$  within  $SB$ . In order to determine whether  $r_i$  needs to interact with the surface,  $r_i$  checks if no neighbors exist in  $A(l_e)$  or if  $d_e \leq \frac{\sqrt{3}d_u}{2}$ . If the condition is satisfied,  $r_i$  executes the uniform conformation algorithm, otherwise, executes the self-configuration algorithm.

##### A. Self-configuration

Self-configuration is related to deploying a swarm of robots into equilateral triangle lattices based on the local interaction. In order to form a triangle,  $r_i$  selects the first neighbor  $r_{s1}$  located the shortest distance from itself. The second neighbor  $r_{s2}$  is selected such that the total distance from  $p_{s1}$  to  $p_i$  passing through  $p_{s2}$  is minimized. Then,  $r_i$  forms  $\mathbb{T}_i$  with  $N_i$ , and computes the target point  $p_{ti}$  by  $\varphi_{interaction}$ . Self-configuration enables a robot swarm to have a multitude of equilateral triangular lattices, denoted by  $\sum_{i=1}^n \mathbb{E}_i$ . Specifically,  $r_i$  dynamically changes the neighbors within  $SB$  at each time, enabling the robots to configure themselves without having adjacent triangles partly overlapping each other (see Fig. 3).

Now we examine the effect of changing neighbors in configuring  $\mathbb{E}_i$ . We use Lyapunov's theory with a scalar function given by

$$f_{sc,i} = \sum_{\mathbb{T}_i} (d_k - d_u)^2 + f_{l,i} \quad (15)$$

where  $f_{l,i}$  is given by (13) and  $\sum_{\mathbb{T}_i} (d_k - d_u)^2$  is defined as the constant value associated with  $\mathbb{T}_i$  at each time (see (9)). Thus, from (13), the scalar function of (15) is always positive definite except  $d_i \neq d_r$  and  $\alpha_i \neq 60$ . (If  $\mathbb{T}_i$  is equal to  $\mathbb{E}_i$ , it is easily seen that  $\sum_{\mathbb{T}_i} (d_k - d_u)^2$  reaches 0, resulted from  $d_r = d_u/\sqrt{3}$ .) The derivative of the scalar function is given by

$$\dot{f}_{sc,i} = \dot{f}_{l,i} = -(d_i - d_r)^2 - (60^\circ - \alpha_i)^2. \quad (16)$$

Eq. (16) is negative definite. Finally, the scalar function  $f_{sc,i}$  is radially unbounded since it tends to infinity as  $\|\mathbf{x}\| \rightarrow \infty$ . Therefore, the equilibrium state is asymptotically stable, implying that  $r_i$  reaches a vertex of  $\mathbb{E}_i$  from an arbitrary  $\mathbb{T}_i$ .

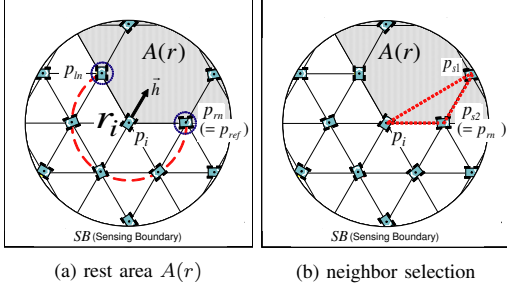


Fig. 4. Illustration of the self-reparation algorithm

Now we show the convergence property for a swarm of  $n$  robots. The  $n$ -order scalar function  $\mathbf{F}_{sc}$  is defined as

$$\mathbf{F}_{sc} = \sum_{i=1}^n f_{sc,i} = \sum_{i=1}^n \sum_{\mathbb{T}_i} (d_k - d_u)^2 + \sum_{i=1}^n f_{l,i}. \quad (17)$$

It is straightforward to verify that  $\mathbf{F}_{sc}$  is positive definite and  $\dot{\mathbf{F}}_{sc}$  is negative definite.  $\mathbf{F}_{sc}$  is radially unbounded since it tends to infinity as  $t$  approaches infinity. Consequently, a swarm of  $n$  robots converges into  $\mathbb{E}_i$  for their  $N_i$ .

### B. Uniform Conformation

Now we describe how to enable a swarm of robots to conform to the plane border. This means that the positions on the border (or the virtual static robots) need to be incorporated into  $\mathbb{T}_i$ . If the border is detected by  $r_i$ , it defines  $r_{s1}$  located at the shortest distance and check the conditions whether no other neighbors exist in  $A(l_e)$  or  $d_e \leq \frac{\sqrt{3}d_u}{2}$ . If the condition is satisfied,  $r_i$  computes the midpoint  $p_m$  of  $\overline{p_i p_{s1}}$  that is projected onto  $l_e$  and defined as  $p_v$  (see Fig. 2-(b)). Now  $p_v$  is considered as  $p_{s2}$ , and  $N_i$  is defined as  $\{p_{s1}, p_v\}$ . It is readily evident that  $r_i$  can compute  $p_{ti}$  by  $\varphi_{interaction}$ .

Note that  $p_v$  is a virtual, static point. Furthermore, it is difficult to identify the border since each robot has their  $SB$ . Therefore, it is almost impossible for them to exactly form  $\mathbb{E}_i$ . To take this effect into account, we introduce another measure for  $\mathbb{T}_i$  given by

$$\mathbf{D}_{i,e} = \begin{bmatrix} 0 & (dist(p_i, p_{s1}) - d_u)^2 \\ (dist(p_i, p_{s2}) - d_u)^2 & 0 \end{bmatrix}. \quad (18)$$

$\mathbf{D}_{i,e}$  is applied only when any robots located in close to the plane border interact with  $p_v$ .

## V. SELF-REPARATION

Robots attempt to reach a uniform spatial density, but probably holes remain in a converged distribution. This is because each robot determines their direction of movement based on the current position of neighbors. To repair the holes,  $r_i$  changes its neighbors. Let  $P_u$  denote the set of robot positions located within the range of  $d_u$ .  $r_i$  defines its heading  $\vec{h}$  with respect to the local coordinates. Let  $ang(\vec{m}, \vec{n})$  denote the angle between two arbitrary vectors  $\vec{m}$  and  $\vec{n}$ . As shown in Fig. 4-(a),  $r_i$  selects the reference neighbor  $p_{ref}$  in  $P_u$  such that the value of  $ang(\vec{h}, \overline{p_i p_{ref}})$

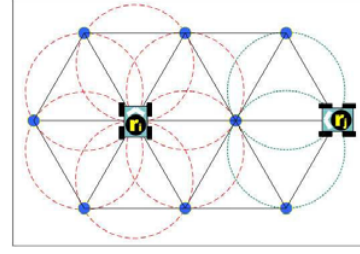


Fig. 5.  $r_i$  and  $r_j$  form  $\sum_{m=1}^6 (\mathbb{E}_i)_m$  and  $\sum_{m=1}^2 (\mathbb{E}_j)_m$ , respectively.

is minimized.  $r_i$  then checks if any neighbor exists in the area obtained by rotating  $\overline{p_i p_{ref}}$  60 degrees clockwise. If there exists one,  $r_i$  checks the next neighbor by sweeping another 60 degree clockwise.  $r_i$  continues to check until it finds a hole, then the last neighbor is defined as  $p_{ln}$ . Similarly,  $r_i$  attempts to find neighbors by rotating  $\overline{p_i p_{ref}}$  counterclockwise and locate the last neighbor  $p_{rn}$ . The reparation area  $A(r)$  is defined as the area between  $\overline{p_i p_{rn}}$  and  $\overline{p_i p_{ln}}$  in  $SB$ , where no element of  $P_u$  exists. As illustrated in Fig. 4-(b),  $r_i$  selects the first neighbor located the shortest distance away from  $p_i$  in  $A(r)$  as  $p_{s1}$ . The second position is defined such that the total distance from  $p_{s1}$  to  $p_i$  can be minimized through either  $p_{rn}$  or  $p_{ln}$ . As a result,  $p_{ti}$  can be determined by  $\varphi_{interaction}$ .

Like the surface tension of liquids caused by intermolecular forces, as illustrated in Fig. 5, the self-reparation will have each robot attempt to reach the maximum possible number of desired configurations  $\mathbb{E}_i$  within  $SB$  given by

$$\max \left[ \sum_{m=1}^s (\mathbb{E}_i)_m \right] \quad (19)$$

where  $s$  is greater than or equal to 1 and less than or equal to 6, as the desired configuration is a hexagon composed of 6 equilateral triangle lattices. Therefore, a collective configuration reaches a swarm of robots with  $\max[\sum (\mathbb{E}_i)]$  while filling up holes.

To obtain  $\max[\sum_{m=1}^s (\mathbb{E}_i)_m]$ , our algorithm changes the neighbors according to the condition whether  $r_i$  forms  $\mathbb{E}_i$  at any time. Therefore, we can modify (15) as follows:

$$f_{sr,i} = \begin{cases} \sum_{m=1}^c (f_{l,i})_m + f_{sc,i} & \text{if } \mathbb{T}_i = \mathbb{E}_i \\ f_{sc,i} & \text{otherwise} \end{cases} \quad (20)$$

where  $f_{sc,i}$  is given in (15) and  $c$  is less than  $\max[s]$ . Using (20), (19) can be re-written as

$$f_{sr,i} = \min \left[ \sum_{m=1}^s (f_{l,i})_m \right]. \quad (21)$$

Here, (21) enables  $r_i$  to reach the minimum energy level by maximizing the number of  $\mathbb{E}_i$ .

Next, let the internal energy, increasing or decreasing during self-configuration, be denoted as  $q_i$  given by

$$q_i = \sum_{m=1}^c (f_{l,i})_m. \quad (22)$$

When  $\mathbb{T}_i$  is equal to  $\mathbb{E}_i$ ,  $q_i$  forces  $r_i$  to locally interact toward forming another equilateral triangular lattice by changing neighbors. We assume that  $q_i$  starts with any nonnegative value  $q_i(0)$  and evolves according to the following equation:

$$\dot{q}_i = \sum_{m=1}^c (\dot{f}_{l,i})_m. \quad (23)$$

Note that  $q_i$  forces  $r_i$  to minimize  $f_{sc,i}$ . If  $q_i$  decreases, we can predict that  $r_i$  moves toward  $\min[\sum_{m=1}^s (f_{l,i})_m]$ . By doing this repeatedly, the holes will get eliminated.

To show that each robot converges into  $\max[\sum_{m=1}^s (\mathbb{E}_i)_m]$  while increasing the number of neighboring robots after a finite number of activation steps, we use Lyapunov's theory and show the convergence of  $r_i$  using (20) and (22), with a scalar function defined as:

$$f_{sr,i} = f_{sc,i} + q_i. \quad (24)$$

Recall that  $q_i(0)$  is initialized to a nonnegative value and evolves according to (23). Moreover,  $q_i$  is defined in such a way that it increases when  $f_{l,i}$  lacks. Whenever  $\mathbb{T}_i = \mathbb{E}_i$ ,  $q_i$  is set to  $q_i(0)$ . On the other hand,  $f_{sc,i}$  is positive definite by (15). Since  $f_{sc,i} > 0$  and  $q_i > 0$ , it is clear that  $f_{sr,i} > 0$ . Next, differentiating  $f_{sr,i}$  gives

$$\dot{f}_{sr,i} = \dot{f}_{sc,i} + \dot{q}_i. \quad (25)$$

$$(\dot{f}_{sr,i} = \dot{f}_{sc,i} + \sum_{m=1}^c (\dot{f}_{l,i})_m = \dot{f}_{l,i} + \sum_{m=1}^c (\dot{f}_{l,i})_m)$$

which can be simplified to

$$\dot{f}_{sr,i} = \sum_{m=1}^s (\dot{f}_{l,i})_m. \quad (26)$$

It is easy to see that  $\dot{f}_{sr,i}$  is negative definite. Moreover, the scalar function  $f_{sr,i}$  is radially unbounded since it tends to infinity as  $\|\mathbf{x}\| \rightarrow \infty$  even though  $q_i$  remains a positive constant. Therefore, based on Lyapunov's theory, the position of  $r_i$  converges into  $\sum_{m=1}^s (\mathbb{E}_i)_m$ .

Now we show the convergence property for a swarm of  $n$  robots. The  $n$ -order scalar function  $\mathbf{F}_{sr}$  is defined as

$$\mathbf{F}_{sr} = \sum_{i=1}^n f_{sr,i} = \sum_{i=1}^n f_{sc,i} + \sum_{i=1}^n q_i. \quad (27)$$

It is straightforward to verify that  $\mathbf{F}_{sc}$  is positive definite. Next, differentiating  $\mathbf{F}_{sc}$  gives

$$\dot{\mathbf{F}}_{sr} = \sum_{i=1}^n \dot{f}_{sc,i} + \sum_{i=1}^n \dot{q}_i. \quad (28)$$

$\dot{\mathbf{F}}_{sr}$  is negative definite and radially unbounded since it tends to infinity as  $t$  approaches infinity. Consequently, a swarm of  $n$  robots converges into  $\sum_{i=1}^n (\max[\sum_{m=1}^s (\mathbb{E}_i)_m])$ .

## VI. SIMULATION RESULTS

We performed simulations to investigate the convergence and robustness properties of the proposed algorithm. The standard deviation of the error in the sonar readings is assumed to be 10%. Our algorithm terminates when all robots converge into the distance  $d_u \pm 1\%$  with their neighbors.

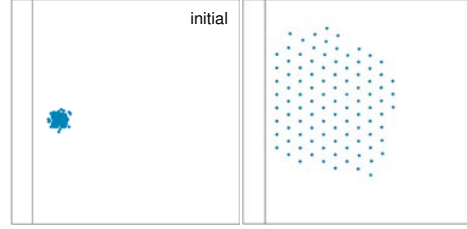


Fig. 6. Dispersion of 100 robots over a flat surface

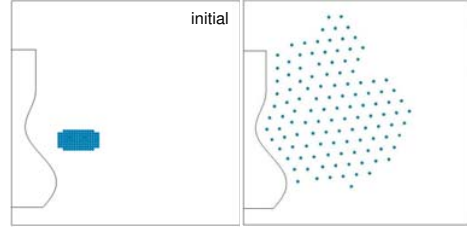


Fig. 7. Dispersion of 120 robots over a curved surface

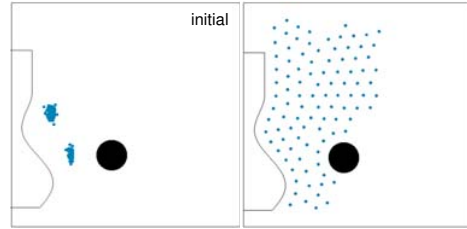
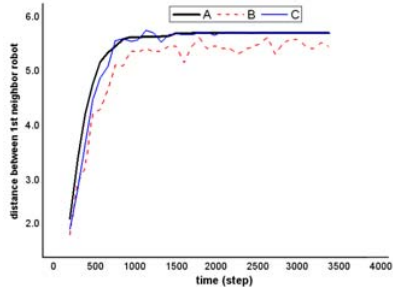


Fig. 8. Dispersion of 100 robots over a curved surface with an obstacle

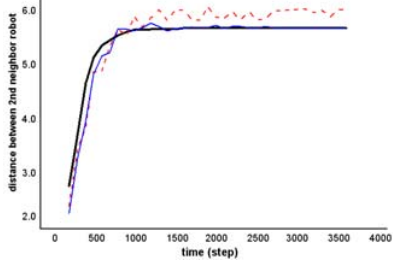
Fig. 6 shows that 100 robots configure themselves into a uniform density pattern over a flat surface. It is observed that the robots could converge into  $\mathbb{E}_i$  conforming to the surface. Fig. 7 demonstrates that 120 robots configured themselves into a curved surface. Most of the robots that do not detect the surface conformed to it through the local interaction. Fig. 8 presents a more complex geographic condition with a circular obstacle. It is also observed that the robots could converge into  $\mathbb{E}_i$ , conforming to the surfaces of the obstacle as well as the border.

Fig. 9 shows the mean values of distance between 120 robots and their  $N_i$  in Fig. 7 according to the iteration step. Here, the black bold line, the blue line, and the red dashed line indicate the case of no sensor error, the Kalman filtered sensor readings, and non-filtered sensor readings, respectively. As can be seen from the figure, each robot could converge into  $\mathbb{E}_i$  in spite of measurement errors.

Finally, robustness is verified against robot failures under the same surface condition as Fig. 7. Five robots unexpectedly failed in Fig. 10-(a), and the same number of holes appeared in the deployment. Each robot checks the existence of holes within  $SB$ . If there are holes around it, the algorithm allows each of the robots change their position to fill the holes. Fig. 10-(b) presents the results of redeployment with 95 robots. In addition, the lost robots were replaced by



(a) distance variation between  $r_i$  and  $r_{s1}$



(b) distance variation between  $r_i$  and  $r_{s2}$

Fig. 9. Distance variations between  $r_i$  and  $N_i$  during self-configuration ('A'-line: no sensor error, 'B'-line: non-filtered, 'C'-line: Kalman filtered)

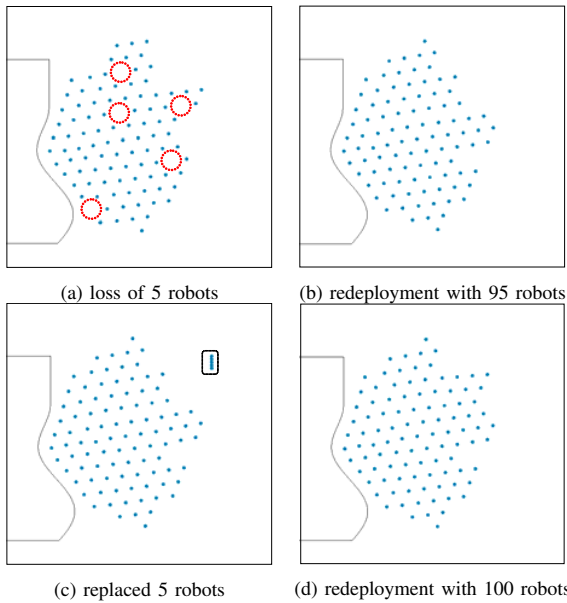


Fig. 10. Robustness against loss of 5 robots over a curved surface

the same number of new robots in Fig. 10-(c). Fig. 10-(d) shows the results of redeployment with 100 robots. From the results, the adaptive self-configuration algorithm has proven effective in improving the robustness of uniform coverage over a designated area.

## VII. CONCLUSION

The adaptive self-configuration problem was addressed to disperse a swarm of robots in a geographically constrained plane. From the practical point of view, we did not use

robot identifiers, common coordinates, global orientation, and direct communication. Robots computed their position without requiring memories of past actions or states, helping cope with transient errors. Also, sensors were subject to strict range and accuracy limitations. Under such conditions, we proposed distributed algorithms that enable robots to configure themselves into triangular lattices adapting to the environment. Among all the possible types of regular polygons, the equilateral triangle lattice can reduce the computational burden and is highly scalable, and less influenced by neighboring robots. The proposed local interaction, where robots were allowed to interact with only two dynamically selected neighbors, is computationally efficient, as they utilize only position information of other robots. By collecting such local behavior of each robot, a uniformly spaced swarm of robots was organized to fill in the environment. Moreover, the proposed algorithm improved the robustness of uniform coverage control over a designated area against robot failures. The convergence of the algorithm was proven mathematically, and also verified through extensive simulations.

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