Termination of Higher-Order Rewrite Systems

Munehiro Iwami
School of Information Science
Japan Advanced Institute of Science and Technology

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Abstract

Higher-order rewriting is a natural extension of first-order term rewriting systems to reason with higher-order equations. Higher-order rewrite systems have been widely used as a model of higher-order functional and logic programming languages and as a basis of higher-order theorem provers.

In term rewriting systems, several sufficient conditions for proving the termination property have been successfully developed in particular cases. These techniques can be classified into two approaches: semantic methods and syntactic methods. Simplification orderings are representatives of syntactic methods. Many simplification orderings, for instance, the recursive path ordering, the improved recursive decomposition ordering and so on, have been proposed for term rewriting systems. The improved recursive decomposition ordering is one of the most powerful simplification orderings.

In higher-order rewrite systems, Jouannaud and Rubio gave a definition of recursive path ordering for higher-order rewrite systems by introducing an ordering on type structure recently.

In this thesis, we study the termination of higher-order rewrite systems by syntactic approaches.

First, we extend the improved recursive decomposition ordering to higher-order rewrite systems for proving termination. Our extension is inspired from Jouannaud and Rubio's idea. This ordering is called the *higher-order improved recursive decomposition ordering*. Further, we show that this ordering is more powerful than the Jouannaud and Rubio's ordering.

Next, we introduce the notion of *simplification orderings* for higher-order rewrite systems. More precisely, we define the simplification ordering on algebraic terms where an algebraic term is η-long β-normal form without λ-abstractions. By this definition, we can analyze the termination of higher-order rewrite systems in abstract level. Further, we define a new recursive path ordering for higher-order rewrite systems, called the *higher-order recursive path ordering*. Our ordering extends Jouannaud and Rubio's ordering, which does not allow comparing two type incompatible terms. We show through several

examples that our ordering can be applied to prove termination of higher-order rewrite systems to which Jouannaud and Rubio's ordering cannot be applied.

Finally, we extend the persistent property of termination to order sorted term rewriting systems. First-order term rewriting systems are special cases of higher-order rewrite systems. So we analyze the termination of first-order term rewriting systems using the notion of persistence. Zantema showed that termination is persistent for term rewriting systems without collapsing or duplicating rules. We show that Zantema's result can be extended to order sorted term rewriting systems, i.e, termination is persistent for order sorted term rewriting systems without collapsing, decreasing or duplicating rules.

Key Words: **higher-order rewrite system, term rewriting system, termination, simplification ordering, improved recursive decomposition ordering, recursive path ordering, persistence**