Efficient Algorithms for Geometric Graph Classes

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Abstract

It is said that every NP-hard problem has no efficient algorithm. However, many NP-hard problems on general graphs can be solved efficiently if we restrict graphs to a geometric graph class. For example, interval graphs form one of the geometric graph class. Coloring problem which is well known NP-hard problem can be solved in linear time on interval graphs. A variety of geometric graph classes have been proposed and studied. In this paper, we treat with some problems for geometric graph classes. These problems are random generation, enumeration, and graph reconstruction, mainly.

We treat with unlabeled graphs to avoid redundancy. We propose random generation and enumeration algorithms for connected proper interval graphs. We use counting for random generation algorithms, so we first give the number of connected proper interval graphs of n vertices. Based on the number, we present a simple algorithm that generates a connected proper interval graph uniformly at random up to isomorphism. Next we propose an enumeration algorithm of connected proper interval graphs. This algorithm is based on the reverse search, and it outputs each connected proper interval graph in O(1) time. Then we propose random generation and enumeration algorithms for connected bipartite permutation graphs. These algorithms are extension of the algorithms of proper interval graphs.

The graph reconstruction conjecture is a long-standing open problem in graph theory. There are many algorithmic studies related it besides mathematical studies, such as deck checking, legitimate deck, preimage construction, and preimage counting. We study these algorithmic problems limiting the graph classes to interval graphs, permutation graphs, and distance-hereditary graphs. Since we can solve graph isomorphism problem for these graph classes in polynomial time, deck checking for these graph classes are easily done in polynomial time. Since the number of interval graphs that can be obtained from a graph by adding a vertex and edges incident to it can be exponentially large, developing polynomial time algorithms for legitimate deck, preimage construction, and preimage counting on these graphs are not trivial. We present that these problems are solvable in polynomial time on these graph classes.

Key Words: **graph algorithm, geometric graph class, counting, random generation, enumeration, graph reconstruction**