Nonclassical logics with identity connective and their algebraic characterization

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Abstract

In this thesis, we investigate various kinds of nonclassical logics by the property of identity connective. Around 1970, R. Suszko proposed the sentential calculus with identity (**SCI** for short) to realize some philosophical ideas of L. Wittgenstein's *Tractatus*. In **SCI**, besides the logical value, he formalized the *referent* of sentences by using identity connective. Inspired by his idea, we introduce a weak system, i.e., propositional calculus with identity (**PCI** for short), which is obtained from **SCI** by deleting two axioms which express the reflexivity and transitivity of identity. As an extension of the *simulation* property of **SCI**, we reconstruct various kinds of nonclassical logics on **PCI**, including two types of logics, namely classical logics with additional operators and weak logics with various kinds of weak implications, e.g., strict/relevance/linear implication. In fact, in this thesis we show that the following logics can be translated to some extensions of **PCI**; classical modal logics **K**, **KT**, **KB**, **K4**, **KD**, **K5**, **S4** and **S5** with necessary operator $\Box$, Angell's analytic containment logic **AC** with relevance entailment $\rightsquigarrow$, Corsi's weak logic **F** with strict implication $\rightharpoonup$ and Girard's classical linear logic **GL** with linear implication $\supset$. In particular, the modal logic **K** is shown to be translated into an extension $\mathbf{PCI}_{\mathrm{K}}$ of **PCI**. Then we will focus on the algebraic property of $\mathbf{PCI}_{\mathrm{K}}$-algebras, which offer the algebraic semantics of extensions of $\mathbf{PCI}_{\mathrm{K}}$. We will give a necessary and sufficient condition for a subvariety of $\mathbf{PCI}_{\mathrm{K}}$-algebras to have *equationally definable principal congruences* (EDPC for short) property.

Keywords : EDPC, identity connective, nonclassical logic, non-Fregean logic, SCI, Suszko, PCI