Algebras and Frames for Modal Logics

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Abstract

In this thesis, we investigate modal logics semantically by using both algebraic semantics and general Kripke type semantics. We will discuss several topics on modal logic. Though the topic varies, there is a unique underlying motif through the whole thesis, i.e. *the duality between algebras and frames*.

Kripke type semantics for modal logics has made a great success in these years. This is mainly due to the fact that Kripke type semantics offers us intuitively comprehensible and easily manageable, mathematical models for modal logics. On the other hand, while algebraic structures lack these properties, they have one quite important merit which Kripke type semantics lacks. That is, every modal logic is complete with respect to algebraic semantics.

To supplement this defect, Kripke type semantics based on *general Kripke frames* was introduced. This semantics bridges between original Kripke type semantics and algebraic semantics. In fact, by the Stone duality, we have a nice correspondence between general Kripke frames and algebras. Through this duality, it becomes possible to get important results on general Kripke frames from results on the corresponding class of algebras, which are obtained by using the fruits of universal algebra.

The first topic of our thesis is pseudo-Euclidean logics. For fixed non-negative integers m and n, let E_k be the logic which is obtained from the smallest normal (classical) modal logic **K** by adding the axiom $\Diamond^k\phi \to \Box^m\Diamond^n\phi$, where $k \geq 0$. We will give a complete answer to the question when $E_k \supseteq E_{k'}$ holds.

Second, we discuss intuitionistic modal logics. For Kripke type semantics, we discuss *finite model property* of intuitionistic modal logics by *filtration method*. For algebraic semantics, we have succeeded to give a description of subdirectly irreducible algebras for various kinds of modal Heyting algebras. By using the duality theory, this result can be translated into a result on a description of irreducible (finite) Kripke frames.

Finally, we introduce a new type of products of modal logics, called *normal products*. Normal products resemble products familiar to researcher of measure theory and topology, and are defined as a generalization of *products of algebras of sets*. Our products of modal logics can be defined either by means of *normal products of general frames*, or by means of *normal products of modal algebras*. Since our notion of products is based highly on the duality theory, it has such a nice property as follows; the product of two general frames is isomorphic to the dual of the product of the corresponding dual algebras. This brought us a desired effect that the definition of the normal product of modal logics L_1 and L_2 is not affected by the choice of classes of general frames (or, modal algebras) which determine L_1 and L_2. Note that this is not the case for usual products of modal logics.

The notion of normal products is quite natural from the view point of duality theory. Therefore this enables us to extend the notion of products to other logics like intuitionistic modal logics.

Key Words: **modal logics, intuitionistic modal logics, duality between algebras and frames, subdirectly irreducible algebras, products of modal logics**