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Description	

Power Allocation for Irregularly Modulated MIMO Signaling with Iterative Frequency Domain Detector

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***Abstract*—A new power allocation method for irregularly modulated signaling in single carrier point-to-point multiple input multiple output (MIMO) systems with an *iterative* frequency-domain (FD) *soft* cancellation (SC) minimum mean squared error (MMSE) equalization is proposed. The proposed method explicitly takes into account the convergence properties of the iterative equalizer while transmission power is minimized. The proposed scheme is based on the combination of irregular modulation, different signal constellations within one code word, and convergence constraint precoding (CCP), technique that decouples the spatial interference between streams using singular value decomposition (SVD), and minimizes the transmission power while achieving the target mutual information for each stream after iterations at the receiver side. Numerical results show that the proposed scheme improves the transmission rate adaptivity of the original CCP while ensuring the convergence with reduced transmission power.**

I. Introduction

Conventionally, convergence properties of iterative receiver have not been taken into account when determining the optimal transmission power and rate allocation. Consequently, excessively large transmission power is used to satisfy desired quality of service (QoS) requirements in communication systems. To allow for more efficient transmission with the aid of iterative receiver in terms of transmission power and rate, the convergence properties of the receiver has to be utilized when determining the optimal transmission power.

A successful approach to adapt transmission rate and the convergence properties of iterative receiver was proposed in [1],[2] by introducing irregular modulation for single-input-single-output (SISO) channels. Irregular modulation combines multiple signal constellations with different mapping schemes within one code word [2]. The irregular modulation was later extended to point-to-point multiple-input-multiple-output (MIMO) channels in [3]. However, none of the previous studies have not considered the question how to design power allocation for irregularly modulated signalling.

Recently, transmit power optimization for an iterative receiver utilizing the convergence properties of a receiver has gained considerable attention [4],[5],[6],[7]. Especially, Yuan et. al. [5],[6], address the transmitter power allocation problem in frequency selective SISO channels with frequency-domain (FD) soft cancellation (SC) minimum mean squared error (MMSE) based iterative equalization, assuming the availability of perfect channel state information (CSI) both at the transmitter and the receiver. Their design method utilizes signal-to-noise ratio (SNR) variance charts [8] in the convergence analysis therein to determine the optimal power allocation. Correspondingly, in our recent work [9] we proposed a semi-analytical convergence constraint precoding (CCP) design for optimal power allocation in MIMO systems with iterative FD-SC-MMSE equalizer. In contrast to [5] we utilize the extrinsic information transfer (EXIT) charts [10],[11].

In the CCP based design [9], the transmit power is minimized while keeping a tunnel open between the equalizer and the decoder EXIT functions up to a desired convergence point. It is shown in [9] that the optimal power allocation in CCP can be solved by using convex optimization tools, e.g., interior point methods [12, Ch. 11]. The proposed method is intended to time division duplex (TDD) based single carrier point-to-point multiple input multiple output (MIMO) systems. Due to the reciprocity assumption of channel in TDD system, perfect CSI is assumed to be available at the transmitter and receiver.

In this paper, we extend the framework developed in [9] to irregular modulation with Gray mapping. In contrast to the previous studies of irregular modulation [1],[2],[3], we address the problem how to design a power allocation for irregularly modulated signaling with fixed signal constellation combination ratios. The main contributions of this paper are summarized as follows: We formulate transmission power optimization problem for a irregular modulation. Then, we demonstrate that the performance of proposed method provides improvement in the rate adaptivity compared to the original CCP as well as transmission power efficiency.

II. System Model

We consider a point-to-point wireless communication scenario, where both the transmitter and the receiver are equipped with multiple antennas, N_{T} transmit and N_{R} receive antennas, respectively. The fixed number of data streams N_{D} are multiplexed over N_{T} transmit antennas. For the d^{th} data stream, $d = 1, ..., N_{\mathrm{D}}$, the information bit vector $\mathbf{x}_d \in \mathbb{B}^{N_{\mathrm{X}}}$ is encoded with code rate R_c^d, where N_{X} is the number of information bits, resulting in encoded bit vector $\mathbf{c}^d = [c_1^d, ..., c_j^d, .., c_{N_{\mathrm{C}}}^d]^{\mathrm{T}} \in \mathbb{B}^{N_{\mathrm{C}}^d}$ and $j = 1, ..., N_{\mathrm{C}}$ where $N_{\mathrm{C}}^d = \frac{N_{\mathrm{X}}}{R_c^d}$ is the number of coded bits per stream. After encoding, the encoded bits are bit interleaved with random interleaver resulting bit sequence $\mathbf{c}'^d \in \mathbb{B}^{N_{\mathrm{C}}^d}$. Then, interleaved bit sequence is divided into N_{Z}^d sub-blocks, where N_{Z}^d is the number of sub-blocks per stream with different modulation methods. For the z^{th}, $z = 1, ..., N_{\mathrm{Z}}^d$, sub-block $\mathbf{c}'^d_z \in \mathbb{B}^{\alpha_{d,z} N_{\mathrm{C}}^d}$, where $\alpha_{d,z} \in [0, 1]$ is the ratio of the z^{th} sub-block, [1] a $N_{\mathrm{Q}}^{d,z}$-tuple $\mathbf{c}'^d_{z,n} = [c'^d_{z,nN_{\mathrm{Q}}^{d,z} - N_{\mathrm{Q}}^{d,z}+1}, ..., c'^d_{z,nN_{\mathrm{Q}}^{d,z}}]^{\mathrm{T}}$ of $\mathbf{c}'^d_z$ bits is mapped to a symbol $b_{z,l}^d = \mathcal{M}_z(\mathbf{c}'^d_{z,n}) \in \mathbb{C}$, $l = 1, ..., N_{\mathrm{S}}^{d,z}$, from the $2^{N_{\mathrm{Q}}^{d,z}}-$ary signal constellation using

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[1] we assume $\alpha_{d,z} N_{\mathrm{C}}^d$ to be a integer.

the gray mapping function $\mathcal{M}_{d,z}(\cdot)$. $N_{\text{S}}^{z,d}$ is the number of modulation symbols for the z^{th} sub-block and the d^{th} stream. We assume that the power of each modulation symbol is normalized to one. Furthermore, we assume that for the d^{th} stream $\sum_{z=1}^{N_{\text{Z}}} \alpha_{d,z} = 1$. Let us define the modulated data stream vector for the z^{th} sub-block of the d^{th} stream as $\mathbf{b}_z^d = [b_{z,1}^d, ..., b_{z,l}^d, ..., b_{z,N_{\text{S}}^{z,d}}^d]^{\text{T}} \in \mathbb{C}^{N_{\text{S}}^{z,d}}$. Correspondingly, the entire data stream vector for the d^{th} stream is given by $\mathbf{b}^d = [\mathbf{b}_1^{d\text{T}}, ..., \mathbf{b}_z^{d\text{T}}, ..., \mathbf{b}_{N_{\text{Z}}}^{d\text{T}}]^{\text{T}} \in \mathbb{C}^{N_{\text{S}}}$, where $N_{\text{S}} = \sum_{z=1}^{N_{\text{Z}}} N_{\text{S}}^{z,d}$. After irregular modulation, each data stream is transformed into the frequency domain by the unitary discrete Fourier transform (DFT) matrix $\mathbf{F}_{N_{\text{F}}} \in \mathbb{C}^{N_{\text{F}} \times N_{\text{F}}}$, with the elements $f_{m,l} = \frac{1}{\sqrt{N_{\text{F}}}} \exp^{\frac{\sqrt{(-1)}2\pi ml}{N_{\text{F}}}}$, where $m, l = 0, ..., N_{\text{F}}-1$ with N_{F} being the length of DFT [2]. Then, each data stream is multiplied with its associated power allocation matrix, $\mathbf{P}_d^{\frac{1}{2}} = \text{diag}([P_{d,1}^{\frac{1}{2}}, ..., P_{d,l}^{\frac{1}{2}}, .., P_{d,N_{\text{F}}}^{\frac{1}{2}}]^{\text{T}}) \in \mathbb{R}^{N_{\text{F}}}$ where $P_{d,l}^{\frac{1}{2}}$ element corresponds to the square root of the power allocated on the l^{th} frequency bin, as well as beamformer weights $\mathbf{V}^d \in \mathbb{C}^{N_{\text{T}} N_{\text{F}} \times N_{\text{F}}}$ where $\mathbf{V}^d = [\mathbf{V}]_{:,(d-1)N_{\text{F}}+1:dN_{\text{F}}}$. The beamforming matrix $\mathbf{V} \in \mathbb{C}^{N_{\text{T}} N_{\text{F}} \times N_{\text{D}} N_{\text{F}}}$ can be obtained with the singular value decomposition (SVD) as shown in [9]. Finally, each stream is transformed into time domain by multiplying the inverse DFT matrix $\mathbf{F}_{N_{\text{F}}}^{-1}$ and cyclic prefix is appended to signal before transmitting them over the channel.

Now, after guard period removal, a space-time presentation of the signal vector $\mathbf{r} \in \mathbb{C}^{N_{\text{R}} N_{\text{F}}}$ received by the N_{R} received antennas is given by

$$\mathbf{r} = \mathbf{H}\mathbf{F}_{N_{\text{T}}}^{-1}\mathbf{T}\mathbf{F}_{N_{\text{D}}}\mathbf{b} + \mathbf{v}, \tag{1}$$

where $\mathbf{v} \in \mathbb{C}^{N_{\text{R}} N_{\text{F}}}$ is a white additive independent identically distributed (i.i.d) Gaussian noise vector with variance σ^2 per dimension. Correspondingly, $\mathbf{b} \in \mathbb{C}^{N_{\text{D}} N_{\text{F}}}$ denotes the transmitted stream vector over the N_{T} transmit antennas $\mathbf{b} = [\mathbf{b}^1, ..., \mathbf{b}^d, ..., \mathbf{b}^D]^{\text{T}}$. The matrices $\mathbf{F}_{N_{\text{D}}} = \mathbf{I}_{N_{\text{D}}} \otimes \mathbf{F}_{N_{\text{F}}} \in \mathbb{C}^{N_{\text{D}} N_{\text{F}} \times N_{\text{D}} N_{\text{F}}}$ and $\mathbf{F}_{N_{\text{T}}} = \mathbf{I}_{N_{\text{T}}} \otimes \mathbf{F}_{N_{\text{F}}} \in \mathbb{C}^{N_{\text{T}} N_{\text{F}} \times N_{\text{T}} N_{\text{F}}}$ are block diagonal DFT matrices where $\otimes$ indicates the Kronecker product. The precoder matrix $\mathbf{T} \in \mathbb{C}^{N_{\text{T}} N_{\text{F}} \times N_{\text{D}} N_{\text{F}}}$ can be further factorized as $\mathbf{T} = \mathbf{V}\mathbf{P}^{\frac{1}{2}}$, where diagonal matrix $\mathbf{P}^{\frac{1}{2}} = \text{diag}(\mathbf{P}_1^{\frac{1}{2}}, ..., \mathbf{P}_d^{\frac{1}{2}}, ..., \mathbf{P}_{N_{\text{D}}}^{\frac{1}{2}}) \in \mathbb{R}^{N_{\text{D}} N_{\text{F}} \times N_{\text{D}} N_{\text{F}}}$. $\mathbf{H} \in \mathbb{C}^{N_{\text{R}} N_{\text{F}} \times N_{\text{T}} N_{\text{F}}}$ is circulant block matrix, comprised of channel submatrices $\mathbf{H}^{r,t} \in \mathbb{C}^{K_S \times K_S}$ between the t^{th} transmit and the r^{th} receive antennas, $t = 1, ..., N_{\text{T}}$ and $r = 1, ..., N_{\text{R}}$, which are also circulant matrices, as $\mathbf{H}^{r,t} = \text{circ}\left\{\left[h_1^{r,t}, h_2^{r,t} \dots h_L^{r,t}, \mathbf{0}_{1 \times N_{\text{F}}-L}\right]^{\text{T}}\right\}$. The operator circ$\{\ \}$ generates matrix that has a circulant structure of its argument vector. L denotes the length of the channel, and $h_l^{r,t}$, $l = 1, ..., L$, the fading gains of multipath channel between t^{th} transmit antenna and the r^{th} receive antenna. Average signal-to-noise ratio per receiver antenna is defined as $\text{SNR} = \frac{\tilde{P}_u}{2\sigma^2}$, where, $\tilde{P}_u$ is the average transmitted symbol energy.

III. Iterative Equalizer and EXIT Functions

In this paper, we utilize the EXIT charts in which the exchange of extrinsic mutual information is tracked between the equalizer and the decoding blocks. Particularly, we use the relationship between the mutual information and the variance of log-likelihood ratios (LLRs) that can obtained using J-function [10]. Thus, we express in this section the variance of LLRs and the mutual information at the output of the equalizer, respectively. However, due to space limitations we can not describe in detail the equalizer that contains FD SC-MMSE and soft-demapping algorithms. Therefore, we briefly summarize the equalizer in the appendix.

Since SVD based transmit beamforming is utilized in the transmission, the transmitted streams can be perfectly decoupled from each other in the spatial domain, when the perfect CSI is available at the transmitter. As a result of the diagonalization procedure, detailed in [9], the channel at the output of equalizer becomes fully diagonal in spatial domain. However, due to multipath propagation some residual inter-symbol interference (ISI) remains at the output of the equalizer. Naturally, the amount of residual ISI depends on the channel realization, SNR, as well the amount of *a* priori information from the channel decoder.

Let us define LLR vector at the output of the equalizer for the d^{th} stream $\hat{\mathbf{L}}_d = [\hat{\mathbf{L}}_{d,1}^{\text{T}}, ..., \hat{\mathbf{L}}_{d,z}^{\text{T}}, ..., \hat{\mathbf{L}}_{\text{d},\text{N}_Z}^{\text{T}}]^{\text{T}} \in \mathbb{R}^{N_{\text{C}}^d}$ where $\hat{\mathbf{L}}_{d,z} = [\hat{\mathcal{L}}(c_{z,1}'^d), ..., \hat{\mathcal{L}}(c_{z,i}'^d), .., \hat{\mathcal{L}}(c_{z,\alpha_{d,z}N_{\text{C}}^d}'^d)]^{\text{T}} \in \mathbb{R}^{\alpha_{d,z}N_{d,\text{C}}}$, where $i = 1, ..., \alpha_{d,z}N_{\text{C}}^d$. The PDF of $\hat{\mathbf{L}}_{d,z}$, can be approximated by a Gaussian distribution satisfying the consistency requirement, i.e., $\hat{\mathbf{L}}_{d,z} \sim \mathcal{N}^{\mathbb{R}}(\hat{\mu}_{d,z}\mathbf{1}, \hat{\sigma}_{d,z}^2\mathbf{I})$, where $\hat{\sigma}_{d,z}^2 = 2\hat{\mu}_{d,z}$. Thus, the LLRs at the output of the soft- demapper, $\hat{\mathcal{L}}(c_{z,i}'^d)$, can be rewritten as

$$\hat{\mathcal{L}}(c_{z,i}'^d) \quad = \quad \hat{\mu}_{d,z}\bar{c}_{z,i}'^d + [\mathbf{w}_{d,z}]_i \tag{2}$$

where $\mathbf{w}_{d,z} \in \mathbb{R}^{\alpha_{d,z}N_{\text{C}}^d}$ and $\mathbf{w}_{d,z} \sim \mathcal{N}^{\mathbb{R}}(\mathbf{0}_{\alpha_{d,z}N_{\text{C}}^d}, \hat{\sigma}_{d,z}^2\mathbf{I}_{\alpha_{d,z}N_{\text{C}}^d})$ and $\bar{c}_{z,i}'^d = 2c_{z,i}'^d - 1$,the mean of LLRs $\hat{\mu}_{d,z}$ is given by [9]

$$\hat{\mu}_{d,z} = \frac{2\zeta_{d,z}}{1 - \zeta_{d,z}\Delta_{d,z}} \tag{3}$$

and the variance of LLRs, $\hat{\sigma}_{d,z}^2$ is simply obtained as

$$\hat{\sigma}_{d,z}^2 = \frac{4\zeta_{d,z}}{1 - \zeta_{d,z}\Delta_{d,z}}. \tag{4}$$

For complex modulations we have to divide (4) by four resulting to

$$\hat{\sigma}_{d,z}^2 = \frac{\zeta_{d,z}}{1 - \zeta_{d,z}\Delta_{d,z}}. \tag{5}$$

The scalar $\Delta_{d,z}$ is the average residual interference energy for the z^{th} modulation method and d^{th} stream and $\zeta_{d,z}$ is signal-to-interference-plus-noise ratio (SINR) in (17) that can be rewritten by following [9] as

$$\zeta_{d,z} = \frac{1}{N_{\text{F}}} \sum_{b=1}^{N_{\text{F}}} \frac{S_{d,b}^2 P_{d,b}}{\sigma^2 + S_{d,b}^2 P_{d,b}\Delta_{d,z}} \tag{6}$$

where $S_{d,b}$ is singular value for the b^{th} frequency bin of the d^{th} stream. As in [13], we approximate LLRs for each bit position to be Gaussian distributed with the same mean and the same variance values. This approximation becomes less accurate when $2^{N_{\text{Q}}^{d,z}}$-ary > 4 [13].

To formulate the actual optimization problem in the next section, we divide the EXIT functions of the equalizer and the decoder into discrete extrinsic mutual information points according to *a priori* values which we index with *a priori*

[2]We assume $N_{\text{F}} = N_{\text{S}}$.

index, $k = 1, ..., N_{\text{K}}$, with N_{K} being the number of *a priori* points per stream and sub-block. The EXIT functions characterize the input/output relations of the equalizer and the decoder $\forall k$, measured by the mutual information. The average mutual information with irregular modulation at the output of the equalizer for the d^{th} stream with k^{th} *a priori* index between transmitted interleaved coded bits $\mathbf{c}'^{d}$ and the LLRs $\hat{\mathbf{L}}_d$ is the linear combination of the EXIT functions of $\hat{I}^E_{d,k,z}$ of the N_{Z} sub-blocks [2]

$$\hat{I}^E_{d,k} = \sum_{z=1}^{N_{\text{Z}}} \alpha_{d,z} \hat{I}^E_{d,k,z}. \tag{7}$$

$\hat{I}^E_{d,k,z}$ is the average mutual information at the output of equalizer for the z^{th} sub-block at the k^{th} *a priori* point between transmitted interleaved coded bits $\mathbf{c}'^{d}_{z}$ and the LLRs $\hat{\mathbf{L}}_{d,z}$, given by

$$\hat{I}^E_{d,k,z} = \hat{\mathrm{f}}(\hat{I}^A_{d,z,k} | \mathbf{S}_d, \mathbf{P}^{\frac{1}{2}}_d, \sigma^2). \tag{8}$$

$\hat{I}^A_{d,k,z} \in [0, 1]$ is the equalizer *a priori* mutual information of the d^{th} stream for the z^{th} sub-block at the k^{th} *a priori* index, $\hat{\mathrm{f}}()$ is the monotonically increasing EXIT function of the equalizer and $\mathbf{S}_d = \text{diag}([S_{d,1}, ..., S_{d,l}, .., S_{d,N_{\text{F}}}]^{\text{T}}) \in \mathbb{R}^{N_{\text{F}} \times N_{\text{F}}}$ contains the singular values of SVD of the FD channel matrix for the d^{th} stream.

The decoder output extrinsic information for the d^{th} stream at with k^{th} *a priori* index is given by

$$\mathring{I}^E_{d,k} = \mathring{\mathrm{f}}(\mathring{I}^A_{d,k}) \tag{9}$$

where $\mathring{I}^A_{d,k} \in [0, 1]$ is the decoder *a priori* mutual information of the d^{th} stream at the k^{th} *a priori* index and $\mathring{\mathrm{f}}()$ is the monotonically increasing EXIT function of the channel decoder.

IV. Power Allocation Method for Irregular Modulation

The goal of the proposed power allocation scheme is to guarantee a convergence tunnel between the EXIT functions of the equalizer and the decoder up to a desired convergence point while minimizing the transmit power. Thus, the mutual information at the output of the equalizer at each *a priori* point has to be always larger than the mutual information at the input of channel decoder up to the target convergence point.

Mathematically, the transmission power optimization problem for irregularly modulation with fixed irregularity ratios, $\alpha_{d,z}$, is formulated as follows:

$$\begin{aligned} &\text{minimize} \quad \text{tr}\{\mathbf{P}\} \\ &\text{subject to} \sum_{z=1}^{N_{\text{Z}}} \alpha_{d,z} \hat{I}^E_{d,k,z} \geqslant \underbrace{\mathring{\mathrm{f}}^{-1}(\mathring{I}^E_{d,k}) + \epsilon_{d,k}}_{=\theta_{d,k}} \forall d, \forall k \end{aligned} \tag{10}$$

where $\mathring{\mathrm{f}}^{-1}()$ denotes the inverse EXIT function of the channel decoder and $\epsilon_{d,k}$ is small positive value scalar. To control the width of the convergence tunnel, we define also the acceptable gap between the equalizer and the decoder EXIT functions with the positive scalar $\epsilon_{d,k}$ and require that $\sum_{z=1}^{Z} \alpha_{d,z} \hat{I}^E_{d,k,z} \geqslant \mathring{\mathrm{f}}^{-1}(\mathring{I}^E_{d,k}) + \epsilon_{d,k}$.

As shown in [9], the mutual information at the output of the equalizer for the d^{th} stream can be closely approximated by using J-function [14] as a function of the variance of the log-likelihood ratios (LLR) $\hat{\sigma}^2_{d,k}$. Therefore, we can transform the right hand side values of the mutual information constraints in (10) to the target LLR variance values at the output of the equalizer by using the inverse J-function [14][3]

$$\hat{\sigma}^{2\text{target}}_{d,k} \approx \sum_{z=1}^{N_{\text{Z}}} \alpha_{d,z} \frac{1}{4} \left(-\frac{1}{H_1^z} \log_2(1 - \theta_{d,k}^{\frac{1}{H_3^z}})\right)^{\frac{1}{H_2^z}} \tag{11}$$

where the parameter values for H_1^z,H_2^z and H_3^z can be found for different modulation methods in [11],e.g., QPSK: $H_1 = 0.3073$,$H2 = 0.8935$,$H3 = 1.1064$ and and 8-PSK: $H_1 = 0.2516$,$H_2 = 0.7274$,$H_3 = 1.2392$. Since the left hand side mutual information constraints in (10) can be also transformed to the variance of LLRs by inverse J-function, we can replace $\hat{I}^E_{d,k,z}$ in (10) with $\hat{\sigma}^2_{d,k,z}$ by defining (5) for the k^{th} *a priori* point. Finally, we can express the transmission power optimization for irregularly modulated symbols with fixed $\alpha_{d,z}$ as

$$\begin{aligned} &\min. \sum_{d=1}^{N_{\text{D}}} \sum_{b=1}^{N_{\text{F}}} P_{d,b} \\ &\text{s. t.} \sum_{z=1}^{N_{\text{Z}}=1} \alpha_{d,z} \frac{\frac{1}{N_{\text{F}}} \sum\limits_{b=1}^{N_{\text{F}}} \frac{S^2_{d,b} P_{d,b}}{\sigma^2 + S^2_{d,b} P_{d,b} \Delta_{d,k,z}}}{1 - \frac{\Delta_{d,k,z}}{N_{\text{F}}} \sum\limits_{b=1}^{N_{\text{F}}} \frac{S^2_{d,b} P_{d,b}}{\sigma^2 + S^2_{d,b} P_{d,b} \Delta_{d,k,z}}} \geqslant \hat{\sigma}^{2\text{target}}_{d,k} \\ &\forall d, k \\ &P_{d,b} \geqslant 0 \forall d, b \end{aligned} \tag{12}$$

where the scalar $\Delta_{d,k,z}$ is the average residual interference energy for the z^{th} modulation method and d^{th} stream at the k^{th} *a priori* point.

By observing (12), it can be noticed that optimization problem is not a convex. Therefore, the globally optimum solution can not be guaranteed and locally optimum solutions can be only achieved. Note that in the case of $N_{\text{Z}} = 1$ the problem in (12) can be reformulated to a convex optimization problem as shown in [9].

V. Numerical Results

The simulation parameters used are summarized as follows: The number of receiver and transmit antennas $N_{\text{T}} = 2$, $N_{\text{R}} = 2$, respectively, streams $N_{\text{D}} = 2$, $N_{\text{F}} = 512$, $\epsilon_{d,k} = 0.02$ for $\forall k, \forall d$, QPSK, 8-PSK with Gray mappings, and a systematic repeat accumulate (RA) codes [15] with two different code rates, 1/3 and 1/4 were used. Rayleigh fading channel with 5-path equal gains over 50 channel realizations was assumed in the simulations. Matlab optimization toolbox was used in the transmission power optimization. Figure 1a illustrates the comparison of EXIT functions of the proposed technique and the original CCP with fixed code rates. The proposed method has a mixture of QPSK and 8-PSK for the both data streams where ratios $\alpha_{1,1} = 0.3$ and $\alpha_{1,2} = 0.7$ for $d = 1$ and $\alpha_{2,1} = 0.4$ and $\alpha_{2,2} = 0.6$ for $d = 2$, respectively. The convergence points were set at 0.59 for $d = 1$ and 0.49 for $d = 2$ both with $\epsilon_{d,k} = 0.02$. The solid and dashdot curves represent the EXIT functions when $d = 1$ and $d = 2$, respectively. As can be seen, the convergence results of the proposed method and the original CCP method are nearly the same. The information rate lower bound over the streams

[3]This holds for complex modulations.

for this particular configuration is defined by fixed QPSK transmission (=1.17 [bits/s/Hz]) whereas the upper bound given by fixed 8-PSK transmission (=1.75 [bits/s/Hz]). By using the previously given $\alpha_{d,z}$, the proposed method achieves 1.55 [bits/s/Hz] information rate. Thus, the proposed scheme can flexibly vary its data rate within the aforementioned bounds by changing irregularity ratios, $\alpha_{d,z}$. Correspondingly, the resulting SNRs for the original CCP with 8-PSK, the proposed scheme with the mixture of QPSK and 8-PSK, and the original CCP with QPSK are computed SNR= 4.35 dB, SNR= 5.57 dB and SNR= 1.6 dB, respectively. Hence, the data rate adaptation can be made more flexible with less transmission power by using the proposed method compared to the original CCP. Figure 1b shows an example when the convergence points were set at 0.98 for $d = 1$ and 0.98 for $d = 2$ both with $\epsilon_{d,k} = 0.02$. It can be observed that by using the both proposed technique, the right most point of the EXIT chart (1,1) can be achieved. Consequently, nearly error-free performance can be achieved with the proposed method, given that sufficient number of iterations is performed.

VI. Conclusions

This paper proposes a new semi-analytical power allocation method for irregularly modulated point-to-point MIMO signaling with iterative FD SC-MMSE equalization, in which the convergence properties of the equalizer are taken into account. The proposed scheme is based on the combination of irregular modulation, different signal constellations within one code word, and convergence constraint precoding (CCP), technique that decouples the spatial interference between streams using singular value decomposition (SVD), and minimizes the transmission power while achieving the target mutual information for each stream after iterations at the receiver side. We show that the transmission power allocation can be formulated as a optimization problem when irregular modulation is used. Simulation results show that the proposed power allocation method improves the rate adaptivity of the original CCP technique while ensuring the convergence with reduced transmission power.

Appendix A
Frequency Domain SC-MMSE and Soft-demapping

A. Soft-cancellation

The output of soft-cancellation, $\hat{\mathbf{r}} \in \mathbb{C}^{N_R N_F}$, is given by

$$\hat{\mathbf{r}} = \mathbf{F}_{N_R}\mathbf{r} - \mathbf{\Gamma}\mathbf{T}\mathbf{F}_{N_D}\tilde{\mathbf{b}} \tag{13}$$

where $\mathbf{F}_{N_R} = \mathbf{I}_{N_R} \otimes \mathbf{F}_{N_F} \in \mathbb{C}^{N_R N_F \times N_R N_F}$, $\mathbf{\Gamma} \in \mathbb{C}^{N_R N_F \times N_T N_F}$ is the block-wise diagonal frequency domain channel matrix and $\tilde{\mathbf{b}} = \left[\tilde{\mathbf{b}}^{1\mathrm{T}}, ..., \tilde{\mathbf{b}}^{d\mathrm{T}}, ..., \tilde{\mathbf{b}}^{D\mathrm{T}}\right]^{\mathrm{T}} \in \mathbb{C}^{N_D N_S}$ where $\tilde{\mathbf{b}}^d = \left[\tilde{\mathbf{b}}_1^{d\mathrm{T}}, ..., \tilde{\mathbf{b}}_z^{d\mathrm{T}}, ..., \tilde{\mathbf{b}}_{N_Z}^{d\mathrm{T}}\right]^{\mathrm{T}} \in \mathbb{C}^{N_S}$. $\tilde{\mathbf{b}}_z^d = \left[\tilde{b}_{z,1}^d, ..., \tilde{b}_{z,s}^d, ..., \tilde{b}_{z,N_S^{z,d}}^d\right]^{\mathrm{T}} \in \mathbb{C}^{N_S^{z,d}}$, $s = 1, ..., N_S^{z,d}$, where $\tilde{b}_{z,s}^d$ [4] is the soft-estimate estimate of the l^{th} transmitted symbol of the d^{th} stream and the z^{th} sub-block. The first moment of soft-symbol estimates, $\tilde{b}_{z,s}^d = E\left\{b_{z,s}^d\right\}$ is obtained as

$$\tilde{b}_{z,s}^d = E\left\{b_{z,s}^d\right\} = \sum_{b_i \in 2^{N_Q^{d,z}}} b_i P(b_{z,s}^d = b_i) \tag{14}$$

[4]We assume $N_S = N_F$.

where symbol a priori probability $P(b_{z,s}^d = b_i)$ in (14) can be calculated from [16]

$$P(b_{z,s}^d = b_i) = \left(\frac{1}{2}\right)^{N_Q^{d,z}} \prod_{q=1}^{N_Q^{d,z}} \left(1 - \overline{c}_{i,q} \tanh\left(\lambda_{z,s,q}^d/2\right)\right), \tag{15}$$

with $\overline{c}_{i,q} = 2c_{i,q} - 1$ and $\lambda_{s,q}^d$ being a priori LLR of the bit $c_{i,q}$, provided by decoder.

B. MMSE Filtering

To detect all irregularly modulated sub-blocks properly per stream, FD MMSE filtering has to be performed N_Z times per each data stream. For the z^{th} sub-block of the d^{th} stream, we consider only the limited number of symbols at the output of FD MMSE filter as follows $\hat{\mathbf{b}}_{d,z} = [\mathbf{y}_{d,z}]_{N_S^{d,z}(z-1)+1:N_S^{d,z}z}$ where

$$\mathbf{y}_{d,z} = \frac{1}{\zeta_{d,z}\overline{\ddot{\beta}}_{d,z} + 1}(\mathbf{F}_{N_F}^{-1}\mathbf{T}_d^{\mathrm{H}}\mathbf{\Gamma}^{\mathrm{H}}\mathbf{\Sigma}_{\hat{r},d,z}^{-1}\hat{\mathbf{r}} + \zeta_{d,z}\tilde{\mathbf{b}}^d), \tag{16}$$

where $\mathbf{T}^d = [\mathbf{T}]_{:,(d-1)N_F+1:dN_F}$, $\overline{\ddot{\beta}}_{d,z} = \mathrm{avg}\left\{\ddot{\mathbf{b}}_z^d\right\}$ with $\ddot{\mathbf{b}}_z^d = \left[|\tilde{b}_{z,1}^d|^2 \dots |\tilde{b}_{z,s}^d|^2 \dots |\tilde{b}_{z,N_S^{z,d}}^d|^2\right]^{\mathrm{T}} \in \mathbb{C}^{N_S^{z,d}}$, avg$\{\cdot\}$ indicates operator that calculates the arithmetic mean from its argument vector elements. The scalar $\zeta_{d,z}$ is given by

$$\zeta_{d,z} = \frac{1}{N_F}\mathrm{tr}\left\{\mathbf{T}^{d^{\mathrm{H}}}\mathbf{\Gamma}^{\mathrm{H}}\mathbf{\Sigma}_{\hat{r},d,z}^{-1}\mathbf{\Gamma}\mathbf{T}^d\right\} \tag{17}$$

where $\mathbf{\Sigma}_{\hat{r},d,z} \in \mathbb{C}^{N_R N_F \times N_R N_F}$, is the covariance matrix of the output of soft-cancellation, given by

$$\mathbf{\Sigma}_{\hat{r},d,z} = \mathbf{\Gamma}\mathbf{T}\mathbf{\Delta}_{d,z}\mathbf{T}^{\mathrm{H}}\mathbf{\Gamma}^{\mathrm{H}} + \sigma^2\mathbf{I}. \tag{18}$$

By assuming the impact of ISI over the sub-blocks per stream to be minor to the performance, we approximate the residual interference energy matrix given SVD based transmission, $\mathbf{\Delta}_{d,z} \in \mathbb{R}^{N_D N_F \times N_D N_F}$, by

$$\mathbf{\Delta}_{d,z} = \mathrm{diag}([\Delta_{1,z}, ..., \Delta_{d,z}, ..., \Delta_{N_D,z}] \otimes \mathbf{1}_{N_S}) \tag{19}$$

where $\Delta_{d,z} = \mathrm{avg}\left\{\mathbf{1}_{N_S^{d,z}} - \ddot{\mathbf{b}}_z^d\right\}$. Note that the streams are perfectly decoupled from each other with SVD based transmission when channel state information is perfect.

C. Soft-Demapping

The LLRs for the z^{th} sub-block are expressed as [16]

$$\hat{\mathcal{L}}_{z,m,v}^d = \ln\frac{\sum_{\forall c_i : c_{i,v}=0} \exp(-\omega_{z,m,i}^d + \sum_{v' \neq v} \frac{1}{2}\tilde{c}_{i,v'}\lambda_{z,m,v'}^d)}{\sum_{\forall c_i : c_{i,v}=1} \exp(-\omega_{z,m,i}^d + \sum_{v' \neq v} \frac{1}{2}\tilde{c}_{i,v'}\lambda_{z,m,v'}^d)} \tag{20}$$

where $\lambda_{z,m,v'}^d$ being a priori LLR of the bit for the v'^{th} bit position of the m^{th} symbol of the z^{th} sub-block provided by decoder, $m = 1, ..., N_S^{d,z}$, $v = 1, ..., N_Q^{d,z}$, and $\tilde{c}_{i,v'}$ is given by

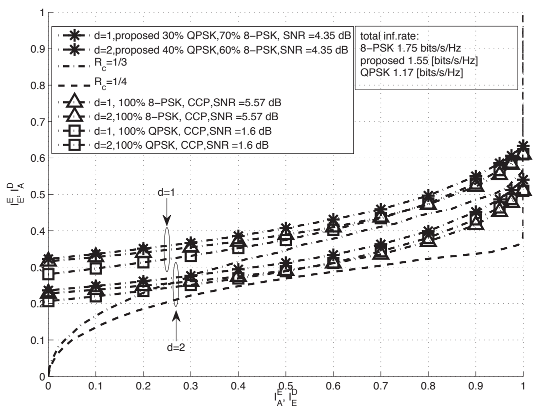
$$\tilde{c}_{i,v'} = \begin{cases} -1, c_{i,v'} = 0 \\ +1, c_{i,v'} = 1. \end{cases} \tag{21}$$

The scalar $\omega_{z,m}^d$ is defined as

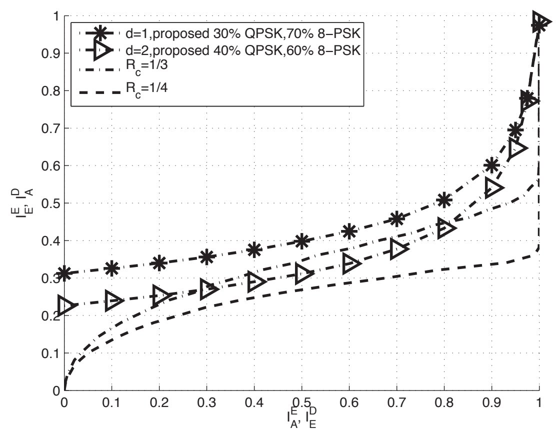
$$\omega_{z,m,i}^d = \frac{|[\hat{\mathbf{b}}_{d,z}]_m - \phi_{d,z}\mathcal{M}_z(\dot{\mathbf{c}}_i)|^2}{\upsilon_{d,z}} \tag{22}$$

where $\upsilon_{d,z} = \phi_{d,z} - \phi_{d,z}^2$, $\dot{\mathbf{c}}_i$ is candidate bit vector, and

$$\phi_{d,z} = \frac{\zeta_{d,z}}{\zeta_{d,z}\ddot{\overline{\beta}}_{d,z} + 1}. \tag{23}$$



(a) Convergence points set at 0.59 for $d = 1$ (solid) and at 0.49 for $d = 2$ (dashdot).



(b) Convergence points set at 0.98 for $d = 1$ (solid) and at 0.98 for $d = 2$ (dashdot), SNR= 12.4 dB

Fig. 1: $N_\mathrm{T} = N_\mathrm{R} = 2, N_\mathrm{D} = 2$.

REFERENCES

[1] F. Schreckenbach and G. Bauch, "Irregular signal constellations, mappings and precoder," in *Proc. IEEE Int. Symp. Inform. Theory and its Applications (ISITA)*, Parma, Italy, Oct.10–13 2004, pp. 1372–1376.

[2] ——, "Bit-interleaved coded irregular modulation," *European Trans. Telecommun.*, no. 17, pp. 269–282, 2006.

[3] Y. Zhang, Y. Zhu, D. Yang, and X. Zhang, "A new irregular modulation technique for turbo-blast systems," in *Proc. IEEE Veh. Technol. Conf. (VTC)*, Dublin, Ireland, 2007, pp. 1515–1519.

[4] D. Shepherd, Z. Shi, M. Reed, and F. Schreckenbach, "Optimization of unequal power coded multiuser DS-CDMA using extrinsic information transfer charts," in *Proc. Conf. Inform. Sciences Syst. (CISS)*, Princeton, USA, Mar. 2006.

[5] X. Yuan, H. Li, L. Ping, and X. Lin, "Precoder design for ISI channels based on iterative LMMSE equalization," in *Proc. Int. Symb. on Turbo Codes*, Lausanne, Switzerland, Sept.1–5 2008, pp. 198–203.

[6] H. Li, X. Yuan, X. Lin, and L. Ping, "On water-filling precoding for coded single-carrier systems," *IEEE Commun. Lett.*, vol. 13, no. 1, pp. 34–36, Jan. 2009.

[7] D. Shepherd, F. Brannstrom, F. Schreckenbach, Z. Shi, and M. Reed, "Adaptive optimization of an iterative multiuser detector for turbo-coded CDMA," *IEEE Trans. Wirel. Commun.*, vol. 7, no. 1, pp. 4284–4293, Nov. 2008.

[8] X. Yuan, Q. Guo, X. Wang, and L. Ping, "Evolution analysis of low-cost iterative equalization in coded linear systems with cyclic prefix," *IEEE J. Select. Areas Commun.*, vol. 26, no. 2, pp. 301–310, Feb. 2008.

[9] J. Karjalainen, A. Tölli, T. Matsumoto, and M. Juntti, "On convergence constrainend precoder design for frequency domain MIMO detector," in *Proc. IEEE Int. Symp. Inform. Theory (ISIT)*, Seoul, Korea, June 2009.

[10] S. T. Brink, "Convergence behaviour or iterativively decoded parallel concatenated codes," *IEEE Trans. Commun.*, vol. 49, no. 10, pp. 1727–1737, Oct. 2001.

[11] K. Kansanen and T. Matsumoto, "An analytical method for MMSE MIMO turbo equalizer EXIT chart computation," *IEEE Trans. Wirel. Commun.*, vol. 6, no. 1, pp. 328–339, Jan. 2007.

[12] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, UK: Cambridge University Press, 2004.

[13] V. Ramon, C. Herzet, and L. Vandendorpe, "A semi-analytical method for predicting the performance and convergence behaviour of a multiuser turbo-equalizer/demapper," *IEEE Trans. Signal Processing*, vol. 55, no. 5, pp. 1104–117, Mar. 2007.

[14] F. Brännström, L. K. Rasmussen, and A. J. Grant, "Convergence analysis and optimal scheduling for multiple concatenated codes," *IEEE Trans. Inform. Theory*, vol. 51, no. 9, pp. 3354–3364, Sept. 2005.

[15] A. Abbasfar, D. Divsalar, and K. Yao, "Accumulate repeat accumulate codes," *IEEE Trans. Commun.*, vol. 55, no. 4, pp. 692–702, Apr. 2007.

[16] M. Tüchler, A. C. Singer, and R. Koetter, "Minimum mean squared error equalisation using a priori information," *IEEE Trans. Signal Processing*, vol. 50, no. 3, pp. 673–683, Mar. 2002.