Title	On convergence constrained precoder design for iterative frequency domain MIMO detector
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Citation	IEEE International Symposium on Information Theory, 2009. ISIT 2009: 1278-1282
Issue Date	2009-07
Type	Conference Paper
Text version	publisher
URL	http://hdl.handle.net/10119/9109
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Description	

ISIT 2009, Seoul, Korea, June 28 - July 3, 2009

On Convergence Constrained Precoder Design for Iterative Frequency Domain MIMO Detector

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***Abstract*—This paper proposes a novel linear precoder design technique for single carrier single-user multiple input multiple output (MIMO) systems with frequency-domain (FD) *soft* cancellation (SC) minimum mean squared error (MMSE) *iterative* equalization where the convergence properties of the equalizer are taken into account. The proposed precoder design technique, convergence constrained precoding (CCP), minimizes the transmission power while it achieves the target mutual information for each stream after the iterations at the receiver side. We show that the optimality criterion for the proposed design can be formulated as a convex optimization problem. The results demonstrate that our proposed technique outperforms the existing linear precoding techniques by ensuring the convergence with a reduced transmission power. Furthermore, we show that with CCP we can adjust transmission according to convergence properties of the iterative equalizer in a more flexible way than, e.g., minimum sum mean squared error (MinSumMSE) and maximum information rate (MaxRate) precoding.**

I. Introduction

Despite the benefits of linear precoding with linear receivers presented, e.g., in [1],[2], a fundamental question associated with iterative receivers arises that how sensitive (or insensitive) the convergence property of iterative receiver is to the precoder design criterion. We [3],[4] have recently demonstrated that precoder strategy has a significant impact on the convergence properties of the iterative receiver. It was shown in [5] and [6] that the convergence properties of a receiver can be taken into account while optimizing the transmitter power allocation. Particularly, Yuan et. al. [6], address the transmitter power allocation problem in frequency selective single-input-single-output (SISO) channels with minimum mean squared error (MMSE) based iterative equalization, assuming the availability of perfect channel state information (CSI) both at the transmitter and the receiver. Signal-to-noise ratio (SNR) variance charts [7] are used in the convergence analysis therein to determine the optimal power allocation strategy.

The problem of designing a precoder while assuming an iterative receiver and taking into account its convergence properties has not been thoroughly investigated in multiple-input-multiple-output (MIMO) channels. In this paper, we propose a new semi-analytical convergence constrained precoding (CCP) design technique for single carrier single-user MIMO systems motivated by the extrinsic information transfer (EXIT) charts [8],[9] using mutual information rather than SNR as proposed in [7]. We also minimize the transmit power while keeping the convergence tunnel open between equalizer and decoder EXIT functions up to a desired convergence point. Similarly as in [6], equalization is performed by using an iterative frequency-domain (FD) soft cancellation (SC) and MMSE filtering based equalizer. Perfect CSI is assumed to be available at the transmitter and the receiver.

The main contributions of this paper are summarized as follows: We employ semi-analytical EXIT chart as in [9] to design linear precoding scheme for MIMO systems. More specifically, in contrast to previous work in [6], the spatial interference between transmitted streams can be perfectly decoupled by performing singular value decomposition (SVD) based beamformer at the transmitter. Thus, the problem reduces to a power allocation problem over the streams and frequency bins. We show that the power allocation optimization problem can be categorized as a convex optimization problem, for which the globally optimum solution can be found over the streams and frequency bins. The proof of the global optimality of the problem is also provided. Furthermore, we provide an algorithm to perform transmitter power optimization. To justify our semi-analytical design approach, we show the comparison between the semi-analytical results and conventional histogram based EXIT measurements [8]. Finally, we demonstrate through simulations the advantageous point of the proposed CCP technique in terms of flexibility in power allocation over the known precoding techniques, e.g, minimum sum mean squared error (MinSumMSE) [10] and maximum information rate (MaxRate) [11] algorithms.

II. System Model

Consider a point-to-point wireless communication scenario, where both the transmitter and the receiver are equipped with multiple antennas, T transmit and R receive antennas, respectively. The fixed number D of data streams are multiplexed over T transmit antennas. After guard period removal,[1] a space-time presentation of the signal vector $\mathbf{r} \in \mathbb{C}^{RK_S \times 1}$ received by the R received antennas is given by

$$\mathbf{r} = \mathbf{H}\mathbf{F}_T^{-1}\mathbf{T}\mathbf{F}_D\mathbf{b} + \mathbf{v}, \tag{1}$$

where $\mathbf{v} \in \mathbb{C}^{RK_S \times 1}$ is a white additive independent identically distributed (i.i.d) Gaussian noise vector with variance σ^2 per dimension, with K_S being the length of discrete Fourier transform (DFT). Correspondingly, $\mathbf{b} \in \mathbb{C}^{DK_S \times 1}$ denotes the transmitted stream vector over the T transmit antennas $\mathbf{b} = [\mathbf{b}^1, ..., \mathbf{b}^d, ..., \mathbf{b}^D]^{\mathrm{T}}$ with $\mathbf{b}^d \in \mathbb{C}^{K_S \times 1}$ being $\mathbf{b}^d = [b_1^d, ..., b_k^d, ..., b_{K_s}{}^d]^{\mathrm{T}}$, where let $d = 1, ..., D$ and $k = 0, ..., K_s - 1$ denote the number of data streams and the symbol indices, respectively. $\mathbf{F}_D \in \mathbb{C}^{DK_S \times DK_S}$ and $\mathbf{F}_T \in \mathbb{C}^{TK_S \times TK_S}$ are block diagonal DFT matrices given by $\mathbf{F}_D = \mathbf{I}_D \otimes \mathbf{F}_S$ and $\mathbf{F}_T = \mathbf{I}_T \otimes \mathbf{F}_S$, respectively. $\mathbf{I}_T \in \mathbb{R}^{T \times T}$ and $\mathbf{I}_D \in \mathbb{R}^{D \times D}$ are identity matrices and the symbol $\otimes$ indicates the Kronecker product. $\mathbf{F}_S \in \mathbb{C}^{K_S \times K_S}$ is the unitary

This work was supported in part by Finnish Funding Agency for Technology and Innovation (Tekes), Nokia, Nokia Siemens Networks, Elektrobit, Uninord, and Academy of Finland.

[1] We restrict ourselves to the case where the length of guard period is larger than or as large as the channel memory length.

DFT matrix with the elements $f_{m,k} = \frac{1}{\sqrt{K_S}} \exp^{\frac{j2\pi mk}{K_S}}$, where $m, k = 0, ..., K_S - 1$. The precoder matrix $\mathbf{T} \in \mathbb{C}^{TK_S \times DK_S}$ can be further factorized as $\mathbf{T} = \mathbf{V}\mathbf{P}^{\frac{1}{2}}$. The diagonal matrix $\mathbf{P}^{\frac{1}{2}} \in \mathbb{R}^{DK_S \times DK_S}$ is the power allocation matrix with diagonal elements corresponding to the square root of the power allocated on the each frequency bin. $\mathbf{V} \in \mathbb{C}^{TK_S \times DK_S}$ is the transmit beamforming matrix. The circulant block matrix, $\mathbf{H} \in \mathbb{C}^{RK_S \times TK_S}$, comprised of channel submatrices $\mathbf{H}^{r,t} \in \mathbb{C}^{K_S \times K_S}$ between the t^{th} transmit and the r^{th} receive antennas, $r = 1, ..., R$, which are also circulant, as $\mathbf{H}^{r,t} = \text{circ}\left\{\left[h_1^{r,t}, h_2^{r,t} \dots h_L^{r,t}, \mathbf{0}_{1\times K_{S-L}}\right]^{\mathrm{T}}\right\}$. The operator circ{ } generates matrix that has a circulant structure of its argument. L denotes the length of the channel, and $h_l^{r,t}$, $l = 1, ..., L$, the fading gains of multipath channel between t^{th} transmit antenna and the r^{th} receive antenna. Average signal-to-noise ratio per receiver antenna is defined as $SNR = \frac{\tilde{P}_u}{2\sigma^2}$, where, $\tilde{P}_u$ is the average transmitted symbol energy.

III. Convergence Constrained Precoder Design

The goal of the CCP based design is to guarantee a convergence tunnel between the EXIT functions of the equalizer and the decoder up to a desired convergence point while minimizing the transmit power. Let us firstly divide the EXIT functions of equalizer and decoder into the discrete extrinsic mutual information points according to *a priori* values which we index with *a priori* index, $k = 1, ..., K$, with K being the number of the *a priori* points per stream.The equalizer extrinsic mutual information for the d^{th} stream at the k^{th} *a priori* index is given by, a $D+1$ dimensional function, as [2]

$$\hat{I}_{d,k}^{E} = \hat{\mathrm{f}}(\mathbf{r}, \left[\hat{I}_{1,k}^{A}, ..., \hat{I}_{d,k}^{A}, ..., \hat{I}_{D,k}^{A}\right]^{\mathrm{T}}) \tag{2}$$

where $\hat{I}_{d,k}^{A} \in [0,1]$ is the equalizer *a priori* mutual information of the d^{th} stream at the k^{th} *a priori* index and $\hat{\mathrm{f}}()$ is the EXIT function of the equalizer. Note that Appendix A summarizes the algorithm for iterative FD-SC-MMSE equalizer. The decoder output extrinsic information for the d^{th} stream with k^{th} *a priori* index is given by

$$\mathring{I}_{d,k}^{E} = \mathring{\mathrm{f}}(\mathring{I}_{d,k}^{A}) \tag{3}$$

where $\mathring{I}_{d,k}^{A} \in [0,1]$ is the decoder *a priori* mutual information of the d^{th} stream at the k^{th} *a priori* index and $\mathring{\mathrm{f}}()$ is the EXIT function of the channel decoder. In order to control the width of the convergence tunnel, we define also the acceptable gap between the equalizer and the decoder EXIT functions with the positive scalar $\epsilon_{d,k}$ and require that $\hat{I}_{d,k}^{E} \geqslant \mathring{\mathrm{f}}^{-1}(\mathring{I}_{d,k}^{E}) + \epsilon_{d,k}$. $\mathring{\mathrm{f}}^{-1}()$ denotes the inverse EXIT function of the channel decoder.

Now, the optimization problem can be formulated as follows:

$$\begin{array}{ll} \text{minimize} & \mathrm{tr}\{\mathbf{T}\mathbf{T}^{\mathrm{H}}\} \\ \text{subject to} & \hat{I}_{d,k}^{E} \geqslant \mathring{\mathrm{f}}^{-1}(\mathring{I}_{d,k}^{E}) + \epsilon_{d,k} \quad \forall d, \forall k \end{array} \tag{4}$$

where $\mathrm{tr}\{\}$ is the matrix trace operator, and the operator $^{\mathrm{H}}$ is Hermitian transpose of the vector/matrix.

The probability density function of the log-likelihood ratios (LLR) $\hat{\mathcal{L}}_d$ of the equalizer output for the d^{th} stream can be approximated by a Gaussian distribution satisfying the consistency requirement, i.e., $\hat{\mathcal{L}}_d \sim \mathcal{N}(\frac{\hat{\sigma}_d^2}{2}, \hat{\sigma}_d^2)$ [9]. Then, the EXIT function of iterative equalizer can be calculated by using J-function [12] with single parameter $\hat{\sigma}_d^2$. For the d^{th} stream, it is given by [3]

$$\hat{I}_{d,k}^{E} \approx J(\hat{\sigma}_{d,k}^2) \approx (1 - 2^{H_1 \hat{\sigma}_{d,k}^{2H_2}})^{H_3} \tag{5}$$

where $\hat{\sigma}_{d,k}^2$ is the variance of LLRs for the d^{th} stream at the iterative equalizer output at the k^{th} *a priori* index. For quaternary phase shift keying (QPSK) with Gray mapping, the constants are [9] $H_1 = 0.3073, H_2 = 0.8935$ and $H_3 = 1.1064$. By using (5) the variance of $(d,k)^{th}$ LLR at the output of equalizer becomes

$$\hat{\sigma}_{d,k}^2 \approx \tau(\kappa_1/2)^2 \tag{6}$$

where $\kappa_1 = \kappa_2^{\frac{1}{2H_2}}$ with the scalar $\kappa_2 = (\log_2(1 - \hat{I}_{d,k}^{E\,\frac{1}{H_3}}))^{-H1}$ and $\tau = 1$ for QPSK [9].

The mutual information constraints in (4) can be converted by using (6) into the equivalent constraints on the variance of LLRs by replacing $\hat{I}_{d,k}^{E}$ with $\mathring{\mathrm{f}}^{-1}(\mathring{I}_{d,k}^{E}) + \epsilon_{d,k}$. Thus, the LLR's variance at the input of the decoder, $\mathring{\sigma}_{d,k}^2$, for the d^{th} stream at the k^{th} *a priori* index can be solved. Thus, it turns out that the original constraints in the precoder optimization in (4) can be reformulated as the constraints on the variances of LLRs at the decoder input and equalizer output for each data stream.

Let us focus on the calculation of variance of LLRs at the output of the equalizer. The iterative FD equalizer output, given by (13) in Appendix A, is assumed to be Gaussian distributed, $\hat{\mathbf{b}}_d|\mathbf{b}_d \sim \mathcal{CN}(\mu_d \mathbf{b}_d, \mu_d(1-\mu_d))$. The scalar $\mu_d = \bar{\beta}_d \pi_d \varphi_d$ is the mean of the MMSE equalizer output where the scalars $\bar{\beta}_d, \pi_d$ and φ_d are defined in Appendix A. Thus, by using the average energy of the soft symbol, $\bar{\bar{\beta}}_d = 1 - \Delta_d$ with Δ_d being residual average interference energy after soft-cancellation for the d^{th} stream, the variance of LLRs for the d^{th} stream at the output of the equalizer can be expressed as

$$\hat{\sigma}_d^2 = \frac{\mu_d^2}{\mu_d(1-\mu_d)} = \frac{\hat{\zeta}_d}{1 - \Delta_d \hat{\zeta}_d}. \tag{7}$$

The scalar $\hat{\zeta}_d$ denotes signal-to-interference ratio (SINR) at the output of the frequency domain MMSE filter, given by

$$\hat{\zeta}_d = \mathrm{tr}\{\mathbf{\Gamma}\mathbf{T}_d\mathbf{T}_d^{\mathrm{H}}\mathbf{\Gamma}^{\mathrm{H}}(\mathbf{\Gamma}\mathbf{T}\mathbf{\Delta}\mathbf{T}^{\mathrm{H}}\mathbf{\Gamma}^{\mathrm{H}} + \sigma^2\mathbf{I})^{-1}\}. \tag{8}$$

The matrices $\mathbf{\Gamma}, \mathbf{T}_d$ and $\mathbf{\Delta}$ are defined in Appendix A.

Singular value decomposition (SVD) based transmit beamforming is utilized in the transmission. The transmitted streams can be perfectly decoupled from each other in the spatial domain when perfect CSI is available at the transmitter. As a result of the diagonalization procedure, given in Appendix B, the channel at the output of equalizer becomes fully diagonal. Hence, the SINR for the d^{th} stream at the k^{th} *a priori* index in (8) can be rewritten by using (16) as

$$\hat{\zeta}_{d,k} = \frac{1}{K_S}\sum_{b=1}^{K_S} \frac{S_{d,b}^2 P_{d,b}}{\sigma^2 + S_{d,b}^2 P_{d,b}\Delta_{d,k}} \tag{9}$$

[2]It is assumed that streams are coupled to each others.

[3]Block lengths are assumed to be large enough.

where the scalar $P_{d,b}$ is the power allocated to the d^{th} stream in the b^{th} frequency bin, and the scalar $S_{d,b}$ is its associated singular value. Note that $\Delta_{d,k}$ is defined similarly as Δ_d for the k^{th} *a priori* index.

We can now rewrite the optimization problem in (4) as

$$\begin{array}{ll} \text{minimize} & \sum_{d=1}^{D}\sum_{b=1}^{K_S} P_{d,b} \\ \text{subject to} & \frac{\hat{\zeta}_{d,k}}{1-\Delta_{d,k}\hat{\zeta}_{d,k}} \geqslant \mathring{\sigma}^2_{d,k} \quad \forall d, \forall k \end{array} \tag{10}$$

By (9), the constraints in (10) can be rewritten as

$$\frac{1}{K_S}\sum_{b=1}^{K_S}\frac{S^2_{d,b}P_{d,b}}{\sigma^2+S^2_{d,b}P_{d,b}\Delta_{d,k}} \geqslant \frac{\mathring{\sigma}^2_{d,k}}{1+\mathring{\sigma}^2_{d,k}\Delta_{d,k}}. \tag{11}$$

It is shown below that the power allocation over the streams and the frequency bins is a convex optimization problem. It is easy to see that the objective function in (10) is a linear function of the transmit power. Hence, the objective function is convex. Correspondingly, the left hand side of (11) is a concave function. It can be proven by analyzing Hessian of $\hat{\zeta}_{d,k}$ in (9) [16]. It is easy to see that Hessian is a diagonal matrix with non-positive entries on the diagonal where the b^{th} diagonal element of Hessian of $\hat{\zeta}_{d,k}$ in (9) is given by

$$\left[\nabla^2\hat{\zeta}_{d,k}\right]_{b,b} = -\frac{1}{K_S^2}\frac{2S^4_{d,b}\sigma^2\Delta_{d,k}}{(\sigma^2+S^2_{d,b}P_{d,b}\Delta_{d,k})^3} \leqslant 0. \tag{12}$$

The last inequality in (12) follows from the fact that $S^4_{d,b}\sigma^2\Delta_{d,k} \geqslant 0$ and $(\sigma^2+S^2_{d,b}P_{d,b}\Delta_{d,k}) \geqslant 0$. Therefore, the problem of the power allocation over the data streams as well as over the frequency bins is a convex optimization problem. Thus, it can be solved easily by using standard optimization tools, e.g interior-point methods [16, Ch. 11].

Algorithm 1 described in Table 1 summarizes the power allocation algorithm for iterative FD SC-MMSE equalizer in point-to-point MIMO system, where $\mathcal{M}()$ denotes soft mapping operation which calculates the first order moment of the soft-symbol estimates. The inverse J-Function $J^{-1}()$ is defined similarly as in [12].

Algorithm 1 Power allocation algorithm for iterative frequency domain equalizer in point-to-point MIMO

1) Generate *a priori* vectors, $\tilde{\mathbf{c}}_{d,k} \in \mathbb{R}^{QK_S\times 1}$, for all k and d, $\tilde{\mathbf{c}}_{d,k} = \frac{\eta^2_{d,k}}{2}\mathbf{c}_{d,k} + \mathbf{n}_{d,k}$, where Q is the number of bits per symbol, $\eta^2_{d,k} = J^{-1}(\hat{I}^A_{d,k})$, $\mathbf{n}_{d,k} \in \mathbb{R}^{QK_S\times 1}$,$\mathbf{n}_{d,k} \sim N(0,\eta^2_{d,k})$ and $\mathbf{c}_{d,k} \in \mathbb{R}^{QK_S\times 1}$ where the elements takes values $\{\mp 1\}$
2) $\tilde{\mathbf{b}}_{d,k} = \mathcal{M}(\tilde{\mathbf{c}}_{d,k})$, $\tilde{\mathbf{b}}_{d,k} \in \mathbb{C}^{K_S\times 1}$, $k = 1, ..., K$, $d = 1, ..., D$
3) Calculate $\Delta_{d,k} = 1 - avg\{\tilde{\mathbf{b}}_{d,k}\}$ $k = 1, ..., K$, $d = 1, ..., D$
4) Calculate $\mathring{\sigma}^2_{d,k}$ by Eq.(6), $k = 1, ..., K$, $d = 1, ..., D$
5) minimize $\sum_{d=1}^{D}\sum_{b=1}^{K_S} P_{d,b}$ by Eq. (11) subject to $\frac{1}{K_S}\sum_{b=1}^{K_S}\frac{S^2_{d,b}P_{d,b}}{\sigma^2+\Delta_{d,k}S^2_{d,b}P_{d,b}} \geqslant \frac{\mathring{\sigma}^2_{d,k}}{1+\mathring{\sigma}^2_{d,k}\Delta_{d,k}}$ $\forall d, k$
6) Check the power budget $\sum_{d=1}^{D}\sum_{b=1}^{K_S} P_{d,b} \leqslant \rho_u$. If the power budget is not satisfied relax the code parameters and re-start the optimization from Step (4).

In order to obtain the EXIT functions with the proposed precoder design, Algorithm 1 has to be first performed to obtain optimal transmission power allocation. After this, the variance of the LLRs is calculated by using (7) with the obtained optimal transmit power allocation. Finally, mutual information at the output of iterative equalizer for each stream is computed by using the J-function in (5).

IV. Numerical Results

The simulation parameters used are summarized as follows. The number of receiver and transmit antennas $R = 2, T = 2$, respectively, streams $D = 2$, $\epsilon_{d,k} = 0.02$ for $\forall k, \forall d$, QPSK with Gray mapping, and a systematic repeat accumulate (RA) codes [13] with two different code rates, 1/3 and 1/4 were used. A static complex valued channel 5-path was assumed in the simulation scenarios considered. The channel realizations are shown in Table I for $T = R = 2$. MinSumMSE [10] and MaxRate [11] based linear precoding schemes were also used for comparison. Due to orthonormal beamforming matrix $\mathbf{V}$, the EXIT function of the iterative equalizer for the considered stream become independent of the rest of the streams in all the precoding schemes considered in the simulations. Hence, the EXIT analysis is two-dimensional. To obtain the EXIT trajectories, random interleaver with a size of 74240 bits is used. Standard convex optimization tools were used in performing Step 5) of Algorithm 1. The optimization constraints were derived according to Algorithm 1 using each decoder's EXIT curve. In the simulations, we declared convergence point to be the last K^{th} *a priori* constraint point for each stream. The convergence points are changed by simulation case-by-case whereas the rest $K-1$ *a priori* constraints points are kept fixed.

Since this paper's proposed scheme, CCP, is based on the utilization of the equalizer EXIT function approximation given in [9], it is important to verify the accuracy of the approximation. In fact, it was shown in [9] that the J-function based approximation is useful without precoding. Hence, we verify the accuracy by comparing the approximated EXIT function of the equalizer with the one obtained by histogram measurements, both with precoding. Figure 1 shows the results of the verification simulations, where $T = R = 2, D = 2$ and $SNR = -1$ dB. The EXIT curves were obtained by using 600 blocks per each a priori index, and K_S=9014. In addition, we set the convergence points at 0.54 and 0.4 for the stream $d = 1$ and $d = 2$, respectively. It is found that the two curves are almost identical, indicating that the J-function approximation works properly also with precoding. Furthermore, the EXIT trajectory is also depicted in the same figure. As can be seen, the trajectories slightly deviate from the EXIT functions. The reason for this is that EXIT functions assume the length of codeword to be infinite, which is not always the case in the trajectory evaluation. Finally, the accuracy of the EXIT analysis results was verified a series of Monte Carlo bit error rate (BER) simulations. However, due to the space limitations, we were not able to include results into this paper. Thus, we refer those results by mentioning that Monte Carlo BER and the estimated BER from EXIT charts are consistent with each other except when $d = 2$ and the number of iterations is over 11. The reason for this is again the infinite codeword length assumption in EXIT analysis.

Figure 2 compares EXIT functions of different precoding schemes for $T = R = 2, D = 2$ where each precoding scheme is required to achieve at least the desired convergence points. The convergence points were set at 0.7 and 0.5 for $d = 1$ and $d = 2$, respectively. It is found that the proposed CCP scheme can achieve the desired convergence points and simultaneously minimize the gap between the two curves with 2.5 dB and 3.5 dB less transmission power compared to MinSumMSE and MaxRate, respectively. By contrast, the both MinSumMSE and MaxRate criterions do not provide any way to control the gap between equalizer and decoder EXIT functions. Even though the beamforming matrix decouples perfectly the streams from each other in the spatial domain in all the considered precoding schemes, inter-symbol interference (ISI) components still remain in the received signal. Furthermore, additional stream-wise ISI is introduced by the power allocation. The impact of the power allocation can be observed when the received signal is rewritten with the help of SVD, as summarized as $\mathbf{r} = \mathbf{F}_D^{-1}\mathbf{S}\mathbf{P}^{\frac{1}{2}}\mathbf{F}_D\mathbf{b} + \mathbf{F}_D^{-1}\mathbf{D}^{\mathrm{H}}\mathbf{F}_R\mathbf{v}$, where the product of matrices, $\mathbf{F}_D^{-1}\mathbf{S}\mathbf{P}^{\frac{1}{2}}\mathbf{F}_D$, is blockwise circulant matrix and where $\mathbf{D} \in \mathbb{C}^{RK_S \times RK_S}$ is left-hand eigenvectors of the frequency domain channel matrix .

In order to demonstrate the advantageous points of the proposed precoder design, let us now consider the case where convergence point is set at 0.98 for $d = 1$ and at 0.98 for $d = 2$ both with $\epsilon_{d,k} = 0.02$. Figure 3 compares the convergence properties of the precoding methods when $T = R = D = 2$ and SNR is fixed to 5.9 dB for all the schemes. Notice that the area between equalizer and decoder curves can be minimized while keeping $\epsilon_{d,k} = 0.02$, for the both streams with the proposed precoding scheme. Moreover, by using the proposed precoding scheme, the right most point of the EXIT chart (1,1) can be achieved which implies that nearly error-free performance can be achieved, given that sufficient number of iterations corresponding to the $\epsilon_{d,k}$, is performed. By contrast, the MaxRate and the MinSumMSE criterion are not able to achieve the right most point of the EXIT chart with the both streams.

V. Conclusions

We have proposed a novel linear precoder design technique for singe-user MIMO with iterative FD SC-MMSE equalization, where account is taken of convergence properties the equalizer. The proposed precoder design technique, convergence constraint precoding (CCP) minimizes the transmission power for each stream while achieving the target mutual information after the iterations at the receiver side. It has been shown that the optimality criterion for the proposed design can be formulated as a convex optimization problem, for which the existence of the globally optimum solution of the power allocation over the data streams and frequency bins can be guaranteed. We have also demonstrated that our proposed technique outperforms the existing linear precoding techniques in the sense of convergence assurance and with reduced transmission power.

Appendix A
Frequency Domain SC-MMSE and Soft-demapping

Due to the space limitations only the key equations for the iterative frequency domain soft-cancellation and MMSE receiver are provided. However, more detailed explanations can be found [3] [4]. Similarly, as in [14], the residual interference energy within each stream is approximated to be time invariant. Therefore, the MMSE filter output can be written as [3]

$$\hat{\mathbf{b}}_d = \overline{\dot{\beta}}_d \pi_d (\mathbf{F}_S^{-1}\mathbf{T}_d^{\mathrm{H}}\mathbf{\Gamma}^{\mathrm{H}}\mathbf{\Sigma}_{\hat{r}}^{-1}\hat{\mathbf{r}} + \varphi_d \tilde{\mathbf{b}}^d) \quad (13)$$

where the real scalar $\overline{\dot{\beta}}_d$ is obtained as $\overline{\dot{\beta}}_d = \mathrm{avg}\left\{\dot{\mathbf{b}}_d\right\}$. The vector $\dot{\mathbf{b}}^d \in \mathbb{C}^{K_S \times 1}$ is given by $\dot{\mathbf{b}}^d = \left[E\left\{|b_1^d|^2\right\} \dots E\left\{|b_{k_S}^d|^2\right\} \dots E\left\{|b_{K_S}^d|^2\right\}\right]^{\mathrm{T}}$ with $b_{k_S}^d$ being k_S^{th} transmitted symbol of the d^{th} stream.The real scalar π_d is given by $\pi_d = \frac{1}{\varphi_d \overline{\ddot{\beta}}_d + 1}$ with being $\overline{\ddot{\beta}}_d = \mathrm{avg}\left\{\ddot{\mathbf{b}}^d\right\}$. The vector $\ddot{\mathbf{b}}^d \in \mathbb{C}^{K_S \times 1}$ is given by $\ddot{\mathbf{b}}^d = \left[|\tilde{b}_1^d|^2 \dots |\tilde{b}_{k_S}^d|^2 \dots |\tilde{b}_{K_S}^d|^2\right]^{\mathrm{T}}$ with $\tilde{b}_{k_S}^d$ being soft estimate of k_S^{th} transmitted symbol of the d^{th} stream. $\tilde{\mathbf{b}}^d \in \mathbb{C}^{K_S \times 1}$ represents the soft estimate of the d^{th} transmitted stream and it is given by $\tilde{\mathbf{b}}^d = \left[\tilde{b}_1^d \dots \tilde{b}_{k_S}^d \dots \tilde{b}_{K_S}^d\right]^{\mathrm{T}}$. Reference [14] describes in detail the first two moments of soft-symbol estimates, $\tilde{b}_{k_S}^d = E\left\{b_{k_S}^d\right\}$ and $E\left\{|b_{k_S}^d|^2\right\}$. The operator avg{} calculates the vector-wise average from its argument vector. The matrix $\mathbf{T}^d \in \mathbb{C}^{TK_S \times K_S}$ is precoder matrix of d^{th} stream and it contains all rows from the $(d-1)K_S + 1$ to dK_S-th columns in $\mathbf{T}$. The scalar, φ_u^d, is given by

$$\varphi_d = \frac{1}{K_S}\mathrm{tr}\left\{\mathbf{T}^{d^{\mathrm{H}}}\mathbf{\Gamma}^{\mathrm{H}}\mathbf{\Sigma}_{\hat{r}}^{-1}\mathbf{\Gamma}\mathbf{T}^d\right\}. \quad (14)$$

The covariance matrix of the output of soft-cancellation, $\mathbf{\Sigma}_{\hat{r}} \in \mathbb{C}^{RK_S \times RK_S}$, is given by $\mathbf{\Sigma}_{\hat{r}} = \mathbf{\Gamma}\mathbf{T}\mathbf{\Delta}\mathbf{T}^{\mathrm{H}}\mathbf{\Gamma}^{\mathrm{H}} + \sigma^2\mathbf{I}$. The residual interference energy, $\mathbf{\Delta} \in \mathbb{R}^{DK_S \times DK_S}$, after soft-cancelation is defined similarly as in [14]. $\mathbf{\Gamma} \in \mathbb{C}^{RK_S \times TK_S}$ is the block-wise diagonal frequency domain channel matrix. The output of soft-cancellation, $\hat{\mathbf{r}} \in \mathbb{C}^{RK_S \times 1}$, is given by $\hat{\mathbf{r}} = \mathbf{F}_R\mathbf{r} - \mathbf{\Gamma}\mathbf{T}\mathbf{F}_D\tilde{\mathbf{b}}$. The vector $\tilde{\mathbf{b}} \in \mathbb{C}^{DK_S \times 1}$ is given by $\tilde{\mathbf{b}} = \left[\tilde{\mathbf{b}}^1 \dots \tilde{\mathbf{b}}^d \dots \tilde{\mathbf{b}}^D\right]^{\mathrm{T}}$. Finally, the soft-demodulator computes equalizer output LLRs by assuming Gaussian distributed MMSE filter output for QPSK as follows [15] $\hat{\mathcal{L}}_d[2k_S - 1, 1] = \frac{\sqrt{8}}{1-\mu_d}\Re(\hat{\mathbf{b}}_d)$ and $\hat{\mathcal{L}}_d[2k_S, 1] = \frac{\sqrt{8}}{1-\mu_d}\Im(\hat{\mathbf{b}}_d)$, $k_S = 1, ..., K_S$.

Appendix B
Diagonalization Procedure

In this section it is shown that a point-to-point MIMO channel is fully diagonalized at the output of equalizer. Firstly, matrix inversion lemma and a matrix manipulation[4] is applied into (7). Then, SINR over the all the streams is given by

$$\hat{\zeta} = \mathrm{tr}\{(\mathbf{I}_{DK_S} + \mathbf{\Gamma}^{\mathrm{H}}\mathbf{T}^{\mathrm{H}}\mathbf{\Sigma}_v^{-1}\mathbf{\Gamma}\mathbf{T}\mathbf{\Delta})^{-1}\mathbf{T}^{\mathrm{H}}\mathbf{\Gamma}^{\mathrm{H}}\mathbf{\Sigma}_v^{-1}\mathbf{\Gamma}\mathbf{T}\}. \quad (15)$$

where $\mathbf{\Sigma}_v = \sigma^2\mathbf{I} \in \mathbb{R}^{RK_S \times RK_S}$ and $\mathbf{I}_{DK_S} \in \mathbb{R}^{DK_S \times DK_S}$. Now, the block-wise SVD has to be computed at the transmitter for the pre-whitened channel matrix $\mathbf{\Gamma}^{\mathrm{w}} = \mathbf{\Sigma}_v^{-\frac{1}{2}}\mathbf{\Gamma}$ with $\mathbf{\Sigma}_v^{-\frac{1}{2}}$ being the square root of the matrix $\mathbf{\Sigma}_v^{-1}$. Hence, permutation matrices $\mathbf{\Theta}_R \in \mathbb{R}^{RK_S \times RK_S}$ and $\mathbf{\Theta}_T \in \mathbb{R}^{TK_S \times TK_S}$ are introduced. By using $\mathbf{\Theta}_R$ and $\mathbf{\Theta}_T$, the space-frequency

[4] $(\mathbf{I} + \mathbf{A}\mathbf{B})^{-1} = \mathbf{I} - \mathbf{A}(\mathbf{I} + \mathbf{B}\mathbf{A})^{-1}\mathbf{B}$

TABLE I: Channel coefficients.

r, t	$h_1^{r,t}$	$h_2^{r,t}$	$h_3^{r,t}$	$h_4^{r,t}$	$h_5^{r,t}$
1,1	-0.5011 + 0.5863i	0.0971 + 0.1784i	0.0283 + 0.5156i	0.2561 + 0.5342i	-0.3368 - 0.6815i
2,1	0.3282 + 0.6185i	0.2156 - 0.0657i	0.2715 + 0.4530i	0.0343 - 0.1345i	-0.1076 + 0.2743i
1,2	-0.1167 + 0.0838i	-0.2615 + 0.0212i	0.3575 + 0.6740i	-0.1428 + 0.4413i	-0.1300 + 0.0488i
2,2	0.0527 - 0.1101i	-0.2916 - 0.2309i	-0.5856 - 0.2035i	-0.2139 - 0.3449i	-0.2386 - 0.1878i

channel matrix $\mathbf{\Gamma}_u$ can be re-written as a block diagonal matrix $\hat{\mathbf{\Gamma}} = \mathbf{\Theta}_R \mathbf{\Gamma}^{\mathrm{w}} \mathbf{\Theta}_T$ where each block of the block diagonal matrix is with dimensions $R \times T$. The SVD can now be performed for each block of matrix, $\hat{\mathbf{\Gamma}}$, separately resulting block diagonal unitary matrices $\hat{\mathbf{U}} \in \mathbb{C}^{RK_S \times RK_S}$ and $\hat{\mathbf{V}} \in \mathbb{C}^{TK_S \times TK_S}$ with each block containing left and right singular matrices of the matrix $\hat{\mathbf{\Gamma}}$, respectively. Moreover, the diagonal matrix, $\hat{\mathbf{S}} \in \mathbb{R}^{RK_S \times TK_S}$ contains the singular values of matrix $\hat{\mathbf{\Gamma}}$. Finally, we can express on singular value matrix $\mathbf{S} \in \mathbb{R}^{RK_S \times TK_S}$, and unitary singular matrices, $\mathbf{U} \in \mathbb{C}^{RK_S \times RK_S}$ and $\mathbf{V} \in \mathbb{C}^{TK_S \times TK_S}$, in the original matrix structure as $\mathbf{U} = \mathbf{\Theta}_R^{\mathrm{T}} \hat{\mathbf{V}} \mathbf{\Theta}_R$, $\mathbf{V} = \mathbf{\Theta}_T^{\mathrm{T}} \hat{\mathbf{V}} \mathbf{\Theta}_T$, $\mathbf{S} = \mathbf{\Theta}_R^{\mathrm{T}} \hat{\mathbf{S}} \mathbf{\Theta}_T$. In this way, (15) can be re-written in fully diagonalized form as

$$\hat{\zeta} = \mathrm{tr}\{(\mathbf{I}_{DK_S} + \mathbf{\Sigma}_v^{-1} \mathbf{S}^2 \mathbf{P} \mathbf{\Delta})^{-1} \mathbf{S}^2 \mathbf{P} \mathbf{\Sigma}_v^{-1}\}. \quad (16)$$

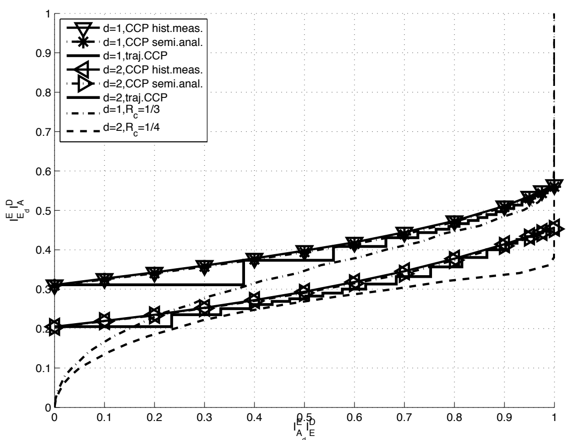

Fig. 1: Verification simulations: $T = R = 2, D = 2, SNR = -1$ dB.

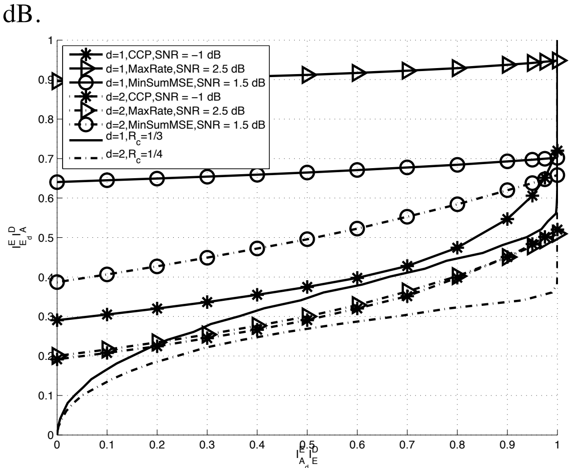

Fig. 2: Convergence points set at 0.7 for $d = 1$ and at 0.5 for $d = 2$,$T = R = 2, D = 2$ (solid lines $d = 1$, dash-dotted lines $d = 2$).

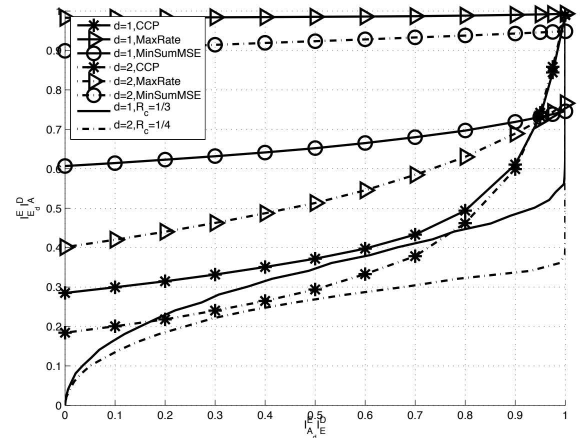

Fig. 3: Convergence points set at 0.98 for $d = 1$ and at 0.98 for $d = 2$,$T = R = 2, D = 2$, $SNR = 5.9$ dB (solid lines $d = 1$, dash-dotted lines $d = 2$).

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