Frequency Domain Joint-over-Antenna MIMO Turbo Equalization

Juha Karjalainen, Kimmo Kansanen, Nenad Veselinović, and Tad Matsumoto
Centre for Wireless Communications
University of Oulu, Finland
phone:+358 8 553 7637, fax: +358 8 553 2845
e-mail: {juha.karjalainen, kimmo.kansanen, nenad.veselinovic, tadashi.matsumoto}@ee.oulu.fi

***Abstract*— This paper proposes a novel iterative frequency domain multiple input multiple output (MIMO) signal detection technique for the reception of overloaded spatially multiplexed MIMO transmission for single carrier signalling in frequency-selective channels. In the presence of spatial correlation as well as the case where there are more transmit antennae than receive antennae (refered to as overloaded transmission in this paper) the performance of antenna-by-antenna (AA)-based MIMO minimum mean square error (MMSE) receivers are, in general, degraded. Therefore, we propose joint-over-antenna (JA) detection-based iterative frequency domain technique to improve the performance of receiver in the presence of overloaded antenna transmission and spatial correlation. The proposed receiver is based on the combination of maximum aposteriori probability (MAP) algorithm, and soft-cancellation (SC) and MMSE filtering for turbo-coded single carrier point-to-point MIMO systems. In the proposed receiver MIMO signal detection is comprised of two stages; At the first stage, inter-symbol interference (ISI) is suppressed with MMSE filtering. At the second stage, co-antenna interference (CAI) is suppressed by the MAP algorithm and transmitter antennae are decomposed each other. Simulation results show that performance gain of JA compared to AA technique is significant in the presence of spatial correlation as well as in overloaded cases. The performance comparison is made in terms of frame error rate (FER) with different antenna configurations and different spatial correlations in point-to-point MIMO frequency-selective fading channels.**

I. Introduction

The motivation of this paper is that in many of the applications, because of the practical reasons it is not always reasonable to assume balanced multiple input multiple output (MIMO) configurations ($T = R$, where T and R are the number of transmit and receiver antennas, respectively), which results the most likely in $R < T$ (overloaded cases); Even if the same number of antennae can be located at the both transmitter and receiver sides, antennae are the most probable spatially correlated. Therefore, to achieve robustness in MIMO transmission performance the overloaded cases and/or in the presence of high spatial correlation is of significant importance.

Broadband single carrier point-to-point MIMO communication requires receiver to be able to reduce distortions caused by intersymbol interference (ISI) and co-antenna interference (CAI). One of the most promising techniques that can achieve excellent performance is soft-cancellation (SC) and minimum mean squeare error (MMSE) filtering based iterative receiver [1].

Recently, several frequency-domain processing techniques for MMSE MIMO turbo equalization have been proposed, which are known to be able to significantly reduce the complexity [2], [3], [4]. The frequency domain techniques proposed by [2] and [4] aim to detect signals transmitted from the multiple antennas on an antenna-by-antenna (AA) basis. Therefore, it is expected that their perfomances are degraded in the presence of spatially correlated antennae as well as overloaded transmission. Recently, [5] proposed a joint-over antenna (JA) signal detection technique based on SC-MMSE for space-time trellis coded multiuser MIMO systems. However, its computation is in the time domaim, for which the cubic order complexity of due to the matrix inversion for MMSE is still required. In this paper, we investigate spatially multiplexed (SM) point-to-point-MIMO transmission with the frequency domain AA and JA detection techniques. Impact of the overloaded transmission and spatial correlation on performances of the both schemes are evaluated and compared through simulations in frequency selective MIMO channels with spatial correlation.

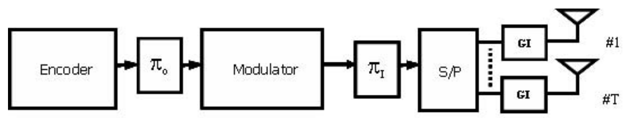

Fig. 1. Transmitter for vertically encoded spatially multiplexed point-to-point MIMO.

II. System Model

Figure 1 shows a transmitter model for the considered vertically encoded spatially multiplexed point-to-point MIMO system with T transmit antennas. The considered system uses cyclic-prefix single carrier burst transmission. Since the cyclic-prefix burst transmission technique is very well known [6], details are not described in this paper. After guard period removal,[1] a space-time presentation of the signal, $\mathbf{r}$, $\mathbf{r} \in \mathbb{C}^{RK\times 1}$, received by R received antennas is given by,

$$\mathbf{r} = \hat{\mathbf{H}}\mathbf{b} + \mathbf{v}, \tag{1}$$

where $\mathbf{v}$, $\mathbf{v} \in \mathbb{C}^{RK\times 1}$, is white additive i.i.d Gaussian noise vector with variance σ^2, K is the length of Discrete Fourier Transform (DFT), $\mathbf{b}, \in \mathbb{C}^{TK\times 1}$ defines transmitted layers over, T transmit antennae

$$\mathbf{b} \quad = \quad [\mathbf{b}^t, ..., \mathbf{b}^T]^\dagger \tag{2}$$

where $\mathbf{b}^t, \in \mathbb{C}^{K\times 1}$

$$\mathbf{b}^t \quad = \quad [b_1^t, ..., b_K^t]^\dagger \tag{3}$$

contains transmitted symbols of t^{th}, $t = 1, ..., T$, layer, the sum power over all layers is normalized to one for each symbol time instant, $\dagger$ determines vector/matrix transpose operator. A circulant block channel matrix, $\hat{\mathbf{H}}$, $\hat{\mathbf{H}} \in \mathbb{C}^{RK\times TK}$, is then given by

$$\hat{\mathbf{H}} = \begin{bmatrix} \hat{\mathbf{H}}^{1,1} & \dots & \hat{\mathbf{H}}^{1,T} \\ \vdots & \ddots & \vdots \\ \hat{\mathbf{H}}^{R,1} & \dots & \hat{\mathbf{H}}^{R,T} \end{bmatrix}, \tag{4}$$

[1]We restrict ourselves to study only signal processing of received signal where the length of guard period is at least channel memory.

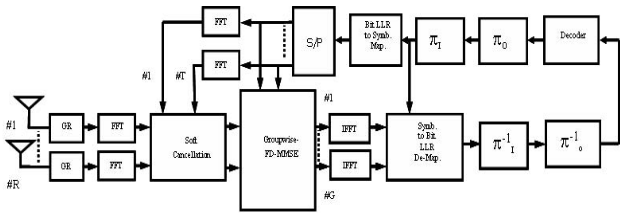

Fig. 2. Iterative frequency domain JA MIMO receiver.

where, $\hat{\mathbf{H}}^{r,t}$, $\hat{\mathbf{H}}^{r,t} \in \mathbb{C}^{K \times K}$ being channel submatrice between the t^{th} transmit and r^{th} receive antenna pair, $r = 1, ..., R$, is circulant, given as

$$\hat{\mathbf{H}}^{r,t} = circ\left\{ \left[h_0^{r,t}, h_1^{r,t} \dots h_L^{r,t} \right]^{\dagger} \right\}. \tag{5}$$

L is the length of channel memory, and $h_l^{r,t}$, $l = 0, ..., L-1$ describe the fading gains of multipath channel for t^{th} transmit and r^{th} receive antenna pair. In the each transmitter receiver antenna pair the sum power of fading gains is normalized to one.

It is well known that the circulant matrices can be diagonalized by the unitary DFT matrix $\mathbf{F}$, $\mathbf{F} \in \mathbb{C}^{K \times K}$, [7] with the elements $f_{m,k}$ = $\exp^{\frac{j2\pi mk}{K}}$, $m, k = 0, ..., K-1$. Similarly, the circulant block matrices can be block-diagonalized by using block diagonal DFT matrices. The block-diagonalization process for, $\hat{\mathbf{H}}$ is summarized below:

$$\hat{\mathbf{H}} = \mathbf{F}_R^{-1} \mathbf{\Gamma} \mathbf{F}_T, \tag{6}$$

where $\mathbf{\Gamma}$ is corresponding diagonal block matrix, $\mathbf{\Gamma} \in \mathbb{C}^{RK \times TK}$, $\mathbf{F}_R^{-1} = \frac{1}{K}\mathbf{F}_R^{\ddagger}$ is the unitary block Inverse Discrete Fourier Transform (IDFT) matrix, $\mathbf{F}_R^{-1} \in \mathbb{C}^{RK \times RK}$, ‡ indicates Hermitian transpose, $\mathbf{F}_R$ is block-diagonal DFT matrix, $\mathbf{F}_R \in \mathbb{C}^{RK \times RK}$, given by

$$\mathbf{F}_R = \mathbf{I}_R \otimes \mathbf{F} \tag{7}$$

for R received antennae, where $\otimes$ indicates Kronecker product. $\mathbf{F}_T \in \mathbb{C}^{TK \times TK}$, given by

$$\mathbf{F}_T = \mathbf{I}_T \otimes \mathbf{F} \tag{8}$$

for the transmit antennae, where $\mathbf{I}_T$ is the identity matrix, $\mathbf{I}_T \in \mathbb{C}^{T \times T}$. Signal-to-noise ratio per receiver antenna is defined as ratio of information bit power and noise power as follows

$$\frac{E_b}{N_o} = \frac{P_s}{2\sigma^2 R_c Q}, \tag{9}$$

where, transmitted symbol energy $P_s = \frac{1}{K} Trace\left\{E\left\{\mathbf{b}^{\ddagger}\mathbf{b}\right\}\right\}$, Q number of bits per symbol, and R_c the coding rate of error correcting code.

III. Iterative Frequency Domain Equalization

The iterative frequency domain JA receiver depicted in Fig. 2 exchange iteratively extrinsic information between two soft input soft output (SfISfO) stages; equalization and channel decoder. The extrinsic information exchange follows the turbo principle. The aim of the equalization stage is to mitigate ISI and CAI. The equalization stage consists of two stages; groupwise filtering and joint-detection. Correspondingly, the channel decoder task is to generate soft decisions of the decoded bits in the time domain based on the a priori information for the coded bits and the trellis structure of the consituent codes.

A. Frequency Domain Groupwise MMSE Filter

In the groupwise filtering the residual ISI is supressed within the same transmit antenna group, and CAI is suppressed between different transmit antenna groups. The number of jointly detected antennas, G, and the number of transmit antennas T, defines number of jointly detected antenna groups, J, as follows: $J = [T/G]$. The frequency domain groupwise filter coefficients, $\mathbf{\Omega}^j$, $\mathbf{\Omega}^j \in \mathbb{C}^{RK \times GK}$, for j^{th} jointly detected antenna group and virtual antenna matrix $\mathbf{A}^j$, $\mathbf{A}^j \in \mathbb{C}^{GK \times GK}$, of equivalent Gaussian channel are determined according to the following MMSE criterion:

$$[\mathbf{\Omega}^j, \mathbf{A}^j] = \arg\min_{\mathbf{\Omega}^j, \mathbf{A}^j} E\left\{ \left\| \mathbf{F}_G^{-1} {\mathbf{\Omega}^j}^{\dagger} \hat{\mathbf{r}}^j - \mathbf{S}(n) {\mathbf{A}^j}^{\dagger} \mathbf{b}^j \right\|^2 \right\} \tag{10}$$

where, $\mathbf{S}(n)$, $\mathbf{S}(n) \in \mathbb{R}^{GK \times GK}$, is a time-varying sampling matrix having ones at main diagonal at a time of intrest with rest of the elements being zeros, $\mathbf{b}^j$, $\mathbf{b}^j \in \mathbb{C}^{GK \times 1}$, is j^{th},$j = 1 \cdots J$, desired transmit antenna group to be jointly detected. $\mathbf{F}_G$, $\mathbf{F}_G \in \mathbb{C}^{GK \times GK}$, is defined as $\mathbf{F}_G = \mathbf{I}_G \otimes \mathbf{F}$, where $\mathbf{I}_G$ is the identity matrix, $\mathbf{I}_G \in \mathbb{C}^{G \times G}$. The vector, $\hat{\mathbf{r}}^j$, $\hat{\mathbf{r}}^j \in \mathbb{C}^{RK \times 1}$, combines the output of the soft-canceller with desired transmit antenna group signals as follows

$$\hat{\mathbf{r}}^j = \hat{\mathbf{r}} + \mathbf{F}_R \hat{\mathbf{H}}^j \mathbf{S}(n) \tilde{\mathbf{b}}^j, \tag{11}$$

where $\tilde{\mathbf{b}}^j$, $\tilde{\mathbf{b}}^j \in \mathbb{C}^{GK \times 1}$ is j^{th} soft estimate group of layers to be jointly detected. The block-circulant channel matrix $\hat{\mathbf{H}}^j$, $\hat{\mathbf{H}}^j \in \mathbb{C}^{RK \times GK}$, corresponds to j^{th} transmit antenna group to be jointly detected. $\hat{\mathbf{r}}$, $\hat{\mathbf{r}} \in \mathbb{C}^{RK \times 1}$, is the residual interference signal in the frequency domain after cancellation of known signal components from received signal as follows

$$\hat{\mathbf{r}} = \mathbf{F}_R \mathbf{r} - \mathbf{\Gamma} \mathbf{F}_T \tilde{\mathbf{b}}. \tag{12}$$

$\tilde{\mathbf{b}}$, $\tilde{\mathbf{b}} \in \mathbb{C}^{TK \times 1}$ represents soft estimate of transmitted layers

$$\tilde{\mathbf{b}} \quad = \quad [\tilde{\mathbf{b}}^t, ..., \tilde{\mathbf{b}}^T]^{\dagger}. \tag{13}$$

The virtual antenna matrix $\mathbf{A}^j$ is given by

$$ {\mathbf{A}^j}^{\dagger} = \begin{bmatrix} \mathbf{A}^{1,1\dagger} & \dots & \mathbf{A}^{1,G\dagger} \\ \vdots & \ddots & \vdots \\ \mathbf{A}^{G,1\dagger} & \dots & \mathbf{A}^{G,G\dagger} \end{bmatrix} \tag{14}$$

with submatrix $\mathbf{A}^{u,v\dagger}$, $\mathbf{A}^{u,v\dagger} \in \mathbb{C}^{K \times K}$, being the virtual channel matrix between the u^{th} virtual transmit and the v^{th} transmit antenna pair, given by

$$\mathbf{A}^{u,v\dagger} = diag\left\{ \left[a_0^{k,k^{\ddagger}} \dots a_0^{K,K^{\ddagger}} \right]^{\dagger} \right\}. \tag{15}$$

$a_0^{k,k}$ is the first path of the virtual equivalent channel matrix and $\ddagger$ indicates complex conjugation. The optimization problem of (10) can be re-written as

$$[\mathbf{\Omega}^j, \mathbf{A}^j] = \arg \min_{\mathbf{\Omega}^j, \mathbf{A}^j} E\left\{ \left\| {\mathbf{G}^j}^{\ddagger} \mathbf{y}^j \right\|^2 \right\}, \tag{16}$$

where ${\mathbf{G}^j}^{\ddagger}$, ${\mathbf{G}^j}^{\ddagger} \in \mathbb{C}^{GK \times (R+G)K}$, is given by

$${\mathbf{G}^j}^{\ddagger} = \begin{bmatrix} \mathbf{F}_G^{-1} {\mathbf{\Omega}^j}^{\ddagger} & -\mathbf{S}(n) {\mathbf{A}^j}^{\ddagger} \end{bmatrix}, \tag{17}$$

and $\mathbf{y}^j$, $\mathbf{y}^j \in \mathbb{C}^{(R+G)K \times 1}$, by

$$\mathbf{y}^j = \begin{bmatrix} \hat{\mathbf{r}}^j \\ \mathbf{b}^j \end{bmatrix}. \tag{18}$$

$\mathbf{\Omega}^j$ and $\mathbf{A}^j$ are subject to constraint in order to avoid the trivial solution $[\mathbf{\Omega}^j, \mathbf{A}^j] = [0, 0]$. The path constraint is imposed to (16) in the same way as in [5]. The solution to this optimization problem of is obtained by following the derivation given in [5]. The filter coefficients in (16) can be re-written as

$$\mathbf{G}^j = \left[\mathbf{G}^1 \dots \mathbf{G}^G\right], \tag{19}$$

where $\mathbf{G}^g$, $\mathbf{G}^g \in \mathbb{C}^{(R+G)K\times K}$ is determined as

$$\mathbf{G}^g = \mathbf{\Sigma}_y^{j^{-1}} \mathbf{E}^g (\mathbf{E}^{g^\ddagger} \mathbf{\Sigma}_y^{j^{-1}} \mathbf{E}^g)^{-1}, \tag{20}$$

where $\mathbf{E}^g$, $\mathbf{E}^g \in \mathbb{R}^{(R+G)K\times K}$, is given by

$$\mathbf{E}^g = [\mathbf{0}_{RK}^g \mathbf{I}_K \mathbf{0}_K^g]^\dagger. \tag{21}$$

$\mathbf{O}_{RK}^g$ is zero matrix, $\mathbf{O}_{RK}^g \in \mathbb{R}^{((g-1)K+(RK))\times K}$, and $\mathbf{I}_K$ is identity matrix $\mathbf{I}_K \in \mathbb{R}^{K\times K}$ and $\mathbf{O}_K^g$ is identity matrix $\mathbf{O}_K^g \in \mathbb{R}^{((G-g)K)\times K}$. $\mathbf{\Sigma}_y^{j^{-1}}$ is obtained by applying the block-matrix inversion Lemma [8] , which results in

$$\mathbf{\Sigma}_y^{j^{-1}} = \begin{bmatrix} \mathbf{\Sigma}_{\hat{r}}^{j^{-1}} & -\mathbf{\Sigma}_{\hat{r}}^{j^{-1}} \mathbf{\Phi}^j \\ -\dot{\mathbf{B}}^j \dot{\mathbf{B}}^{j^\ddagger} \mathbf{\Phi}^{j^\ddagger} \mathbf{\Sigma}_{\hat{r}}^{j^{-1}} & \dot{\mathbf{B}}^{j^{-1}} + \mathbf{\Phi}^{j^\ddagger} \mathbf{\Sigma}_{\hat{r}}^{j^{-1}} \mathbf{\Phi}^j \end{bmatrix} \tag{22}$$

with $\mathbf{\Sigma}_{\hat{r}}^j$,$\mathbf{\Sigma}_{\hat{r}}^j \in \mathbb{C}^{RK\times RK}$, being combined covariance matrix of residual and desired signal component, given by

$$\mathbf{\Sigma}_{\hat{r}}^j = \mathbf{\Sigma}_{\hat{r}} + \mathbf{\Phi}^j \mathbf{\Lambda}^j \mathbf{\Phi}^{j^\ddagger}, \tag{23}$$

where $\mathbf{\Phi}^j$, $\mathbf{\Phi}^j \in \mathbb{C}^{RK\times GK}$ is given by

$$\mathbf{\Phi}^j \quad = \quad \mathbf{F}_R \hat{\mathbf{H}}^j \mathbf{S}(n) \tag{24}$$

and $\mathbf{\Lambda}^j$,$\mathbf{\Lambda}^j \in \mathbb{C}^{GK\times GK}$, is written as

$$\mathbf{\Lambda}^j \quad = \quad \ddot{\mathbf{B}}^j - \dot{\mathbf{B}}^j \tag{25}$$

and covariance matrix of residual, $\mathbf{\Sigma}_{\hat{r}}$, $\mathbf{\Sigma}_{\hat{r}} \in \mathbb{C}^{RK\times RK}$ is defined as

$$\mathbf{\Sigma}_{\hat{r}} = \mathbf{\Gamma} \mathbf{F}_T \mathbf{\Lambda} \mathbf{F}_T{}^\ddagger \mathbf{\Gamma}^\ddagger + \sigma^2 \mathbf{I}. \tag{26}$$

The soft-feedback terms in (25) and (26) are defined as follows: $\mathbf{B}^j \in \mathbb{C}^{GK\times GK}$ is obtained as

$$\dot{\mathbf{B}}^j = diag\left\{\left[\dot{\mathbf{b}}^1 \dots \dot{\mathbf{b}}^G\right]^\dagger\right\}, \tag{27}$$

where $\dot{\mathbf{b}}^j \in \mathbb{C}^{K\times K}$ is given by

$$\dot{\mathbf{b}}^g = \left[E\left\{|b_1^g|^2\right\} \dots E\left\{|b_K^g|^2\right\}\right]^\dagger. \tag{28}$$

$E\left\{|b_k^g|^2\right\}$ is determined as

$$E\left\{|b_k^g|^2\right\} = \sum_{b_i\in M} |b_i|^2 P(b_k^g = b_i) \tag{29}$$

with $P(b_k^j = b_i)$ [9] being

$$P(b_k^j = b_i) = \left(\frac{1}{2}\right)^Q \prod_{q=1}^{Q} \left(1 - \bar{c}_{i,q} \tanh\left(\lambda_{k,q}^j/2\right)\right) \tag{30}$$

and $\bar{c}_{i,q} = 2c_{i,q} - 1$. $\lambda_{k,q}^j$ is apriori likelihood information of bit $c_{i,q}$ provided by decoder. $\ddot{\mathbf{B}} \in \mathbb{C}^{GK\times GK}$ is given as

$$\ddot{\mathbf{B}}^j = diag\left\{\left[\ddot{\mathbf{b}}^1 \dots \ddot{\mathbf{b}}^G\right]^\dagger\right\} \tag{31}$$

with $\ddot{\mathbf{b}}^g$, $\ddot{\mathbf{b}}^g \in \mathbb{C}^{GK\times GK}$, being

$$\ddot{\mathbf{b}}^g = \left[|\tilde{b}_1^g|^2 \dots |\tilde{b}_K^g|^2\right]^\dagger. \tag{32}$$

$\mathbf{\Lambda}$, $\mathbf{\Lambda} \in \mathbb{C}^{TK\times TK}$ expresses the mean residual interference energy after soft cancellation as

$$\mathbf{\Lambda} = \dot{\mathbf{B}} - \ddot{\mathbf{B}}, \tag{33}$$

where

$$\dot{\mathbf{B}} = diag\left\{\left[\dot{\mathbf{b}}^1 \dots \dot{\mathbf{b}}^T\right]^\dagger\right\} \in \mathbb{C}^{TK\times TK}, \tag{34}$$

and $\dot{\mathbf{b}}^t \in \mathbb{C}^{K\times K}$ is obtained by using (28). $\ddot{\mathbf{B}}$, $\ddot{\mathbf{B}} \in \mathbb{C}^{TK\times TK}$, is denoted as

$$\ddot{\mathbf{B}} = diag\left\{\left[\ddot{\mathbf{b}}^1 \dots \ddot{\mathbf{b}}^T\right]^\dagger\right\}, \tag{35}$$

where $\ddot{\mathbf{b}}^t \in \mathbb{C}^{K\times K}$ is obtained using (32). The block circulant Hermitean covariance matrix of the feedback soft estimates , $\mathbf{\Delta}$, $\mathbf{\Delta} \in \mathbb{C}^{TK\times TK}$, is denoted as

$$\mathbf{\Delta} \quad = \quad \mathbf{F}_T \mathbf{\Lambda} \mathbf{F}_T^\dagger, \tag{36}$$

In order to avoid the computational complexity increase in (36) caused by the block circulant structure of $\mathbf{\Delta}$, the covariance matrix is approximiated by a diagonal matrix

$$\mathbf{\Delta} \quad \approx \quad diag\left\{\left[\bar{\delta}_1^1 \dots \bar{\delta}^T\right]^\dagger \otimes \mathbf{l}\right\} \tag{37}$$

where $\mathbf{l}$, $\mathbf{l} \in \mathbb{R}^{K\times 1}$,vector that all the elements are ones and scalar $\bar{\delta}^t$ is obtained as

$$\bar{\delta}^t = \frac{1}{K} \sum \left\{\dot{\mathbf{b}}^t - \ddot{\mathbf{b}}^t\right\}. \tag{38}$$

In order to further simplify computational complexity of (23), matrix inversion lemma [8] is applied to (23). As a result, (23) can be re-written as

$$\mathbf{\Sigma}_{\hat{r}}^{j^{-1}} = \mathbf{\Sigma}_{\hat{r}}^{-1} - \mathbf{\Sigma}_{\hat{r}}^{-1} \mathbf{\Phi}^j \left(\mathbf{\Phi}^{j^\ddagger} \mathbf{\Sigma}_{\hat{r}}^{-1} \mathbf{\Phi}^j + \overline{\mathbf{\Lambda}}^{j^{-1}}\right)^{-1} \mathbf{\Phi}^{j^\ddagger} \mathbf{\Sigma}_{\hat{r}}^{-1}, \tag{39}$$

where the diagonal elements of $\overline{\mathbf{\Lambda}}_j$, $\overline{\mathbf{\Lambda}}_j \in \mathbb{C}^{GK\times GK}$ are computed using averaged version of the diagonal values of $\mathbf{\Lambda}_j$. Finally, by using (17) the optimal filter weights for $\mathbf{\Omega}^j$ and $\mathbf{A}^j$ can be expressed as

$$\begin{aligned} \mathbf{\Omega}^j \quad &= \quad \mathbf{\Sigma}_{\hat{r}}^{j^{-1}} \mathbf{\Phi}^j (\overline{\dot{\mathbf{B}}}^{j^{-1}} + \mathbf{\Phi}^{j^\ddagger} \mathbf{\Sigma}_{\hat{r}}^{j^{-1}} \mathbf{\Phi}^j)^{-1} \\ \mathbf{A}^j \quad &= \quad \overline{\dot{\mathbf{B}}}^{j^{-1}} + \mathbf{\Phi}^{j^\ddagger} \mathbf{\Sigma}_{\hat{r}}^{j^{-1}} \mathbf{\Phi}^j, \end{aligned} \tag{40}$$

where the diagonal elements of $\overline{\dot{\mathbf{B}}}^j$, $\overline{\dot{\mathbf{B}}}^j \in \mathbb{C}^{GK\times GK}$ are computed using the averaged version of the diagonal values of (27). By using (11) and (40) and after minor mathematical manipulations the output of groupwise MMSE filter $\mathbf{z}^j$, $\mathbf{z}^j \in \mathbb{C}^{GK\times 1}$ can be be written as

$$\mathbf{z}^j = \mathbf{\Xi}^{j^{-1}} \hat{\mathbf{\Xi}}^j (\mathbf{F}_G^{-1} \mathbf{\Gamma}^{j^\ddagger} \mathbf{\Sigma}_{\hat{r}}^{-1} \hat{\mathbf{r}} + \Upsilon^j \tilde{\mathbf{b}}^j) \tag{41}$$

where $\mathbf{\Gamma}^j$, $\mathbf{\Gamma}^j \in \mathbb{C}^{RK\times GK}$ is obtained by applying (6) with $\mathbf{F}_G$ to $\hat{\mathbf{H}}^j$, and $\mathbf{\Xi}^j$, $\mathbf{\Xi}^j \in \mathbb{C}^{GK\times GK}$ is given by

$$\mathbf{\Xi}^j = \overline{\dot{\mathbf{B}}}^{j^{-1}} + \mathbf{\Upsilon}^j - \mathbf{\Upsilon}^j (\mathbf{\Upsilon}^j + \overline{\mathbf{\Lambda}}^{j^{-1}})^{-1} \mathbf{\Upsilon}^j. \tag{42}$$

$\hat{\mathbf{\Xi}}^j$, $\hat{\mathbf{\Xi}}^j \in \mathbb{C}^{GK\times GK}$, is denoted as

$$\hat{\mathbf{\Xi}}^j = \mathbf{I}_{GK} - \mathbf{\Upsilon}^j (\mathbf{\Upsilon}^j + \overline{\mathbf{\Lambda}}^{j^{-1}})^{-1}, \tag{43}$$

where $\mathbf{I}_{GK}$, $\mathbf{I}_{GK} \in \mathbb{R}^{GK\times GK}$, is identity matrix and $\mathbf{\Upsilon}^j$, $\mathbf{\Upsilon}^j \in \mathbb{C}^{GK\times GK}$, is defined as

$$\mathbf{\Upsilon}^j = \begin{bmatrix} \varphi^{1,1}\mathbf{I}_K & \dots & \varphi^{1,G}\mathbf{I}_K \\ \vdots & \ddots & \vdots \\ \varphi^{G,1}\mathbf{I}_K & \dots & \varphi^{G,G}\mathbf{I}_K \end{bmatrix} \tag{44}$$

with $\varphi^{g,f}$, $f = 1, \dots, G$, being

$$\varphi^{g,f} = \frac{1}{K} Trace\left\{\mathbf{\Gamma}_j^{g^\ddagger} \mathbf{\Sigma}_{\hat{r}}^{-1} \mathbf{\Gamma}_j^f\right\}. \tag{45}$$

$\mathbf{\Gamma}_j^g$, $\mathbf{\Gamma}_j^g \in \mathbb{C}^{RK \times K}$ is gth transmit antenna of j^{th} group of transmit antennas. The filter output, (41), is approxiamated as Gaussian distributed. Therefore, mean value of filter output, $\mathbf{\Phi}^j$, $\mathbf{\Phi}^j \in \mathbb{C}^{GK \times GK}$ is expressed as

$$\mathbf{\Phi}^j = \mathbf{\Xi}^{j^{-1}} \hat{\mathbf{\Xi}}^j \mathbf{\Upsilon}^j. \tag{46}$$

The covariance matrix of the filter output, $\mathbf{\Psi}^j$, $\mathbf{\Psi}^j \in \mathbb{C}^{GK \times GK}$ is defined as

$$\mathbf{\Psi}^j = \mathbf{\Xi}^{j^{-1}} \hat{\mathbf{\Xi}}^j (\dot{\mathbf{\Upsilon}}^j + \ddot{\mathbf{\Upsilon}}^j) \hat{\mathbf{\Xi}}^j \mathbf{\Xi}^{j^{-1}}, \tag{47}$$

where $\dot{\mathbf{\Upsilon}}^j$, $\dot{\mathbf{\Upsilon}}^j \in \mathbb{C}^{GK \times GK}$ is defined as

$$\dot{\mathbf{\Upsilon}}^j = \begin{bmatrix} \dot{\varphi}^{1,1}\mathbf{I}_K & \dots & \dot{\varphi}^{1,G}\mathbf{I}_K \\ \vdots & \ddots & \vdots \\ \dot{\varphi}^{G,1}\mathbf{I}_K & \dots & \dot{\varphi}^{G,G}\mathbf{I}_K \end{bmatrix} \tag{48}$$

with $\dot{\varphi}^{g,f}$ being

$$\dot{\varphi}^{g,f} = \frac{1}{K} Trace \left\{ \mathbf{\Gamma}_j^{g^{\ddagger}} \mathbf{\Sigma}_{\hat{r}}^{-1} \mathbf{\Gamma} \mathbf{\Delta} \mathbf{\Gamma}^{\ddagger} \mathbf{\Sigma}_{\hat{r}}^{-1} \mathbf{\Gamma}_j^f \right\}. \tag{49}$$

$\ddot{\mathbf{\Upsilon}}^j$, $\ddot{\mathbf{\Upsilon}}^j \in \mathbb{C}^{GK \times GK}$ is defined as

$$\ddot{\mathbf{\Upsilon}}^j = \begin{bmatrix} \ddot{\varphi}^{1,1}\mathbf{I}_K & \dots & \ddot{\varphi}^{1,G}\mathbf{I}_K \\ \vdots & \ddots & \vdots \\ \ddot{\varphi}^{G,1}\mathbf{I}_K & \dots & \ddot{\varphi}^{G,G}\mathbf{I}_K \end{bmatrix} \tag{50}$$

with $\ddot{\varphi}^{g,f}$ being

$$\ddot{\varphi}^{g,f} = \frac{\sigma^2}{K} Trace \left\{ \mathbf{\Gamma}_j^{g^{\ddagger}} \mathbf{\Sigma}_{\hat{r}}^{-1} \mathbf{\Sigma}_{\hat{r}}^{-1} \mathbf{\Gamma}_j^f \right\}. \tag{51}$$

B. MAP Detector

The joint detection is performed by using MAP detector that separates transmitted layers within the group to be jointly detected using the spatial domain MAP algorithm [5]. The spatial MAP computes distance metric for each possible bit sequence candidates, $w = 1 \cdots 2^{GQ}$, $s = 1 \cdots GQ$, as follows:

$$\zeta(c_{k,s}^{j,w}) = (\mathbf{z}_k^j - \mathbf{\Phi}_k^j \mathfrak{M}\{\dot{\mathbf{c}}_w\})^{\ddagger} \mathbf{\Psi}_k^{j^{-1}} (\mathbf{z}_k^j - \mathbf{\Phi}_k^j \mathfrak{M}\{\dot{\mathbf{c}}_w\}) \tag{52}$$

where $\mathbf{z}_k^j$, $\mathbf{z}_k^j \in \mathbb{C}^{G \times 1}$, is the k^{th} symbol instant and j^{th} antenna group of Eq. (41), $\mathbf{\Phi}_k^j$, $\mathbf{\Phi}_k^j \in \mathbb{C}^{G \times G}$, k^{th} symbol instant and j^{th} antenna group of Eq. (46), $\mathbf{\Psi}_k^j$, $\mathbf{\Psi}_k^j \in \mathbb{C}^{G \times G}$, k^{th} symbol instant and j^{th} antenna group of Eq. (47), $\mathfrak{M}\{\}$ is mapping function that maps bits to symbols and $\dot{\mathbf{c}}_w$ is the w^{th} candidate bit vector. A Log-likelihood ratio of the extrinsic probability of each bit is given as

$$\lambda_{k,s}^j = ln \frac{\sum\limits_{\forall c_{w,s}=1} e^{(-\zeta(c_{k,s}^{j,w}) + \sum\limits_{s\prime \neq s} lnP(c_{k,s\prime}^j = 1))}}{\sum\limits_{\forall c_{w,s}=0} e^{(-\zeta(c_{k,s}^{j,w}) + \sum\limits_{s\prime \neq s} lnP(c_{k,s\prime}^j = 0))}} \tag{53}$$

where $lnP(c_{k,s\prime}^j = 1)$ and $lnP(c_{k,s\prime}^j = 1)$ are natural logarithms of apriori probabilities of coded bit, $s\prime = 1 \cdots GQ$, and are calculated similarly as given in [10] as follows: $lnP(c_{k,s\prime}^j = 1) = \lambda_{k,s\prime}^j - ln(1 + e^{\lambda_{k,s\prime}^j})$, $lnP(c_{k,s\prime}^j = 0) = -ln(1 + e^{\lambda_{k,s\prime}^j})$. $c_{w,s} = 1$ and $c_{w,s} = 0$ correspond candidate vectors where there is one or zero at s^{th} position, respectively.

C. Computational Complexity

The complexity of iterative freqeuncy domain JA receiver is dominated by either FFT/IFFT or MAP algorithm. This is due to fact that the invertable matrice sizes in MMSE filter are only depend R and G. Therefore, the inversion of $G \times G$ or $R \times R$ matrix is typically lower in complexity than FFT/IFFT for each symbol time instant. Therefore, the receiver complexity per symbol time instant can be roughly approximated as $O(max\left\{log_2(K), M^{TQ}\right\})$. In the case of time domain receiver, [5], the complexity per symbol time instant is roughly dominated by either MMSE filter or MAP $O(max\left\{((2L-1)R)^3, M^{TQ}\right\})$. Therefore, it is obvious that frequency domain approach provides significantly lower complexity that time domain receiver.

IV. Numerical Results

Parameters used in the simulations are as follows: Quadrature Phase Shift Keying (QPSK) modulation, frame length $M = 544$, FFT length $K = 512$, guard period length $P = 32$. Channel is modelled as tapped delay line with $L = 32$, each having equal average powers. Spatial correlation of MIMO channel is modelled as Kronecker product of the spatial correlation matrices at the transmitter and receiver sides. The turbo encoder uses two recursive systematic component codes with a generator polynomial (15,13), and the coded bit sequence are adequately punctured so that overall code rate is 1/2. The Log-MAP algorithm is used in the SfISfO decoder. The number of iterations with the turbo decoder is 8. The numbers of equalizer iterations for the both JA and AA are 8 and 6 for correlated and uncorrelated antennae, respectively. The lengths of the random bit interleaver and the semi-random symbol interleaver are 960 bits and 480 symbols,respectively. Perfect channel state information is assumed in the simulations.

Figure 3 shows FER performance in a balanced antenna configuration, $T = R = 2$, with the different number of iterations. The results show that AA has faster convergence within the first three iterations. However, the JA achieves more iteration gain compered AA when the number of iterations exceed three. Similar phenomena was also observed in the case of higher number of transmitter and receiver antennas with balanced transmission. Figure 4 presents FER performance of AA ($G = 1$) and JA ($G > 1$) in the presence of spatially correlated and uncorrelated transmitter antennae.The matched filter bound (MFB) provides an upper bound of performance. The MFB curves were obtained also through simulations assuming perfect feedback for balanced antenna case. Results show that both AA and JA have around 1.25 dB loss from MFB when antennae are uncorrelated. Evidently, in the presence of spatial correlation JA outperforms AA. It can be expected that the gain by using JA becomes larger when the spatial correlation as well as the number transmitter antennae increase. Figure 5 shows FER the performance in an overloaded antenna configuration. The results show clearly that JA outperforms AA significantly when $T > R$. The performance of AA degrades due to lack in degrees of freedom of MMSE filtering needed to suppress interference caused by overloaded transmit antennae. Correspondingly, by looking at the decay of the curves it is find out that the performance of JA technique is degraded, but the diversity order remains the same in all the overloaded cases tested as in the balanced cases. The possibility of JA to maintain equivalent diversity order is due fact that MMSE can suppress ISI for which it has to have enough degrees of freedom.

V. Conclusions

A new iterative frequency domain joint over-antenna MIMO signal detection concept has been proposed for broadband overloaded

MIMO transmission over frequency-selective channels in this paper. The results show that significant performance gain can be achieved with JA technique compared to AA technique. By using JA technique, it is possible jointly detect all the transmit antennae even if receiver has less antennae than transmitter, maintain the same diversity order as balanced case. The results also indicates that the proposed JA technique is also effective in improving the performance of in the presence of spatial correlation. Numerical results and complexity estimates show that the proposed technique achieves reasonable tradeoff between complexity and performance.

ACKNOWLEDGEMENTS

Part of this work has been performed in the framework of the IST project IST-2003-507581 WINNER, which is partly funded by the European Union. The author J. Karjalainen would like to acknowledge the contributions of their colleagues in WINNER.

REFERENCES

[1] X. Wang and H. V. Poor, "Iterative (turbo) soft interference cancellation and decoding for coded CDMA," *IEEE Trans. Commun.*, vol. 47, no. 7, pp. 1046–1061, July 1999.

[2] M. Yee, M. Sandell, and Y. Sun, "Comparison study of single-carrier and multi-carrier modulation using iterative based receiver for MIMO system," in *Proc. IEEE Veh. Technol. Conf. (VTC)*, Milan, Italy, May 17–19 2004.

[3] M. Tüchler and J. Hagenauer, "Linear time and frequency domain turbo equalisation," in *Proc. IEEE Veh. Technol. Conf. (VTC)*, vol. 4, oct 7-11 2001, pp. 2773–2777.

[4] R. Wohlgenannt, K. Kansanen, D. Tujkovic, and T. Matsumoto, "Outage -based LDPC code design for SC/MMSE turbo equalization," in *Proc. IEEE Veh. Technol. Conf. (VTC)*, Stockholm, Sweden, May 30 – June 1 2005.

[5] N. R. Veselinović, T. Matsumoto, and M. Juntti, "Iterative STTrC-coded multiuser detection and equalization with unknown inteference," *EURASIP J. Wireless Comm. and Netw.*, vol. 2, pp. 309–321, Dec. 2004.

[6] D. Falconer, S. Ariyavisitakul, A. Benyamin-Seeyar, and B. Eidson, "Frequency domain equalization for single-carrier broadband wireless systems," *IEEE Commun. Mag.*, vol. 40, no. 4, pp. 58–66, 2002.

[7] P. Davis, *Circulant Matrices*, 2nd ed. New York, USA: Chelsea Publishing, 1994.

[8] T. Kailath and A. Sayed, *Fast Reliable Algorithms for Matrices with Structure*. Philadelphia,USA: SIAM, 1999.

[9] M. Tüchler, A. C. Singer, and R. Koetter, "Minimum mean squared error equalisation using a priori information," *IEEE Trans. Signal Processing*, vol. 50, no. 3, pp. 673–683, Mar. 2002.

[10] P. Robertson, E. Villebrun, and P. Hoeher, "A comparison of optimal and sub-optimal map decoding algorithms operating in the log domain," in *Proc. IEEE Int. Conf. Commun. (ICC)*, vol. 2, 1995, pp. 1009 –1013.

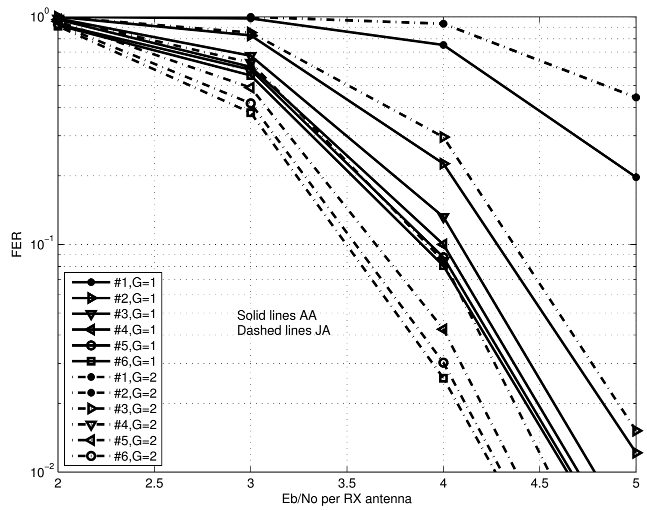

Fig. 3. FER results with the different number of iterations, $T = R = 2$.

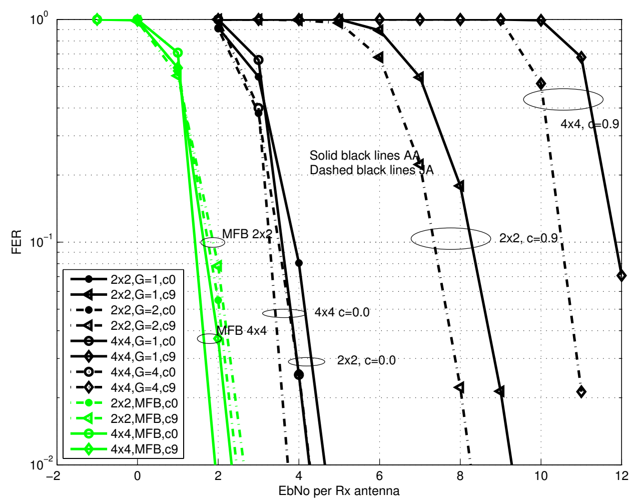

Fig. 4. FER results with spatial correlation.

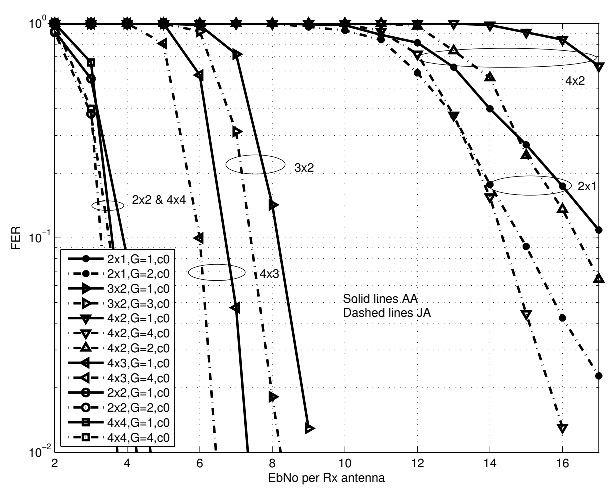

Fig. 5. FER results with overloaded and balanced transmission.