A Computationally Efficient MIMO Turbo-Equaliser

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Abstract— A reduced-complexity version of the soft-interference cancelling MMSE turbo equaliser is proposed for single-carrier MIMO systems. In the receiver exact symbol-by-symbol interference covariance matrice inverses are replaced by time averages acquired through matrix inversion lemma iterations. Consequently, also the requirement to explicitly estimate signal-to-noise ratio is removed. After a number of iterations the equaliser is further simplified by utilising the matched filter approximation. Remaining multipath and multiuser interference, in addition to receiver noise, are accounted for in the approximation. The performance of the receiver is verified through simulations with two MIMO radio network configurations.

I. Introduction

Turbo equalisation [1] is one of the most compelling techniques to realise well-performing high-order equalizers without excessive computational complexity by iteratively exchanging soft information between an equaliser algorithm and a soft-in-soft-out channel decoder algorithm. In this paper we consider a wideband single-carrier system with multiple users and a space-time turbo equaliser as the receiver. The equalisation algorithm is a derivative of the algorithm originally proposed in [2] for turbo detection of coded DS-CDMA signals, applied to channel equalisation in [3] and to MIMO channel equalisation in [4]. The original algorithm as proposed in [3] is based on soft interference cancellation followed by a symbol-by-symbol MMSE filtering. The MMSE filtering contains a matrix inverse whose implementation as such exhibits cubic computational complexity. Various techniques to reduce the computational complexity have been proposed e.g. in [5][6][7].

In the proposed algorithm the bitwise matrix inversion calculated for the MMSE estimator is replaced by a framewise average. The inverse is easily computed with the matrix inversion lemma utilising the residual signal at the output of the soft interference canceller, called the time averating matrix inversion lemma (TAM) approximation in the sequel. The approach utilising a covariance matrix time average has been proposed for a single-user equalisation in [5]. A further simplification of the equalisation algorithm is proposed in [6], where the MMSE filter is replaced by a channel matched filter. We propose to enhance the approximation proposed in [6] to account for remaining interference components after interference cancellation as well as noise in a MIMO case, resulting in a more accurate symbol likelihood calculation. The combination of the TAM and MF approximations through switching in a similar fashion as proposed in [8] provides a powerful and computationally efficient MIMO turbo-equaliser.

The paper is organised as follows. The MIMO system model is presented in Section II. The proposed equaliser algorithm is then presented in Section III and its performance evaluated in Section IV. The utilised matched filter approximation is presented in Section V, and the equaliser performance with switching reported in Section VI. The paper concludes with a summary.

II. System Model

The received space-time signal is given by the linear model

$$\mathbf{r} = \mathbf{H}\mathbf{b} + \mathbf{w} \quad (1)$$

where $\mathbf{b}$ is the multiple users' transmitted channel encoded and interleaved information vector, so that

$$\mathbf{b} = \left[\mathbf{b}^{\mathrm{T}}(1), \ldots, \mathbf{b}^{\mathrm{T}}(n), \ldots, \mathbf{b}^{\mathrm{T}}(N)\right]^{\mathrm{T}} \quad (2)$$

$$\mathbf{b}(n) = \left[b_1(n), \ldots, b_k(n), \ldots, b_K(n)\right]^{\mathrm{T}}, \quad (3)$$

where K is the number of users and N is the length of the transmitted frame. The space-time channel matrix $\mathbf{H}$ is constructed as

$$\mathbf{H} = \left[\bar{\mathbf{H}}(1), \ldots, \bar{\mathbf{H}}(n), \ldots, \bar{\mathbf{H}}(N)\right] \quad (4)$$

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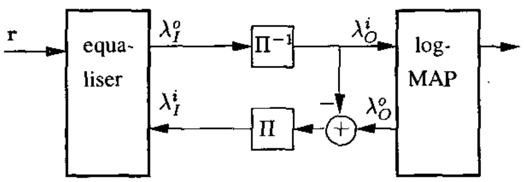

Fig. 1. Turbo equaliser

$$\breve{\mathbf{H}}(n) = \left[\mathbf{0}_K^{(n-1)J}, \mathbf{H}^{\mathrm{T}}(n), \mathbf{0}_K^{(N-n+1)J}\right]^{\mathrm{T}}, \quad (5)$$

where the all-zeros matrix $\mathbf{0}_i^j$ has dimensionality $i \times j$, and

$$\begin{aligned} \mathbf{H}(n) &= [\mathbf{h}_1(n), \ldots, \mathbf{h}_k(n), \ldots, \mathbf{h}_K(n)] & (6) \\ \mathbf{h}_k(n) &= [\mathbf{h}_{k,1}^{\mathrm{T}}(n), \ldots, \mathbf{h}_{k,l}^{\mathrm{T}}(n), \ldots, \mathbf{h}_{k,L}^{\mathrm{T}}(n)]^{\mathrm{T}} & (7) \\ \mathbf{h}_{k,l}(n) &= [h_{k,l,1}(n), \ldots, h_{k,l,j}(n), \ldots, h_{k,l,J}(n)]^{\mathrm{T}}, & (8) \end{aligned}$$

where J is the number of receiver antennas and L is the number of channel taps. The channel matrix has dimensionality $(N + L)J \times NK$. The vector $\mathbf{w}$ is the vector of complex Gaussian received noise samples with variance σ_w^2. It should be noted that the system model can represent either an multiuser or a layered space-time coded multiantenna transmission.

III. SC/MMSE MIMO Turbo Equaliser

The turbo equaliser consists of an equaliser part performing interference cancellation and symbol-by-symbol MMSE filtering, and soft-in-soft-out log-MAP channel decoders for each user. These are separated by deinterleaving and interleaving operations as illustrated in Figure 1. The soft-in-soft-out (SISO) equalisation algorithm presented in [3] has been modified here to be based on the residual signal after interference cancellation

$$\tilde{\mathbf{r}} = \mathbf{r} - \mathbf{H}\hat{\mathbf{b}}, \quad (9)$$

where the soft estimate vector $\hat{\mathbf{b}}$ of the transmitted symbols is the symbol-by-symbol expected value (i.e. MMSE estimate) of the binary transmitted symbols given the *a-priori* symbol likelihood λ_I^i provided by the channel decoder

$$\hat{b}_k(n) = \tanh\left(\frac{\lambda_{I,k}^i(n)}{2}\right), \quad (10)$$

for all k and n. The symbol-by-symbol MMSE filtering applied next operates on a symbol window which contains all symbols interfering with the symbol of interest. A partial channel matrix $\breve{\mathbf{H}}(n)$ for each time step n containing the $1+(n-L)J$th to nJth rows and $1 + (n - L)K$th to $(n + L - 1)K$th columns of the channel matrix $\mathbf{H}$, so that the number of (one users') symbols in the windowed coputation is $2L-1$. Vectors $\tilde{\mathbf{r}}(n)$ and $\mathbf{b}(n)$ cover elements $1 + (n - L)J$ to nJ from $\tilde{\mathbf{r}}$ and $1 + (n - L)K$ to $(n + L - 1)K$ from $\mathbf{b}$, correspondingly. The algorithm formulation is based on combining the residual signal and the symbol of interest for each user as

$$\tilde{\mathbf{r}}_k(n) = \tilde{\mathbf{r}}(n) + \mathbf{h}(n)\hat{b}_k(n). \quad (11)$$

The covariance matrix of $\tilde{\mathbf{r}}_k(n)$, with the residual signal covariance separated from the symbol-by-symbol information, is given by

$$\bar{\mathbf{\Theta}}_k(n) = \breve{\mathbf{H}}(n)\mathbf{\Lambda}(n)\breve{\mathbf{H}}^{\mathrm{H}}(n) + \mathbf{h}_k(n)\hat{b}_k^2(n)\mathbf{h}_k^{\mathrm{H}}(n) + \mathbf{I}\sigma_w^2, \quad (12)$$

where the symbol soft information is assumed uncorrelated and so the covariance matrix is given as

$$\mathbf{\Lambda}(n) = \mathrm{diag}\left\{1 - \hat{\mathbf{b}}(n)\hat{\mathbf{b}}(n)^{\mathrm{T}}\right\}. \quad (13)$$

Now, the binary likelihood of the transmitted symbol can be calculated as

$$\lambda_{I,k}^o(n) = \Re\left\{\frac{4z_k(n)}{1 - \mu_k(n)}\right\}, \quad (14)$$

where

$$\begin{aligned} z_k(n) &= \mathbf{h}_k^{\mathrm{H}}(n)\bar{\mathbf{\Theta}}_k^{-1}(n)\tilde{\mathbf{r}}_k(n) & (15) \\ \mu_k(n) &= \mathbf{h}_k^{\mathrm{H}}(n)\bar{\mathbf{\Theta}}_k^{-1}(n)\mathbf{h}_k(n) & (16) \end{aligned}$$

Calculating the matrix inverse of (12) dominates the complexity of the algorithm. With a direct inverse it exhibits $O((2L - 1)^3)$ complexity, which grows quickly prohibitive as the number of equalised channel taps and receiver antennas increases. We propose to approximate the matrix inverse by a matrix averaged over the framelength N. This is achieved by first dividing the matrix inverse into two parts as

$$\bar{\mathbf{\Theta}}_k(n) = \mathbf{\Theta}(n) + \mathbf{h}_k(n)\hat{b}^2\mathbf{h}_k^{\mathrm{H}}(n), \quad (17)$$

where $\mathbf{\Theta}(n)$ is then approximated as

$$\begin{aligned} \mathbf{\Theta}(n) &= \mathbf{H}(n)\mathbf{\Lambda}(n)\mathbf{H}^{\mathrm{H}}(n) + \mathbf{I}\sigma_w^2 & (18) \\ &= \mathrm{E}\left[\tilde{\mathbf{r}}(n)\tilde{\mathbf{r}}^{\mathrm{H}}(n)\right] & (19) \\ &\approx \frac{1}{N}\sum_{n=1}^{N}\tilde{\mathbf{r}}(n)\tilde{\mathbf{r}}^{\mathrm{H}}(n) & (20) \end{aligned}$$

for all n and k. The need for estimating the receiver noise level is removed due to the noise being included in the residual signal utilised for calculating (20). The sum of outer products in (20) can easily be calculated utilising matrix inversion lemma in a similar fashion as the covariance inverse matrix is propagated in the RLS algorithm. The matrix inversion lemma iteration must be normalised since inaccuracies in $\Theta(n)$ will bias the likelihood calculation of the symbol likelihood in (14). The inverse is then calculated by iterating

$$\begin{aligned} \tilde{\mathbf{r}} &= \tilde{\mathbf{r}}(i) + \mathbf{n} && (21) \\ \Theta_i^{-1} &= \lambda^{-1}\Theta_{i-1}^{-1} - \\ & \frac{\lambda^{-2}\Theta_{i-1}^{-1}\tilde{\mathbf{r}}\tilde{\mathbf{r}}\Theta_{i-1}^{-1}}{(1-\lambda)^{-1} + \lambda^{-1}\tilde{\mathbf{r}}\Theta_{i-1}^{-1}\tilde{\mathbf{r}}} && (22) \end{aligned}$$

over the frame with the initialisation $\Theta_0^{-1} = \epsilon^{-1}\mathbf{I}$, where ϵ is a small positive constant and λ is the forgetting factor. Diagonal weighting to stabilise the iteration can be added with the Gaussian noise vector $\mathbf{n}$ to the residual. When calculated in this form, the residual covariance inverse can then be combined with the symbolwise information for each user k by applying the matrix inversion lemma on $\bar{\Theta}(n)$ as given by (17). In a multiuser transmission or a point-to-point MIMO system the calculated matrix inverse is common to all transmitting users and antennas.

The assumption required by the approximation is that the channel is constant over the frame being processed. For a wideband system where the transmission time of a frame can be made short this assumption is relatively valid. The benefit of the averaging is in performing one $O((2L-1)^2)$ matrix inversion lemma iterations instead of one $O((2L-1)^3)$ matrix inverse per received bit. In the first equalisation iteration when no *a-priori* information is available on the transmitted symbols, the equaliser algorithm is a standard linear MMSE equaliser. Only by changing the input signal from the receiver signal $\mathbf{r}$ to the interference residual $\tilde{\mathbf{r}}$ the algorithm can be applied to all cancellation iterations without further modifications. Each equaliser stage consists then of a common matrix inverter, a channel estimator providing the channel state vectors, and a likelihood generator for each user, which also combines the global matrix inverse with the symbol-by-symbol information. An example block diagram of the structure of a receiver stage is illustrated in Figure 2.

IV. Equaliser Performance

The proposed receiver is tested through computer simulations in two MIMO scenarios, a 2-by-2 case and a 4-by-4 case. The channel is assumed to be known to the receiver in the both cases. The information framelength is 300 bits, and the code is 1/2-rate convolutional code with constraint length 3 (i.e. $N = 600$). A different random interleaver is generated for each user and

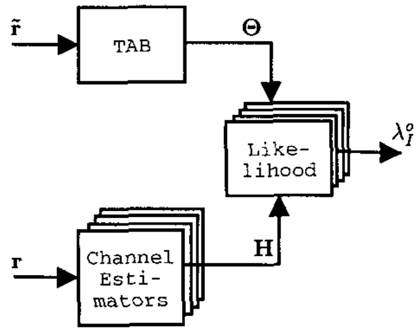

Fig. 2. Equaliser stage block diagram with a single TAB block, K channel estimators and K likelihood processors.

transmitted frame. The channel has 10 Rayleigh fading taps with equal average power that are constant over the duration of a frame. The equaliser performs 4 iterations in the 2-by-2 case and 5 iterations in the 4-by-4 case. In the TAM iterations Gaussian noise at level -3dB relative to received signal power is added to provide stabilising diagonal weighting for the matrix inverse. The suitable level to stabilise the algorithm was found empirically. Forgetting factor for the inverse iteration is set to 0.99.

The performance reference of the simulations is the maximal-ratio-combining (MRC) bound of the channel, which is the performance of a LJth order diversity reception without interference. This is the performance of the algorithm when perfect interference cancellation is performed, and the equaliser becomes a channel matched filter. The MRC bound curve has simply been simulated with a LJth order diversity system with corresponding channel coding and channel conditions.

The simulation results given in Figure 3 and 4 show that the proposed space-time turbo equaliser provides a significant iteration gain up to the third iteration i.e. the third receiver stage is the last to provide significant gain from the previous stage. The performance of the 2-by-2 system is within 0.5-1dB from the MRC bound while the performance gap of the 4-by-4 system to the optimal performance is somewhat larger. The 4-by-4 system seems to suffer from convergence problems at low signal-to-noise region given that the loss from the MRC bound is smaller at high signal-to-noise ratios. The both cases exhibit significant iteration gain in the first three iterations.

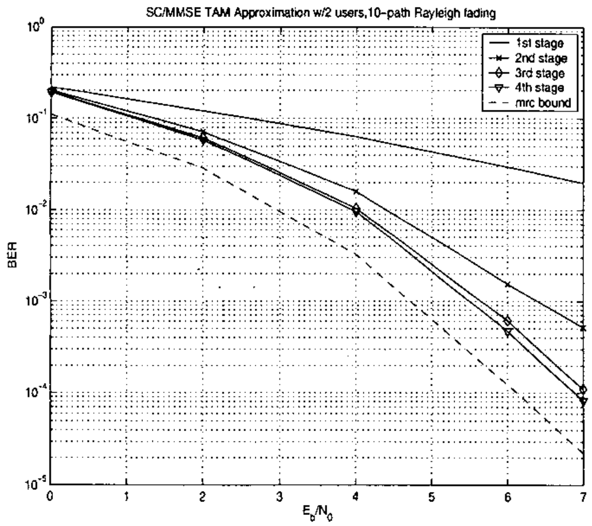

Fig. 3. 2-by-2 System BER with SC/MMSE TAM Approximation

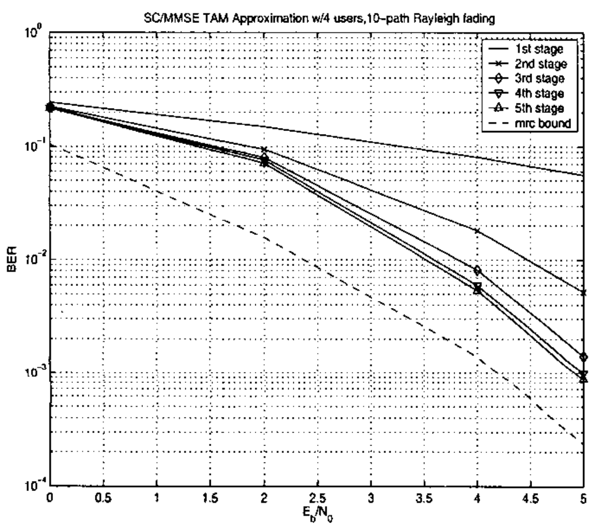

Fig. 4. 4-by-4 System BER with SC/MMSE TAM Approximation

V. Matched Filter Approximation

The approximation presented in Section IV does not provide much iteration gain after three iterations and the third iteration gain is smaller than that of the previous iterations. On the other hand, if the *a-priori* information from the decoders is good enough, even a simpler approximation will suffice to improve the detector performance [7]. It is thus proposed, that a matched filter approximation is performed after the first two iterations, which give a significant iteration gain.

The utilised matched filter approximation is an improved version of that proposed in [6], which does not account for remaining ISI components after interference cancellation. In the proposed algorithm the remaining ISI components after cancellation are approximated as uncorrelated noise and the combined noise and interference power estimated directly from the residual signal $\tilde{\mathbf{r}}$ as

$$\sigma_t^2 = \frac{1}{JN}\tilde{\mathbf{r}}^{\mathrm{H}}\tilde{\mathbf{r}}. \tag{23}$$

The approximated residual signal $\tilde{\mathbf{r}}$ covariance matrix then becomes diagonal, and the multiplication with the matrix inverse can be reduced to a scalar multiplication. The modified algorithm with the MF approximation is computed as the algorithm given in (9)-(16) with (15) and (16) replaced by

$$\begin{aligned} z_{MF}(n) &= \mathbf{h}^{\mathrm{H}}(n)\left[\sigma_t^2 + \mathbf{h}(n)^{\mathrm{H}}\mathbf{h}(n)\right]^{-1} \times \ldots \\ &\quad \ldots \left(\tilde{\mathbf{r}}(n) + \mathbf{h}(n)\hat{\mathbf{b}}(n)\right) \qquad (24) \\ \mu_{MF}(n) &= \mathbf{h}^{\mathrm{H}}(n)\left[\sigma_t^2 + \mathbf{h}(n)^{\mathrm{H}}\mathbf{h}(n)\right]^{-1}\mathbf{h}(n). \qquad (25) \end{aligned}$$

Equation (25) clearly constitutes a channel matched filter for the symbol of interest. No explicit noise level estimation is required.

VI. Performance with MF Approximation

The combined algorithm is tested again with 2-by-2 and 4-by-4 MIMO configurations. The transmission and channel parameters are identical to those utilised in Section IV. Both cases show the MF approximation can provide further gain when the initial provided a-priori information is good. At high signal-to-noise ratios ($\geq$4dB) the performance after a total of five iterations is very close to the MRC bound. In practice, the fourth iteration is the last one to provide any significant iteration gain. At low signal-to-noise ratios the algorithm cannot converge to the MRC bound.

VII. Summary

A simple space-time turbo equaliser receiver algorithm has been proposed, where the total receiver complexity is reduced by accepting a marginal loss in performance. The algorithm provides significant iteration gain in the first three iterations, and can be augumented with a matched filter approximation, which can provide further iteration gain. The receiver performance has been evaluated through simulations in two MIMO scenarios. With last iterations utilising matched filter approximation, the receiver performs very close to optimal at signal-to-noise ratios above 4dB.

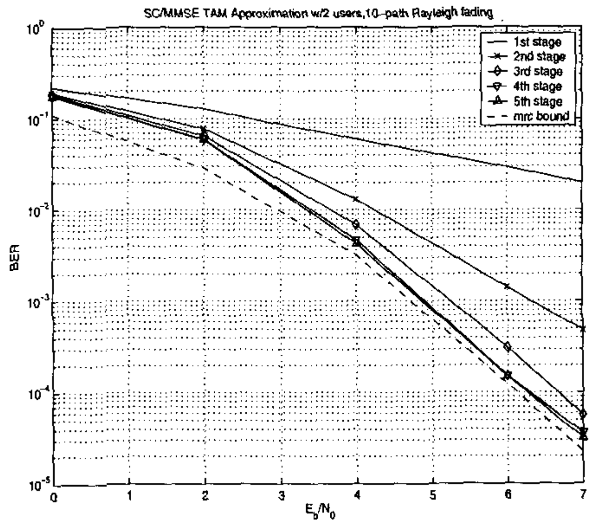

Fig. 5. 2-by-2 System BER with TAM and MF Approximation

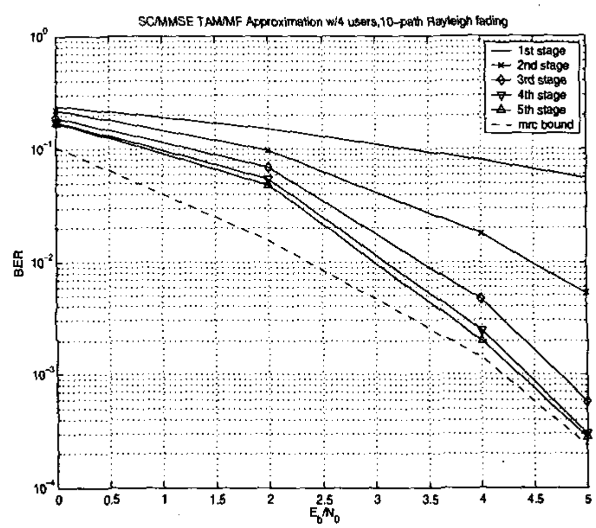

Fig. 6. 4-by-4 System BER with TAM and MF Approximation

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