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Proof theoretical investigations for Visser's logics, classical logic and the first-order arithmetic

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Abstract

In this thesis, we present proof theoretical investigations for Visser's logics, classical logic and the first-order arithmetic. We discuss the following three topics.

First, we introduce sequent calculi for Basic Propositional Logic (BPL) and Formal Propositional Logic (FPL) which are introduced by Visser, and prove the cut-elimination theorems for these by syntactical method. As is well-known, modal logic **S4** corresponds to the intuitionistic propositional logic by the Gödel translation. Visser introduced a logic to which modal logic GL corresponds by the Gödel translation. This logic is FPL. Visser also introduced BPL as a preliminary one for the development of FPL. A cut-free sequent calculus for BPL can be found in Ardeshir's Ph.D.thesis, but it is not satisfactory since it does not satisfies the subformula property. Later, Sasaki introduced another sequent calculus for BPL and prove the cut-elimination theorem. However his system involves an ad hoc expression as $(A \supset B)^+$ which departs from ordinary formulations of sequent calculus and subformula property in this system becomes a weak form. In this thesis, we introduce another sequent calculi for BPL and FPL, both of which satisfy subformula property. Furthermore, we prove the cut-elimination theorem for these by syntactical method.

Next, we introduce another reduction procedure for the first-order classical natural deduction **NK** and prove the strong normalization theorem and the Church-Rosser property. For the first-order classical natural deduction, Prawitz proved the strong normalization theorem for **NK** which is restricted in the sense that $\vee$ and $\exists$ are not treated as primitive logical symbols. For **NK** with full logical symbols, Stålmarck introduced a reduction procedure and proved the strong normalization theorem. However this result is not satisfactory since Stålmarck's reduction does not satisfy the Church-Rosser property. Then we introduce another reduction procedure for **NK** and prove the strong normalization theorem and the Church-Rosser property. This yields the strong normalization theorem with respect to Andou's reduction introduced in 1995 since Andou's reduction steps are expressed by several steps of ours.

Finally, we discuss proof theoretical study for the first-order arithmetic. We treat here is provable well-founded relation of $I\Sigma_k$, where $I\Sigma_k$ is a subsystem of **PA** which is obtained by restricting induction formulae to Σ_k-formulae. Let $\prec$ be a recursive well-ordering of the natural numbers and let $TI(\prec)$ be $\forall x(\forall y(y \prec x \supset \varepsilon(y)) \supset \varepsilon(x)) \to \varepsilon(a)$. Gentzen proved that if $TI(\prec)$ is provable in **PA**, then the order-type of $\prec$ is less than ε_0. Later, Takeuti refined this, i.e., he constructed recursive function f such that if $TI(\prec)$ is provable in **PA**, then $a \prec b \Leftrightarrow f(a) <^* f(b)$ holds where $<^*$ denotes the standard ordering of type ε_0, and there exists an ordinal $\mu < \varepsilon_0$ such that for every a, $f(a) <^* \mu$. Furthermore, Arai weakened the assumption "$\prec$ is well-ordering" to "$\prec$ is an irreflexive and transitive well-founded binary relation". In this thesis, we consider this problem for $I\Sigma_k$ and obtain the similar result to the one for **PA**, in which we can replace ε_0 to ω_{k+2}.

Key Words: sequent calculus, natural deduction, cut-elimination theorem, strong normalization theorem, BPL, FPL, classical logic, first-order arithmetic, provable well-founded relation