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A Proof-Theoretic Study of Non–Classical Logics — Natural deduction systems for intuitionistic substructural logics and implementation of proof assistant system

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Abstract

In the present thesis, we develop a proof-theoretic study of non–classical logics, including substructural logics and modal logics, and introduce a proof assistant system which can be a convenient tool for researchers in mathematical logic.

Proof theory is a mathematical study of formal mathematical arguments or proofs, i.e., proof figures. One of the cornerstones of proof theory is the cut elimination theorem in sequent calculi. Cut elimination theorem says that if a formula is provable then there is a cut-free (i.e. detour-free) proof figure for it. We can derive many interesting logical properties by analyzing cut-free proof figures.

Cut-free proofs in sequent calculi correspond to normal proofs in natural deduction systems. Any basic step for obtaining a normal proof from a given proof is called a 'reduction'. Then, reductions in natural deduction systems correspond to reductions in lambda calculi, by 'Curry–Howard isomorphism'. The following are basic questions with respect to reductions in natural deduction systems.

1. Can we get a normal proof eventually by repeated applications of reductions? (weak and strong normalization, termination)
2. Can we get a unique normal proof for a given proof? (uniqueness)
3. Are there efficient reduction strategies to get a normal proof? (optimal reduction strategy)

In our thesis, we introduce natural deduction systems for basic substructural logics having both additive and multiplicative conjunctions, and prove the strong normaliztion theorem. One particular feature of our theorem is that we give a function which estimates the upper bound of the number of reduction steps in an effective way, for basic substructural logics without contraction rule. Next, we succeed to give an optimal reduction strategy for a certain class of lambda terms.

In most cases, a sequent propositional calculus becomes decidable when the cut elimination theorem holds for it. But we cannot always expect the existence of an efficient algorithm which decides the provability of a given formula. From a practical point of view, most of theorem provers of non–classical logics are unsatisfactory as tools for working logicians. With this in mind, we introduce a proof or refutation algorithm for modal logic S4. For a given formula A, our algorithm gives us a cut-free proof when A is provable, and a finite countermodel when A is not provable.

Moreover, we present a proof assistant system called 'xpe' designed with the purpose of providing a convenient tool for work in logic. It offers not only theorem provers for substructural and modal logics but also proof or refutation systems for modal logics K, KT and S4.

Key Words: **mathematical logic, substructural logics, lambda calculi, automated theorem proving, proof or refutation algorithm**