Petri Net Models for Intuitionistic Linear Logic

Keiko Ishihara
School of Information Science,
Japan Advanced Institute of Science and Technology

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Abstract

The connection between linear logic and Petri nets has recently been a subject of great interest. In these researches, the propositional fragment of intuitionistic linear logic with exponential ! was considered, and Petri nets were related to linear logic as follows: each place of a Petri net is regarded as an atomic proposition of linear logic, and transitions as provability relation.

Soundness and completeness of linear logic are proved for algebraic structures, called quantales. Engberg and Winskel showed soundness of linear logic for quantales induced from Petri nets, but for these quantales completeness was not valid. Soundness states that all provable properties (formulas) in linear logic hold in Petri nets, while completeness states that properties which hold in any Petri net can be proved in linear logic. When both of soundness and completeness are valid in some quantales induced from Petri nets, we can say that a property is provable in linear logic if and only if it is a common property of Petri nets, i.e., it holds in any Petri net.

Recently Engberg and Winskel have also proved completeness of a $\sqcup$-free fragment of linear logic and linear logic with distributivity. One of difficulties in proving completeness for full linear logic lies in distributivity of $\sqcap$ over $\sqcup$, i.e., $A \sqcap (B \sqcup C) \Rightarrow (A \sqcap B) \sqcup (A \sqcap C)$, which does not hold in linear logic. The quantales constructed by Engberg and Winskel are distributive lattices, i.e., distributivity is always valid. Therefore, to prove completeness using their quantales, we have to deal with the $\sqcup$-free fragment or to add the distributivity to linear logic as an axiom. However these are not what we intend to do. Although there should be argument about which of full linear logic or a logic with distributivity is appropriate for representing properties of Petri nets, we here concentrate on proving completeness for full linear logic. To find adequate logics for which the models of Engberg and Winskel are complete is another interesting problem.

In this thesis, we first construct non-distributive quantales, i.e., quantales in which distributivity is not always valid, from Petri nets, and prove completeness of linear logic without exponential for the quantales. In linear logic, exponential ! is added to compensate the absence of the rules of weakening and contraction. For example, $!A$ indicates

that we may extract as many data of type A as we like, i.e., a datum of type $!A$ is a finite collection of data of type A. For Petri nets, we can regard a place with exponential $!$ as a place which can supply arbitrary many but finite resources (tokens, in petri net terminology) by firing transitions. We extend the construction of the quantales to those with exponential, and prove completeness of linear logic for the quantales. It means that properties which hold in any Petri nets with such exponential places can be proved in linear logic.

We also give an impression on the meaning of the logic on the proposed Petri net model, comparing with that by Engberg and Winskel.

Key Words: linear logic, Petri net, quantale, exponential, completeness