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A Ciphertext-Policy Attribute-Based Encryption Scheme with Constant Ciphertext Length

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Abstract: An Attribute-Based Encryption (ABE) is an encryption scheme, where users with some attributes can decrypt ciphertexts associated with these attributes. The length of the ciphertext depends on the number of attributes in previous ABE schemes. In this paper, we propose a new Ciphertext-Policy Attribute-Based Encryption (CP-ABE) with constant ciphertext length. In our scheme, the number of pairing computations is also constant. In addition, the number of additional bits required from CPA-secure CP-ABE to CCA-secure CP-ABE is reduced by 90% with respect to that of the previous scheme.

Keywords: Ciphertext-Policy Attribute-based encryption; Constant Ciphertext Length.


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1 Introduction

A user identity (such as the name, e-mail address and so on) can be used for accessing control of some resources. For example, in Identity-Based Encryption (IBE) schemes such as [9, 12], an encryptor can restrict a decryptor to indicate the identity of the decryptor. An Attribute-Based Encryption (ABE) scheme is an encryption scheme, where users with some attributes can decrypt the ciphertext associated with these attributes. Although IBE schemes have a restriction such that an encryptor only indicates a single decryptor, in ABE schemes, an encryptor can indicate many decryptors by assigning common attributes of these decryptors such as gender, age, affiliation and so on. There are two kinds of ABE, Key-Policy ABE (KP-ABE) and Ciphertext-Policy ABE (CP-ABE). KP-ABE [18, 27] are schemes such that each private key is associated with an access structure. CP-ABE [5, 15, 17, 25, 32] are schemes such that each ciphertext is associated with an access structure. An application of KP-ABE is for a biometric system. If an IBE scheme is used to construct the biometric system, then a user’s information (such as a finger-print, iris data and so on) is registered as the identity of the user. However, these values are somewhat changed since they depend on a user’s condition, on humidity and so on. Therefore, the user is forced to manage secret keys corresponding to all identities. KP-ABE schemes with threshold structures can solve this problem to indicate a threshold value as an error-tolerant value. An application of CP-ABE is for an encrypted storage system. If 1 data is encrypted by using 1 encryption key, then the total number of encryption and decryption keys increases. If plural data are encrypted by using one encryption key, then a fine-grained access control is not achieved. To indicate the set of attributes of a decryptor such as affiliation, the CP-ABE scheme can achieve a fine-grained access control without increasing the number of keys. There are some extended ABE schemes such as ABE schemes with the multi-authority [14, 22], an attribute-based broadcast encryption scheme [23], and a CP-ABE scheme with recipient anonymity [25]. A problem of previous ABE schemes is that the length of the ciphertext depends on the number of attributes. Also, the number of pairing computations depends on the number of attributes. A Predicate Encryption Scheme (PES), where secret keys correspond to predicates, and where ciphertexts are associated with attributes, has been proposed [9, 12], an encryptor can restrict a decryptor to indicate the identity of the decryptor. An Attribute-Based Broadcast Encryption scheme [23], and a CP-ABE scheme with recipient anonymity [25]. A problem of previous ABE schemes is that the length of the ciphertext depends on the number of attributes. This means that, until our work, to the best of our knowledge, there has been no scheme that enables a constant ciphertext length with AND-gates on multi-valued attributes. Our scheme enables Chosen Plaintext Attack (CPA) security. In addition, we construct a Chosen Ciphertext Attack (CCA)-secure CP-ABE scheme with constant ciphertext length by using the conversion method proposed in CN07 [15]. This is the main difference between this paper and the previous version [16].

Organization: The paper is organized as follows: Some definitions are presented in Section 2. The previous scheme is introduced in Section 3. Our scheme with CPA security and the CCA-conversion scheme are described in Section 4. The security proof of our scheme is presented in Section 5. Efficiency comparisons are made in Section 6. The security proof of our CCA-conversion scheme is presented in the Appendix.

2 Preliminary

In this section, some definitions are presented. Note that $x \in \mathbb{R}$ means $x$ is randomly chosen for a set $S$.

2.1 Bilinear Groups and Complexity Assumption

Definition 1. (Bilinear Groups) Bilinear groups and a bilinear map are defined as follows:

1. $G_1$ and $G_T$ are cyclic groups of prime order $p$.
2. $g$ is a generator of $G_1$.
3. $e$ is an efficiently computable bilinear map $e : G_1 \times G_1 \rightarrow G_T$ with the following properties.
   - Bilinearity: for all $u, u', v, v' \in G_1$, $e(uu', v) = e(u, v)e(u', v)$ and $e(u, vv') = e(u, v)e(u, v')$.
   - Non-degeneracy: $e(g, g) \neq 1_{G_T}$ ($1_{G_T}$ is the $G_T$’s unit).

Definition 2. (DBDH assumption)

The Decision Bilinear Diffie-Hellman (DBDH) problem in $G_1$ is a problem, for input of a tuple $(g, g^a, g^b, g^c, Z) \in G_1^4 \times \mathbb{G}_T$, to decide whether $Z = e(g, g)^{abc}$ or not. An algorithm $A$ has advantage $\epsilon$ in solving the DBDH problem in $G_1$ if $Adv_{\text{DBDH}}(A) := \left| \Pr[A(g, g^a, g^b, g^c, e(g, g)^{abc}) = 0] - \Pr[A(g, g^a, g^b, g^c, e(g, g)^{abc}) = 0] \right| \geq \epsilon(n)$, where $e(g, g)^{abc} \in \mathbb{G}_T \setminus \{e(g, g)^{abc}\}$. We say that the DBDH assumption holds in $G_1$ if no PPT algorithm has an advantage of at least $\epsilon$ in solving the DBDH problem in $G_1$.
2.2 Definition of Access Structures

Several access structures such as the threshold structure [27], the tree-based access structure [5, 17], AND-gates on positive and negative attributes with wildcards [15], AND-gates on multi-valued attributes with wildcards [25], and the linear access structure [32] are used in previous ABE schemes. In our scheme, we use AND-gates on multi-valued attributes as follows:

Definition 3. Let $\mathcal{U} = \{\text{att}_1, \ldots, \text{att}_n\}$ be a set of attributes. For $\text{att}_i \in \mathcal{U}$, $S_i = \{v_{i,1}^1, v_{i,2}^1, \ldots, v_{i,n_i}\}$ is a set of possible values, where $n_i$ is the number of possible values for $\text{att}_i$. Let $L = [L_1, L_2, \ldots, L_n]$, $L_i \in S_i$ be an attribute list for a user, and $W = [W_1, W_2, \ldots, W_n]$, $W_i \in S_i$ be an access structure. The notation $L \models W$ expresses that an attribute list $L$ satisfies an access structure $W$, namely, $L_i = W_i$ ($i = 1, 2, \ldots, n$).

The number of access structures is $\prod_{i=1}^n n_i$. For each $\text{att}_i$, an encryptor has to explicitly indicate a status $v_{i,*}$ from $S_i = \{v_{i,1}, v_{i,2}, \ldots, v_{i,n_i}\}$.

2.2.1 Differences between the previous AND-gate structures [15, 25] and ours

If $n_i = 2$ ($i = 1, 2, \ldots, n$), then our structure is the same as the access structures [15] excluding wildcards. In [25], an access structure $W$ is defined as $W = [W_1, W_2, \ldots, W_n]$ for $W_i \subseteq S_i$, and $L \models W$ is defined as $L_i \in W_i$ ($i = 1, 2, \ldots, n$). This means that our access structure is a subset of these in [15, 25].

2.2.2 Adequacy of AND-gate structures

If flexible structures can be achieved (e.g., OR-gate, wildcards, tree-based structures, and so on), then we can achieve fine-grained access control. On the contrary, in our scheme, an encryptor must indicate all attribute, explicitly. We insist that AND-gate structures are the most basic usage, namely, an encryptor indicates a concrete set of attributes, and optionally takes advantage of flexible structures. We have only to accept AND-gate structures, then an efficient CP-ABE scheme can be constructed such as to produce a constant ciphertext length and to reduce the number of additional bits required from CPA-secure CP-ABE to CCA-secure CP-ABE. Furthermore, as a difference of secret sharing [28] (in this case, “AND-gate only” means the unanimous structure, namely, the number of access structures is only 1), the number of access structures is $\prod_{i=1}^n n_i$. We insist that no redundancy, namely without wild cards, is a reasonable restriction.

2.3 Ciphertext-Policy Attribute-Based Encryption Scheme (CP-ABE)

CP-ABE is described using four algorithms, Setup, KeyGen, Encrypt and Decrypt [15].

Definition 4. Ciphertext-Policy Attribute-Based Encryption Scheme

Setup: This algorithm takes as input the security parameter $\kappa$, and returns a public key $PK$ and a master secret key $MK$.

KeyGen: This algorithm takes as input PK, MK and a set of attributes $L$, and returns a secret key $SK_L$ associated with $L$.

Encrypt: This algorithm takes as input PK, a message $M$ and an access structure $W$. It returns a ciphertext $C$ with the property that a user with $SK_L$ can decrypt $C$ if and only if $L \models W$.

Decrypt: This algorithm takes as input PK, C which was encrypted by $W$, and $SK_L$. It returns $M$ if $SK_L$ is associated with $L \models W$.

2.4 Selective Game for CP-ABE

The selective game for CP-ABE has been defined in [15]. This game captures the indistinguishability of messages and the collusion resistance of secret keys, namely, attackers cannot generate a new secret key by combining their secret keys. To capture the collusion resistance, multiple secret key queries can be issued by the adversary $A$ after the challenge phase. This means that $A$ can issue the KeyGen queries $L_1$ and $L_2$ such as $(L_1 \not\models W^*) \land (L_2 \not\models W^*)$ and $(L_1 \cup L_2) \models W^*$. This collusion resistance is an important property of the CP-ABE scheme, which has not been considered in Hierarchical IBE (HIBE) schemes such as in [8]. A weaker definition of CP-ABE has been considered [20], where an adversary cannot obtain secret keys associated with any $\text{att}_i$ such that $\text{att}_i \in L \models W^*$. However, we do not use this weaker definition because it does not guarantee collusion resistance. The selective game for CP-ABE under the CCA is defined as follows:

Definition 5. Selective Game for CP-ABE under the CCA

Init: The adversary $A$ sends the challenge access structure $W^*$ to the challenger.

Setup: The challenger runs Setup and KeyGen, and gives $PK$ to $A$.

Phase 1: $A$ makes KeyGen and Decryption queries. Note that these queries can be repeated adaptively.

KeyGen queries: $A$ sends an attribute list $L$ to the challenger for a KeyGen query, where $L \not\models W^*$. The challenger answers with a secret key for these attributes.

Decryption queries: $A$ sends a ciphertext $C$ encrypted to $W$. If $C$ is an invalid ciphertext, then $A$ loses. The challenger answers the corresponding plaintext $M$. 
Protocol 1. The CPA-secure CP-ABE Scheme [CN07-1] [15]

Phase 2: Same as Phase 1, A sends $L$ to the challenger for a KeyGen query. The challenger answers with a secret key for these attributes. Note that $L \not\subset W^*$, and these queries can be repeated adaptively.

Guess: $A$ outputs a guess $\mu' \in \{0,1\}$.

The advantage of $A$ is defined as $\text{Adv}(A) := |\text{Pr}(\mu' = \mu) - \frac{1}{2}|$.

The selective game for CP-ABE under the CPA is simply converted into the CP-ABE scheme that is proven along with the CCA-type security. Our scheme is proven along with the selective game for CP-ABE under the CPA, and can be converted into the CP-ABE scheme that is proven along with the selective game for CP-ABE under the CCA.

3 Previous CP-ABE

In this section, we summarize a CPA-secure CP-ABE scheme (called the CN07-1 scheme) and a CCA-secure CP-ABE scheme (called the CN07-2 scheme) proposed in [15]. Let $\mathcal{U} = \{-\text{att}_1, \ldots , -\text{att}_n\}$ be a set of negative attributes for a set of attributes $\mathcal{U}$. We refer to attributes $\text{att}_i \in \mathcal{U}$ and their negations $-\text{att}_i$ as literals. Let $W = \bigwedge_{\text{att}_i \in \mathcal{U}} \text{att}_i$ be an access structure, where $I \subseteq \mathcal{U}$ and $\text{att}_i$ is either $\text{att}_i$ or $-\text{att}_i$. The public key elements $T_i, T_{n+i}, T_{2n+i}$ correspond to the three properties of $\text{att}_i$, namely, positive, negative and don’t care.

Protocol 1. The CPA-secure CP-ABE Scheme [CN07-1] [15]

Setup($1^n$): A trusted authority $TA$ chooses a prime number $p$, a bilinear group $\mathbb{G}_1$ with order $p$, a generator $g \in \mathbb{G}_1$, $y \in \mathbb{Z}_p$, and $t_i \in \mathbb{Z}_p (i = 1, 2, \ldots , 3n)$, and computes $Y = e(g, g)^y$ and $T_i = g^{t_i} (i = 1, 2, \ldots , 3n)$. $TA$ outputs $PK = (e, g, Y, T_1, \ldots , T_{3n})$ and $MK = (y, t_1, \ldots , t_{3n})$.

KeyGen($PK, MK, S$): Every $\text{att}_i \not\in S$ is implicitly considered to be a negative attribute. $TA$ chooses $r_i \in \mathbb{Z}_p (i = 1, 2, \ldots , n)$, sets $r = \sum_{i=1}^{n} r_i$, and computes $D = g^{r-r_i}$. $TA$ computes $D_i$ and $F_i$ as follows:

$D_i = \begin{cases} g^{\frac{r}{r_i}} & (\text{att}_i \in S) \\ g^{r_{n+i}} & (\text{att}_i \not\in S) \end{cases}$

$F_i = g^{r_{2n+i}} (\text{att}_i \in \mathcal{U})$

$TA$ outputs $SK = \langle \hat{D}, \{D_i, F_i\}_{i \in [1,n]} \rangle$.

Encrypt($PK, M, W$): Let $W = \bigwedge_{\text{att}_i \in \mathcal{U}} \text{att}_i$. An encryptor chooses $s \in \mathbb{Z}_p$, and computes $C = M \cdot Y^s$ and $\hat{C} = g^s$. The encryptor computes $C_i$ as follows:

$C_i = \begin{cases} T_i^s & (\text{att}_i = \text{att}_i) \\ T_{n+i}^s & (\text{att}_i = -\text{att}_i) \\ T_{2n+i}^s & (\text{att}_i \not\in \mathcal{U} \setminus I) \end{cases}$

The encryptor outputs $C = (W, \hat{C}, \hat{C}, \{C_i\}_{i \in [1, n]}$.

Decrypt($PK, C, SK$): A decryptor computes the pairing $e(C_i, D_i) (\text{att}_i \in \mathcal{U})$ and $e(C_i, F_i) (\text{att}_i \not\in \mathcal{U})$ as follows:

$e(C_i, D_i) = \begin{cases} e(g^{t_i}, g^{r_i}) & (\text{att}_i = \text{att}_i) \\ e(g^{r_{n+i}}, g^{r_{n+i}}) & (\text{att}_i = -\text{att}_i) \end{cases}$

$= e(g, g)^{r-r_i}$

$e(C_i, F_i) = e(g^{r_{2n+i}}, g^{r_{2n+i}}) = e(g, g)^{r-r_i}$

Then $e(C, D) \prod_{i=1}^{n} e(g, g)^{r-r_i} = M \cdot e(g, g)^{r-r_i} = M$ holds.

To compute $e(g, g)^{r-r_i}$, the decryptor has to compute either $e(C_i, D_i)$ or $e(C_i, F_i)$ for each $i$. This means that all $C_i$ are included in a ciphertext, and thus the length of a ciphertext depends on the number of attributes $n$. Moreover, the CN07-1 scheme does not provide for adding new attributes after Setup. If some attributes are added after Setup, then some users (who have already obtained the secret key) can decrypt a ciphertext which one must not be able to decrypt. For example, let $\mathcal{U} = \{\text{att}_1, \text{att}_2\}$, and assume that a user $U$ has secret keys of $\text{att}_1$ and $\text{att}_2$, and that a ciphertext $C$ is associated with $W = \text{att}_1 \land \text{att}_2$. Then, $U$ can decrypt a ciphertext associated with $\text{att}_1 \land \text{att}_2 \land \text{att}_3$ without a secret key of $\text{att}_3$. Concretely, $U$ ignores a part of the ciphertext for $\text{att}_3$. CP-ABE schemes which enable the addition of new attributes after Setup have been proposed in BSW07 [5] and in the 2nd-scheme of NYO08 [25]. If a user wants to decrypt a ciphertext with an access structure including newly added attributes, then the user must once more obtain a new secret key (including newly added attributes) from the trusted authority again. However, the security proof of both schemes contains no reduction, namely, it is proven under the generic group heuristic.

The CN07-1 scheme can be translated into a CCA-secure CP-ABE scheme by using Strongly Existentially Unforgeable (SEU) one-time signatures. This technique is the same as the CHK (Canetti, Halevi and Katz) technique [13] that is a generic construction for a CCA-secure public key encryption using a CPA-secure IBE and an SEU one-time signature. Let SigKeyGen, Sign and Verify be a signature scheme. SigKeyGen is a probabilistic algorithm which outputs a signing/verification key pair $\langle K_s, K_v \rangle$. Sign is a probabilistic algorithm which outputs a signature $\sigma$ from
\(K_x\) and a message \(M\). Verify is a deterministic algorithm which outputs a bit from \(\sigma, K_v\) and \(M\). If Verify outputs 1, this means that \(\sigma\) is a valid signature, and 0 otherwise. The security game of strong existential unforgeability under an adaptive chosen message attack [7] is defined as follows:

**Definition 6. Setup:** The challenger runs \(\text{SigKeyGen}\), and obtains a signing key \(K_s\) and a verification key \(K_v\). The adversary \(A\) is given \(K_v\).

**Sign Queries:** A requests signatures on messages \(M_1, M_2, \ldots, M_q\) \(\in\{0,1\}^*\), where \(q_s\) is the number of queries. The challenger answers \(\sigma_i = \text{Sign}(K_s, M_i)\) for each query. Note that these queries can be repeated adaptively.

**Output:** A outputs a pair \((M^*, \sigma^*)\), and wins the game if \((M^*, \sigma^*) \notin \{(M_1, \sigma_1), \ldots, (M_q, \sigma_q)\}\) and \(\text{Verify}(K_v, \sigma^*, M^*) = 1\).

In the above definition, the forged pair \(M\) could have been signed previously. There are techniques which convert (non-strong) existentially unforgeable signatures into strong existentially unforgeable ones [10, 19, 30]. Especially, Huang et. al [19] have proposed the generic transformation which converts any existentially unforgeable signature into SEU ones by using strong one-time signature schemes. We call this conversion way the HWZ conversion. Note that strong one-time signature schemes can be constructed from any one-way function-based one-time signature. The security game of strong one-time existential unforgeability [19] is simply defined as follows:

**Definition 7. Setup:** The challenger runs \(\text{SigKeyGen}\), and obtains a signing key \(K_s\) and a verification key \(K_v\). The adversary \(A\) is given \(K_v\).

**Sign Queries:** A requests a signature on a message \(M\) \(\in\{0,1\}^*\). The challenger answers \(\sigma = \text{Sign}(K_s, M)\).

**Output:** A outputs a pair \((M^*, \sigma^*)\), and wins the game if \((M^*, \sigma^*) \neq (M, \sigma)\) and \(\text{Verify}(K_v, \sigma^*, M^*) = 1\).

Next, we summarize a CCA-secure CP-ABE scheme (called the CN07-2 scheme). Let \(m\) be the size of \(K_v\), \(K_{v,i}\) be the \(i\)-th bit of \(K_v\), and \(\mathcal{M} = \{1, \ldots, m\}\). Added to the construction of a CPA-secure scheme, a user has secret keys \(G_0^i\) and \(G_1^i\) \((i \in \mathcal{M})\) associated with \(i \in \mathcal{M}\) (this is a secret key of \(K_{v,i} = 0\)) and \(m + i\) (this is a secret key of \(K_{v,i} = 1\)), respectively.

**Protoc 2. The CCA-secure CP-ABE Scheme [CN07-2] [15]**

**Setup(1c):** A trusted authority TA chooses a prime number \(p\), a bilinear group \(G_1\) with order \(p\), a generator \(g \in G_1\), \(y \in R \mathbb{Z}_p\), \(t_i \in R \mathbb{Z}_p\) \((i = 1, 2, \ldots, 3n)\) and \(u_i \in R \mathbb{Z}_p\) \((i = 1, 2, \ldots, 2m)\), and computes \(Y = e(g, y)^{t_{i}}\), \(T_i = g^{u_{i}}\) \((i = 1, 2, \ldots, 3n)\) and \(U_i = g^{u_{i}}\) \((i = 1, 2, \ldots, 2m)\). TA outputs \(PK = (e, g, Y, T_1, \ldots, T_{3n}, U_1, \ldots, U_{2m})\) and \(MK = (y, t_1, \ldots, t_{3n}, u_1, \ldots, u_{2m})\).

**KeyGen(\(PK, MK, S\)):** Every \(i, j \notin S\) is implicitly considered to be a negative attribute. TA chooses \(r_{i} \in R \mathbb{Z}_p\) \((i = 1, 2, \ldots, n)\) and \(\omega_i \in R \mathbb{Z}_p\) \((i = 1, 2, \ldots, m)\), sets \(r = \sum_{i=1}^{n} r_i + \sum_{i=1}^{m} \omega_i\), and computes \(D = g^{y-r}\). TA computes \(D, F_i, G_0^i\) and \(G_1^i\) as follows:

\[
D_i = \begin{cases} 
  g^{r_i} & (\text{att}_i \in S) \\
  g^{-r_i} & (\text{att}_i \notin S)
\end{cases},
F_i = g^{2n+i} (\text{att}_i \in U_0),
G_0^i = g^{r_i},
G_1^i = g^{\omega_i}
\]

**Encrypt(\(PK, M, W\)):** Let \(W = \bigwedge_{\text{att}_i \in I} \text{att}_i\). An encryptor runs \(\text{SigGenKey}\) and obtains a signing/verification key pair \((K_s, K_v)\). The encryptor chooses \(s \in R \mathbb{Z}_p\), and computes \(\hat{C} = M \cdot Y^s\) and \(\hat{C} = g^s\). The encryptor computes \(C_i\) \((i = 1, 2, \ldots, n)\) as follows:

\[
C_i = \begin{cases} 
  T_{s}^i (\text{att}_i = \text{att}_i) \\
  T_{s+i}^i (\text{att}_i = -\text{att}_i) \\
  T_{3n+i}^i (\text{att}_i \in U_0 \setminus I)
\end{cases}
\]

The encryptor computes \(E_i\) \((i = 1, 2, \ldots, m)\) as follows:

\[
E_i = \begin{cases} 
  U_{s}^i (K_{v,i} = 0) \\
  U_{m+i}^i (K_{v,i} = 1)
\end{cases}
\]

The encryptor computes a signature \(\sigma = \text{Sign}(K_s, \{W, \hat{C}, \hat{C}, \{C_i\}_{i \in [1,n]}, \{E_i\}_{i \in [1,m]}\})\).

The encryptor outputs \(C = (W, s, K_v, \hat{C}, \hat{C}, \{C_i\}_{i \in [1,n]}, \{E_i\}_{i \in [1,m]}\))

**Decrypt(\(PK, C, SK\)):** A decryptor checks \(\text{Verify}(K_v, \sigma, \{W, \hat{C}, \hat{C}, \{C_i\}_{i \in [1,n]}, \{E_i\}_{i \in [1,m]}\})\).

If \(\sigma\) is valid, then the decryptor computes the pairing \(e(C_i, D_i)\) \((\text{att}_i \in I)\) and \(e(C_i, F_i)\) \((\text{att}_i \notin I)\) as follows:

\[
e(C_i, D_i) = \begin{cases} 
  e(g^{y-r}, g^{r_i}) & (\text{att}_i = \text{att}_i) \\
  e(g^{2n+i}, g^{r_{3n+i}}) & (\text{att}_i = -\text{att}_i)
\end{cases}
\]

\[
e(C_i, F_i) = e(g^{2n+i}, g^{r_{2m+i}}) = e(g, g)^{r_{s+i}}
\]

Moreover, for each \(i \in \mathcal{M}\), the decryptor computes \(e(E_i, G_0^i)\) \((\text{when } K_{v,i} = 0)\) and \(e(E_i, G_1^i)\) \((\text{when } K_{v,i} = 1)\), and obtains \(e(g,y)^{r_{s+i}}\).

Then \(\hat{C} = \frac{e(C, D) \prod_{i \in I} e(g, g)^{r_{s+i}} \prod_{i \in I} e(g, g)^{r_{s+i}}}{e(g,y)^{r_{s+i}}} = M\) holds.
To enable the CCA-secure scheme, the CN07-2 scheme has to require the additional values \( \{U_i\}_{i \in [1,2m]} \) as PK, \( \{u_i\}_{i \in [1,2m]} \) as MK, \( \{G^0_i, G^1_i\}_{i \in [1,m]} \) as SK, and \( \{E_i\}_{i \in [1,n]}, \sigma, K_u \) as C. Especially, the overhead of the length of ciphertext is \( m|G_1| + \) signature size + \( m \). If we require the BB short signature [7] as a SEU signature, the verification key is \( (u,v) \in G^2_1 \), and the signature is \( (\sigma, r) \in G_2 \times Z_p \). Therefore, when we evaluate that \( m = 161 \times 2 = 322 \) bits, the overhead is \( 322 \times 161 + 321 + 322 = 52485 \) bits and the total ciphertext length is \( (n+1)|G_1| + |G_2| + 52485 = 161(n+1) + 53505 \) bits, where \( |p| = 160 \) bits. Note that any SEU signature scheme can be regarded as a strong one-time signature scheme. Next, we evaluate the overhead when a strong one-time signature is used. In the HWZ conversion [19], Reyzin et al.’s HORS (Hash to Obtain Random Subset) scheme [26] is recommended as a one-time signature scheme to convert a strong one-time signature scheme. The verification key length is 40960 bits, the signing key length is 61440 bits, and the signature length is 4800 bits in the Strong HORS setting recommended in [19]. An efficient strong one-time signature scheme based on a two-tiers signature scheme has been proposed in [4]. The verification key length is 480 bits over a 160-bit elliptic curve group. Therefore, from the viewpoint of the length of ciphertext, the BB short signature is the best one.

4 Our construction

In this section, we propose a constant ciphertext length CP-ABE scheme.

4.1 The difficulty

Our main aim is construction of a constant ciphertext length CP-ABE scheme. Here we explain how difficult is the construction of a constant ciphertext length CP-ABE scheme with access structures included wildcard expression. Under the wildcard setting, an encryptor does not expect what secret keys will be used. Concretely, we can construct a CP-ABE scheme such that a ciphertext will be decrypted using secret keys of a correct set of attributes \( L \), and will not be decrypted using secret keys of an illegal set of attributes \( L' \), where \( L' \cap L = \emptyset \). However, it is difficult to treat a set of attributes \( L'' \) such that \( L \subseteq L'' \) (\( L \) is a proper subset of \( L'' \)), since the encryptor cannot expect a redundancy part \( L'' \setminus L \). This problem can be solved to admit the attribute depended number of ciphertexts. Here we show how to enable the wildcard expression in previous CP-ABE schemes [15, 25] without constant ciphertext length. For \( att_i \) (\( i = 1, 2, \ldots, n \)), we can construct a ciphertext \( C_i \) which can be decrypted by using a corresponding secret key of \( att_i \), and cannot be decrypted by using an illegal secret key. This is the same situation of usual public key encryption scheme, and it is used in the (both) CN07 scheme [15]. In the CN07 scheme, an encryptor computes \( C_i \) (\( i = 1, 2, \ldots, n \)) for \( att_i \) by using one of the public key (\( T_i, T_{n+1}, T_{2n+i} \)). Let \( W \) be an access structure chosen by the encryptor. If \( att_i \in W \) (resp. \( \neg att_i \in W \)), then \( T_i \) (resp. \( T_{2n+i} \)) is used. Otherwise, if \( att_i \not\in W \) (this means the encryptor does not care about \( att_i \)), then \( T_{2n+i} \) is used. A user has three kinds of secret keys (for positive or negative attributes and for wildcards), and decrypts \( C_i \) by using a “positive key” \( (att_i \in W) \) or a “negative key” \( (\neg att_i \in W) \), or a “wildcard key” \( (att_i \not\in W) \). Therefore, every user has \( 2n \) secret keys, respectively. It is easy to construct a extended CN07 scheme with AND-gates on multi-valued attributes with wildcards. However, every users have \( \sum^n_{i=1} n_i \) secret keys, respectively. However, this kind of construction requires the number of \( n \) ciphertexts. On the other hand, each user has only \( n \) secret keys in the NYO08 scheme [25]. However, the number of \( \sum^n_{i=1} n_i \) ciphertexts is required. An access structure \( W \) is defined as \( W = \{W_1, W_2, \ldots, W_n\} \) for \( W_i \subseteq S_i \). If \( v_{i,t} \in W_i \), then \( C_{i,t} \) is correctly computed. Otherwise, if \( v_{i,t} \not\in W_i \), \( C_{i,t} \) is randomly chosen. A user has one state \( v_{i,t} (\ell \in [1,n_i]) \) for each attribute \( att_i \), and can decrypt \( C_{i,t} \) if \( v_{i,t} \in W_i \). If \( S_i = W_i \), then any user can decrypt a ciphertext corresponding to \( att_i \), since all states of \( att_i \) are included in \( W_i \). This means \( att_i \) is indicated as a wildcard, since the encryptor does not care about \( att_i \). From the above considerations, the wildcard expression is achieved to provide the number of \( n \) (or \( \sum^n_{i=1} n_i \)) ciphertexts. Otherwise, in the AND-gates on multi-valued attributes (without wildcard) setting, an encryptor does not have to expect a redundancy part, since an access structure is explicitly described. In addition, the sum of master keys \( t_{i,j} \) (described as \( \sum_{v_{i,j} \in W} t_{i,j} \)) is applied to express an access structure \( W \). This form enables the constant ciphertext length. Although a mapping \( W \rightarrow \sum_{v_{i,j} \in W} t_{i,j} \) is not one-to-one, the condition \( \sum_{v_{i,j} \in W} t_{i,j} \neq \sum_{v_{i,j} \notin W} t_{i,j} \), where \( W \neq W' \), holds with overwhelming probability. See Section 4.3 for details. A generic construction of an identity-based encryption scheme with wildcards (called WIBE for short) from any HIBE scheme has been proposed [2]. However, a user’s secret key size is exponential in the depth of the hierarchy tree. To solve this problem, WIBE schemes also have been constructed based on the Ware’s HIBE [31], the Boneh-Boyen HIBE [6], and the Boneh-Boyen-Goh HIBE [8], respectively. The length of the secret key linearly depends on the maximal hierarchy depth. However, these schemes do not enable the constant ciphertext length, since the length of ciphertext also linearly depends on the maximal hierarchy depth. Next, we discuss the difference between a CP-ABE scheme with AND-gates on multi-valued attributes (without wildcard) and a HIBE scheme. In a HIBE scheme, the user’s identity in depth \( \ell \) is described using the set of identities from root node to own node such that \( I_\ell := ID_1 || ID_2 || \ldots || ID_\ell \). The user with the secret key of \( I_\ell \) can generate a new secret key of \( I'_\ell \), where \( I'_\ell := I_\ell || ID_{\ell'} \). On the other

1Note that, in the original paper [7], the verification key is \( (g_1, g_2, u, v, z) \in \alpha^4 \times \alpha^{5} \). However, \( g_1, g_2 \) and \( z = e(g_1, g_2) \) are regarded as common public values.
hand, the CP-ABE scheme has to require the collusion resistance, namely, attackers cannot generate a new secret key by combining their secret keys. Therefore, proposing a constant ciphertext length CP-ABE scheme with AND-gates on multi-valued attributes is a challenging problem, because a HIBE scheme cannot be regarded as a CP-ABE scheme, since HIBE does not satisfy the collusion resistance property.

4.2 Our schemes

4.3 Construction of secret keys

In our scheme, \( \sum_{v_{ij} \in L} t_{ij} \neq \sum_{v_{ij} \in L'} t_{ij} \) is assumed. If there exist \( L \) and \( L' (L \neq L') \) such that \( \sum_{v_{ij} \in L} t_{ij} = \sum_{v_{ij} \in L'} t_{ij} \), a user with the attribute list \( L' \) can decrypt a ciphertext associated with \( W \), where \( L' \neq W \) and \( L \neq W \). Note that the assumption holds with overwhelming probability

\[
\frac{p^N}{p^N - 1} = \frac{1}{1 - \frac{N}{p}} > 1 - \frac{N^2}{p},
\]

where \( N := \prod_{i=1}^{n} N_i \). Therefore, if each secret key \( t_{ij} \) is chosen at random from \( \mathbb{Z}_p \), then our assumption is natural.

4.4 CCA-conversion scheme

Our scheme can be converted into a CCA-secure CP-ABE scheme by using the conversion method proposed in CN07 [15]. For \( K_v \), let \( V_v = \{ K_v,1, K_v,2, \ldots, K_v,n \} \) be the set of bits of \( K_v, u_{v,0} = u_k (k = 0) \) or \( u_{m+i} \) (if \( K_{v,i} = 1 \)), and \( U_v = g^{u_{v,i}} \).

Protocol 4. Our CCA-Secure CP-ABE Scheme with Constant Ciphertext Length

\textbf{Setup(1c): A trusted authority} \( T_A \) chooses a prime number \( p \), a bilinear group \( (G_1, G_T) \) with order \( p \), a generator \( g \in G_1, h \in G_1, y \in R \mathbb{Z}_p \) and \( l_{ij} \in R \mathbb{Z}_p \) (\( i \in [1,n], j \in [1,n] \)). \( T_A \) computes \( Y = e(g, h)^y, T_{ij} = g^{l_{ij}} \) (\( i \in [1,n], j \in [1,n] \)). \( T_A \) outputs \( PK = (e, g, h, Y, \{ T_{ij} \}_{i,j \in [1,n]}) \) and \( MK = \{ y, \{ l_{ij} \}_{i,j \in [1,n]} \} \). Note that \( \forall L, L' (L \neq L') \), \( \sum_{v_{ij} \in L} t_{ij} = \sum_{v_{ij} \in L'} t_{ij} \) is assumed.

\textbf{KeyGen(\( PK, MK, L \)): A encryptor runs}

\begin{align*}
\text{Encrypt}(PK, M, W): & \text{ An encryptor chooses } s \in R \mathbb{Z}_p, \text{ and computes } C_1 = M \cdot Y^s, C_2 = g^s \text{ and } C_3 = (\prod_{v_{ij} \in W} T_{ij})^s. \text{ The encryptor outputs } C = (W, C_1, C_2, C_3). \\
\text{Decrypt}(PK, C, SK_L): & \text{ A decryptor computes what follows:}
\end{align*}

\[
\frac{C_1 \cdot e(C_3, g^r)}{e(C_2, h^s e(g, g)^{\sum_{v_{ij} \in L} t_{ij}})} = \frac{M \cdot e(g, h)^{y} e(g, g)^{\sum_{v_{ij} \in L} t_{ij}}}{e(g, h)^{y} e(g, g)^{\sum_{v_{ij} \in L} t_{ij}}} = M
\]
assumed. This assumption holds with probability $1 - \frac{(2^n) \prod_{i=1}^{n} n_i}{2^{n^2}}$. If we use the BB short signature as a SET signature scheme, then we can evaluate $m = 322$. We cannot use the same size finite groups of the scheme ($|p| = 160$ bits). Let $n = 10$ and $\prod_{i=1}^{n} n_i = 2^{90}$. We believe that this setting is enough in practice. Then, the length of a prime order $p$ can be $728$ bits. Then the ciphertext length of our CCA-secure scheme is $2\lceil \log_2(1 + 2) \rceil = 2 \times 729 + 4368 = 5826$ bits, whereas the ciphertext length of the CN07-2 scheme is $161(n + 1) + 1020 + 322m + 321 + m = 55276$ bits. This means that our scheme can enable the CCA security with an approximately 4500 bits overhead, whereas the CN07-2 scheme requires an approximately 52500 bits overhead to enable the CCA security. To sum up, the number of additional bits required from CPA-secure CP-ABE to CCA-secure CP-ABE is reduced by 90% with respect to that of previous scheme.

5 Security Analysis

In this section, we prove that our scheme is CPA-secure under the DBDH assumption.

**Theorem 1.** Our scheme satisfies the indistinguishability of messages under the DBDH assumption and the chosen message attack.

**Proof.** We suppose that the adversary $A$ wins the selective CPA game for CP-ABE with the advantage $\epsilon$. Then we can construct an algorithm $B$ that breaks the DBDH assumption with the advantage $\frac{\epsilon}{p}$. Let $N := \prod_{i=1}^{n} n_i$ be the number of expressed access structures. The DBDH challenger selects $a, b, c, z \in R \mathbb{Z}_p$, $\nu \in R \{0, 1\}$, and $g$, where $(g) = G_1$. If $\nu = 0$, then $Z = e(g, g)^{abc}$. Otherwise, if $\nu = 1$, then $Z = e(g, g)^{z}$. The DBDH challenger gives the DBDH instance $(g, g^a, g^b, g^c, Z) \in \mathbb{G}_1^4 \times \mathbb{G}_T$ to $B$. First, $B$ is given the challenge access structure $W^*$ from $A$. Let $W^* = [W_1^*, \ldots, W_n^*]$. $B$ selects $u \in R \mathbb{Z}_p$, and sets $h = g^u$ and $Y = e(g^a, (g^b)^u) = e(g, h)^{ab}$. Moreover, $B$ selects $t_{i,j} \in R \mathbb{Z}_p \ (i \in [1, n], j \in [1, n_i])$, and sets $t_{i,j} = t'_{i,j}$ (in the case where $v_{i,j} = W_i^*$) and $t_{i,j} = bt'_{i,j}$ (in the case where $v_{i,j} \neq W_i^*$), and computes public keys $T_{i,j}$ ($i \in [1, n], j \in [1, n_i]$) as follows:

$$T_{i,j} = g^{t_{i,j}} = \begin{cases} g^{t'_{i,j}} & (v_{i,j} = W_i^*), \\
(g^{t'}_{i,j}) & (v_{i,j} \neq W_i^*) \end{cases}$$

$B$ gives $PK = (e, g, h, Y, \{T_{i,j}\}_{i \in [1, n], j \in [1, n_i]})$ to $A$. For KeyGen query $L$, there exists $v_{i,l}$ such that $v_{i,l} = L_i \land v_{i,l} \neq W_i^*$, since $L \neq W^*$. Therefore, $\sum_{v_{i,l} \in L} t_{i,l} = T_1 + bT_2$, where $T_1, T_2 \in \mathbb{Z}_p$. $B$ can compute $T_1$ and $T_2$, since both $T_1$ and $T_2$ are represented by the sum of $t_{i,j}$. $B$ chooses $\beta \in R \mathbb{Z}_p$, sets $r := \frac{\beta - u}{\beta - 1}$, and computes $SK_L = ((g^\beta)^2 g^{\beta u} (g^a)^{-\frac{\beta u}{\beta - 1}}, g^{\beta u} (g^a)^{-\frac{\beta u}{\beta - 1}})$. We show that $SK_L$ is a valid secret key as follows:

$$(g^\beta)^2 g^{\frac{\beta u}{\beta - 1}} (g^a)^{-\frac{\beta u}{\beta - 1}} = g^{uab} \cdot e^{(g^\beta)^2 (g^a)^{-\frac{\beta u}{\beta - 1}}} = g^{uab} \cdot g^{\frac{\beta u}{\beta - 1}} (g^a)^{-\frac{\beta u}{\beta - 1}} = g^{uab} (T_1 + bT_2)^{\frac{\beta u}{\beta - 1}}$$

If $T_2 = 0 \mod p$, then $B$ aborts. If $T_2 = 0 \mod p$ holds, then there exists $L$ such that $\sum_{v_{i,l} \in L} t_{i,l} = \sum_{v_{i,l} \in W^*} t_{i,l}$ holds. Therefore, this probability is at most $\frac{N^2}{p}$. See Section 4.3 for details. For the challenge ciphertext, $B$ chooses $\mu \in R \{0, 1\}$, computes $C_1 = M_\mu \cdot Z^a$, $C_2 = g^c$ and $C_3 = (g^\beta)^{\sum_{v_{i,l} \in W^*} t_{i,l}}$, and sends $(C_1^*, C_2^*, C_3^*)$ to $A$. Finally, $A$ outputs $\mu' \in \{0, 1\}$. $B$ outputs 1 if $\mu' = \mu$, or outputs 0 if $\mu' \neq \mu$. If $Z = e(g, g)^{abc}$, then $(C_1^*, C_2^*, C_3^*)$ is a valid ciphertext associated with $W^*$. Therefore, $A$ has the advantage $\epsilon$. Hence, $Pr[B \rightarrow 1] = e(g, g)^{abc} = Pr[\mu' = \mu] = e(g, g)^{abc} = \frac{1}{2} + \epsilon$. Otherwise, if $Z = e(g, g)^{z}$, $A$ has no advantage to distinguish a bit $\mu$, since all parts of the challenge ciphertext when $\mu = 0$ and when $\mu = 1$ have the same distributions. Hence, $Pr[B \rightarrow 0] = e(g, g)^{z} = Pr[\mu' \neq \mu] = e(g, g)^{z} = \frac{1}{2}$. It follows that $B$'s advantage in the DBDH game is $\frac{\epsilon}{p}$.  

The CCA-conversion scheme is CCA secure under both the DBDH assumption and a signature scheme is strongly unforgeable. Proof of theorem 2 is given in the Appendix.

**Theorem 2.** Our CCA-conversion scheme satisfies the indistinguishability of messages under the DBDH assumption and the chosen ciphertext attack.

Although a symmetric bilinear map is required in these proofs, our schemes can be proven with an asymmetric bilinear map such as the Weil or Tate pairing $e: \mathbb{G}_1 \times \mathbb{G}_2 \rightarrow \mathbb{G}_T$ over MNT curves [24], where $\mathbb{G}_1$ and $\mathbb{G}_2$ are distinct groups. Then the indistinguishability of messages can be proven under the DBDH assumption over $\mathbb{G}_2$ [3].

6 Comparison

Let $PK$, $MK$, $SK$ and $Ciphertext$ be the size of the public key, of the master key, of the secret key, and the ciphertext length excluding the access structure, respectively. Moreover, Enc. and Dec. are the computational times of encryption and decryption, respectively. We use the terms DBDH, DMBDH [27] and D-Linear [25] to refer to the Decision Bilinear Diffie-Hellman assumption, the Decision Modified Bilinear Diffie-Hellman assumption and the Decision Linear assumption, respectively. The notation $|\mathbb{G}|$
is the bit-length of the element which belongs to $\mathbb{G}$. Let the notations $k\mathbb{G}$ and $ke$ (where $k \in \mathbb{Z}_{>0}$) be the $k$-times calculation over the group $\mathbb{G}$ and pairing, respectively. Let $\mathcal{U} = \{att_1, att_2, \ldots, att_n\}$ be the set of attributes. Let $\gamma_1 (|\gamma_1| = r_1)$ be a set of attributes associated with the ciphertext, and $\gamma_2 (|\gamma_2| = r_2)$ a set of attributes associated with the secret key. Actually, $\gamma_2$ is different for each user. Let $N' := \sum_{i=1}^{n} n_i$ be the total number of possible statements of attributes. The computational time over $\mathbb{Z}_p$ is ignored as usual. Note that SW05 [27] and GPSW06 [18] do not consider the multi-valued attributes. They assign each attribute $att_i$ with a leaf node of an attribute tree. To estimate the same level, we show the result in the case of that each multi-valued attribute $v_{i,j}$ is assigned with a leaf node of an attribute tree. Our scheme is efficient in that the ciphertext length and the costs of decryption do not depend on the number of attributes. In particular, the number of pairing computations is constant. No previous schemes provide these properties. An access structure is constructed by AND-gates on multi-valued attributes defined in section 2.2, which is a subset of the access structures in [25]. To the best of our knowledge, our scheme is the first constant ciphertext length CP-ABE with AND-gates on multi-valued attributes.

Our scheme does not provide recipient anonymity when a symmetric bilinear group is applied. Concretely, for an access structure $W'$, an attacker can run the DDH test $e(C_2, \prod_{v_{i,j} \in W'} T_{i,j}) = e(C_3, g)$. Then, the attacker can de-

<table>
<thead>
<tr>
<th>Table 1. Size of each value</th>
<th>$PK$</th>
<th>$MK$</th>
<th>$SK$</th>
<th>Ciphertext</th>
</tr>
</thead>
<tbody>
<tr>
<td>SW05 [27]</td>
<td>$N'</td>
<td>\mathbb{G}_1 +</td>
<td>\mathbb{G}_T</td>
<td>$</td>
</tr>
<tr>
<td>GPSW06 [18]</td>
<td>$N'</td>
<td>\mathbb{G}_1 +</td>
<td>\mathbb{G}_T</td>
<td>$</td>
</tr>
<tr>
<td>CN07 [15]</td>
<td>$(N' + 1)</td>
<td>\mathbb{G}_1 +</td>
<td>\mathbb{G}_T</td>
<td>$</td>
</tr>
<tr>
<td>BSW07 [5]</td>
<td>$3</td>
<td>\mathbb{G}_1 +</td>
<td>\mathbb{G}_T</td>
<td>$</td>
</tr>
<tr>
<td>NYO08 [25]</td>
<td>$(2N' + 1)</td>
<td>\mathbb{G}_1 +</td>
<td>\mathbb{G}_T</td>
<td>$</td>
</tr>
<tr>
<td>W08 [32]</td>
<td>$2</td>
<td>\mathbb{G}_1 +</td>
<td>\mathbb{G}_T</td>
<td>$</td>
</tr>
<tr>
<td>Our CPA scheme</td>
<td>$(2N' + 3)</td>
<td>\mathbb{G}_1 +</td>
<td>\mathbb{G}_T</td>
<td>$</td>
</tr>
<tr>
<td>Our CCA scheme</td>
<td>$(2N' + 2m + 3)</td>
<td>\mathbb{G}_1 +</td>
<td>\mathbb{G}_T</td>
<td>$</td>
</tr>
</tbody>
</table>

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<tr>
<th>Table 2. Computational time of each algorithm</th>
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<tbody>
<tr>
<td>Enc.</td>
</tr>
<tr>
<td>SW05 [27]</td>
</tr>
<tr>
<td>GPSW06 [18]</td>
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<tr>
<td>CN07 [15]</td>
</tr>
<tr>
<td>BSW07 [5]</td>
</tr>
<tr>
<td>NYO08 [25]</td>
</tr>
<tr>
<td>W08 [32]</td>
</tr>
<tr>
<td>Our CPA scheme</td>
</tr>
<tr>
<td>Our CCA scheme</td>
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<tr>
<th>Table 3. Expressiveness of policy</th>
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<tbody>
<tr>
<td>SW05 [27]</td>
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<tr>
<td>GPSW06 [18]</td>
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<tr>
<td>CN07-1 [15]</td>
</tr>
<tr>
<td>BSW07 [5]</td>
</tr>
<tr>
<td>W08 [32]</td>
</tr>
<tr>
<td>NYO08 [25]</td>
</tr>
<tr>
<td>Our schemes</td>
</tr>
</tbody>
</table>

| Table 4. Performance Results for $n = 3$ |
|-------------------------------|----------------|
|                              | Enc. Time | Dec. Time |
| CN07-1 [15]                  | 0.028sec  | 0.031sec  |
| NYO08 [25]                   | 0.032sec  | 0.078sec  |
| Our CPA scheme               | 0.015sec  | 0.015sec  |
termine whether an encryptor used the policy $W'$ or not. When a DDH-hard bilinear group is applied, namely, the eXternal Diffie-Hellman (XDH) assumption holds, we can show that our scheme enables the property of the hidden encryptor-specified policies in the generic bilinear group model [5, 8, 29]. Let $g_2 \in G_2$ and $g_1 = \psi(g_2) \in G_1$ be generators, where $\psi$ is an efficiently computable isomorphism $G_2 \rightarrow G_1$. We say that the XDH assumption holds if the DDH problem is hard in $G_1$, namely, $\psi^{-1}$ is uncomputable. In the same way as shown in [25], we have only to show that the adversary cannot run the DDH test, even if the adversary is given $g_1$, $g_1^i$, and all $T_{i,j} = g_1^{i,j}$. Under the XDH assumption, where the adversary cannot compute $g_2^{i,j} \in G_2$ from $g_1^{i,j} \in G_1$, this condition holds.

The CN07-1 scheme [15], the NYO08 scheme [25] and ours are implemented with the same access structure \{v_{1,1}, v_{2,1}, v_{3,1}\}, by using the Pairing-Based Cryptography (PBC) Library ver. 0.4.18 [1]. The performance results are shown in Table 4. Our experiment was performed by using a PC with an Intel(R) Core(TM)2 Duo CPU P8400 2.26GHz Windows Vista Home Premium Edition Service Pack 1. The execution of our scheme takes a very small amount of time, which is quite feasible for practical implementation. When $n = 3$, our decryption algorithm is approximately twice as fast as that of the CN07-1 scheme, and approximately five times faster than that of the NYO08 scheme.

7 Conclusion

In this paper, we propose a constant ciphertext length CP-ABE with AND-gates on multi-valued attributes. Moreover, the number of pairing computations is also constant. In addition, the number of additional bits required from CPA-secure CP-ABE to CCA-secure CP-ABE is reduced by 90% with respect to that of previous scheme. To the best of our knowledge, this is the first such construction.

REFERENCES


In this Appendix, we give the proof of Theorem 2:

**Proof.** We suppose that the adversary $A$ wins the selective CCA game for CP-ABE with the advantage $\epsilon$. Then we can construct an algorithm $B$ that breaks the DBDH assumption with the advantage $\frac{\epsilon}{2^{N}}$, where $N := \prod_{i=1}^{n} n_i$ is the number of expressed access structures. The DBDH challenger selects $a, b, c, \in \mathbb{Z}_p$, $\nu \in \mathbb{Z}_p \{0, 1\}$, and $g$, where $\langle g \rangle = G_1$. If $\nu = 0$, then $Z = e(g, g)^{abc}$. Otherwise, if $\nu = 1$, then $Z = e(g, g)^{z}$. The DBDH challenger gives the DBDH instance $(g, g^a, g^b, g^c, Z) \in G_1^4 \times G_T$ to $B$. $B$ runs SigKeyGen, and obtains $(K_*, K_{\nu})$. First, $B$ is given the challenge access structure $W^* \in \mathcal{A}$. Let $W^* = [W^*_1, \ldots, W^*_n]$. $B$ selects $u \in \mathbb{Z}_p$, and sets $h = g^u$ and $Y = e(g^a, (g^u)^{\nu}) = e(g, h)^{ab}$. Moreover, $B$ selects $t'_{i,j} \in \mathbb{Z}_p \{0, 1\}$, and $u_i \in \mathbb{Z}_p \{i \in [1, 2m]\}$, and sets $t_{i,j} = t'_{i,j}$ (in the case where $v_{i,j} = W^*_i$) and $t_{i,j} = u_i$ (in the case where $v_{i,j} \neq W^*_i$), and computes public keys $U_i (i \in [1, 2m])$ and $T_{i,j} (i \in [1, n], j \in [1, n_1])$ as follows:

$$U_i = g^{v_i}$$

$$T_{i,j} = g^{v_{i,j}} = \begin{cases} g^{t'_{i,j}} & (v_{i,j} = W^*_i) \\ (g^{t'_{i,j}})^{v_{i,j} - W^*_i} & (v_{i,j} \neq W^*_i) \end{cases}$$

$B$ gives $PK = (e, g, h, Y, \{T_{i,j}\}_{i \in [1, n], j \in [1, n]}, \{U_i\}_{i \in [1, 2m]})$ to $A$. For KeyGen query $L$, there exists $v_{i,e} \in \mathbb{Z}_p$ such that $v_{i,e} = L_i$ and $v_{i,e} \neq W^*_i$, since $L \neq W^*$. Therefore, $\sum_{v_{i,j} \in L} t_{i,j} = T_1 + bT_2$, where $T_1, T_2 \in \mathbb{Z}_p$. $B$ can compute $T_1$ and $T_2$, since both $T_1$ and $T_2$ are represented by the sum of $t'_{i,j}$. $B$ chooses $\beta \in \mathbb{Z}_p$, sets $r := \frac{\beta - u_a}{T_2}$, and computes $SK_L = ((g^{\beta})^d \cdot g^{\frac{\beta}{T_2}})^{u_2} \cdot g^{\frac{\beta}{T_2}} \cdot (g^\beta)^{\frac{r}{T_2}}$, $G_i = (g^{\frac{\beta}{T_2}} g^{\beta})^{-\frac{r}{T_2}}$. We show that $SK_L$ is a valid secret key as follows:

$$\begin{align*}
(g^{\beta})^d \cdot g^{\frac{\beta}{T_2}} \cdot (g^\beta)^{\frac{r}{T_2}} &= g^{uab} \cdot \frac{\beta - u_a}{T_2} \cdot g^{\frac{\beta}{T_2}} \cdot (g^\beta)^{\frac{r}{T_2}} \\
&= g^{uab} \cdot g^{\frac{\beta}{T_2}} \cdot \frac{\beta - u_a}{T_2} \cdot g^{\frac{\beta}{T_2}} \\
&= g^{uab} \cdot (g^{T_1} \cdot g^{bT_2})^{\frac{\beta - u_a}{T_2}} \\
&= g^{uab} \cdot (g^{T_1 + bT_2})^{\frac{\beta - u_a}{T_2}} \\
&= h^{\beta} \cdot (g^{v_{i,j} \in L} t'_{i,j})^{\beta} \\
\end{align*}$$

and

$$g^{\frac{\beta}{T_2}} \cdot g^{\frac{r}{T_2}} = \frac{\beta - u_a}{T_2} \cdot g^{\frac{r}{T_2}} = g^r,$$

and

$$\begin{align*}
(g^{\frac{\beta}{T_2}} g^{\beta})^{-\frac{r}{T_2}} &= g^{u_2r} \\
(g^{\frac{\beta}{T_2}} g^{\beta})^{-\frac{r}{T_2}} &= g^{u_2r} \\
\end{align*}$$

If $T_2 = 0 \mod p$, then $B$ aborts. If $T_2 = 0 \mod p$ holds, then there exists $L$ such that $\sum_{v_{i,j} \in L} t_{i,j} = \sum_{v_{i,j} \in W^*} t_{i,j}$ holds. Therefore, this probability is at most $\frac{N}{2^T}$. See Section 4.3 for details. Note that this probability does not
depend on \( m \) since all \( u_i \) are not included in \( T_2 \). For Decryption query \( C = (W, \sigma, K_v, C_1, C_2, C_3) \), \( B \) checks \( \sigma \). If \( \sigma \) is invalid, then \( B \) aborts. If \( K_v = K_{v^*} \) (we call this a forge event), then \( B \) gives a random answer to the DBDH challenger. Otherwise, if \( K_v \neq K_{v^*} \), then \( B \) computes \( SK_L \), where \( L \models W \), using the same procedure as a KeyGen query. By using \( SK_L \), \( B \) decrypts \( C \), obtains \( M \), and returns \( M \) to \( A \). For the challenge ciphertext, \( B \) chooses \( \mu \in \{0, 1\} \), computes \( C_1^* = M_\mu \cdot Z^u, C_2^* = g^v, C_3^* = (g^c)^{(\sum_{u_{v^*}, i \in W^*} k_{u_{v^*}, i}) + (\sum_{K_{v^*}, i \in V^*} u_{v^*}, i)} \) and \( \sigma^* = \text{Sign}(K_{v^*}, (W^*, C_1^*, C_2^*, C_3^*)) \), and sends \( (\sigma^*, K_{v^*}, C_1^*, C_2^*, C_3^*) \) to \( A \). Finally, \( A \) outputs \( \mu' \in \{0, 1\} \). \( B \) outputs 1 if \( \mu' = \mu \), or outputs 0 if \( \mu' \neq \mu \). If \( Z = e(g, g)^{abc} \), then \( (C_1^*, C_2^*, C_3^*) \) is a valid ciphertext associated with \( W^* \). Therefore, \( A \) has the advantage \( \epsilon \). Hence, \( \Pr[B \rightarrow 1|Z = e(g, g)^{abc}] \geq \frac{1}{2} + \epsilon - \Pr[\text{forge}|Z = e(g, g)^{abc}] \). Otherwise, if \( Z = e(g, g)^{abc} \), \( A \) has no advantage in distinguishing a bit \( \mu \), since all parts of the challenge ciphertext, when \( \mu = 0 \) and when \( \mu = 1 \) have the same distributions. Hence, \( \Pr[B \rightarrow 0|Z = e(g, g)^{abc}] \geq \frac{1}{2} - \Pr[\text{forge}|Z = e(g, g)^{abc}] \). It follows that \( B \)'s advantage in the DBDH game is \((\frac{1}{2} - \Pr[\text{forge}])(1 - \frac{N^2}{p}) \). Next, we prove that \( \Pr[\text{forge}] \) is negligible. We construct an algorithm \( B' \) which can win the SEU game with probability of at least \( \Pr[\text{forge}] \). \( B' \) obtains \( K_{v^*} \) from the SEU challenger, instead of executing \( \text{SigKeyGen} \) to obtain \( (K_{v^*}, K_v) \). \( B' \) proceeds as \( B \) using the SEU challenger. In the challenge phase of the CCA game, \( B' \) obtains \( \sigma^* \) from the SEU challenger. Therefore, \( B' \) makes at most one signature query. If the event \( \text{forge} \) occurs, namely \( A \) sends a decryption query \( (W, \sigma, K_v, C_1, C_2, C_3) \), where \( K_v = K_{v^*} \), then \( B' \) submits a forge signature \( \sigma \) to the SEU challenger and wins. Therefore, \( \Pr[\text{forge}] \) is negligible, since we assume that the signature scheme is SEU. \( \square \)