Linguistic Multi-Expert Decision Making Involving Semantic Overlapping

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Abstract This paper presents a probabilistic model for linguistic multi-expert decision making (MEDM), which is able to deal with vague concepts in linguistic aggregation and decision-makers' preference information in choice function. In linguistic aggregation phase, the vagueness of each linguistic judgement is captured by a possibility distribution on a set of linguistic labels. A confidence parameter is also incorporated into the basic model to model experts' confidence degree. The basic idea of this linguistic aggregation is to transform a possibility distribution into its associated probability distribution. The proposed linguistic aggregation results in a set of labels having a probability distribution. As a choice function, a target-oriented ranking method is proposed, which implies that the decision-maker is satisfactory to choose an alternative as the best if its performance is as at least "good" as his requirements.

1 Introduction

Multi-expert decision making (MEDM) is a common and important human activity. In practice, the uncertainty, constraints, and even the vague knowledge of the experts imply that the information cannot be assessed precisely in quantitative form, but may be in a qualitative one [7]. A possible way to solve such situation is the use of the fuzzy linguistic approach [19]. Also, the process of activities or decisions usually creates the need for computing with words. One linguistic computational approach is making use of the associated membership function for each label based on the extension principle [4]. Another approach is the symbolic one [5] by means of the convex combination of linguistic labels. In these two approaches, however, the results usually do not match any of the initial linguistic labels, hence an approximation process must be developed to express the result in the initial expression

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domain. This produces the consequent loss of information and lack of precision. To overcome this limitation, a 2-tuple fuzzy linguistic representation model is proposed in [7]. Although such an approach has no loss of information, it does not directly take into account the underlying vagueness of the linguistic labels, i.e., it assumes that any neighboring linguistic labels have no semantic overlapping.

Two approaches have been proposed in an attempt to involve the underlying vagueness of the words in linguistic MEDM problems. Ben-Arieh & Chen [1] have proposed a fuzzy linguistic OWA (FLOWA) operator, which assigns fuzzy membership functions to all linguistic labels by linearly spreading the weights from the labels to be aggregated. The aggregating result changes from a single label to a fuzzy set with membership levels of each label. Tang [17] has introduced a collective linguistic MEDM model to capture the underlying vagueness of linguistic labels based on the semantic similarity relation [18], in which the similarities among linguistic labels are derived from fuzzy relation of linguistic labels. However, such an approach violates the bounded property of the linguistic aggregation. For more detail of the properties of linguistic aggregation, see [5]. Moreover, it assumes that the same label assessed by different experts has the same label overlapping.

According to the *epistemic stance* interpretation in linguistic modeling by Lawry [12], when an expert assesses some alternatives (options) with a linguistic label, it is assumed that he will probably choose other linguistic labels to describe the option. Possibility theory [6] provides a convenient tool to represent experts' uncertain assessments. Furthermore, even if two different experts have assessed an option with the same linguistic label, the appropriateness degree of other linguistic labels may be different according to experts' confidence degree, i.e., to what extent the experts are sure that other linguistic labels are appropriate to describe the option. Finally, our another motivation comes from the fact that experts are not necessarily the decision-makers, but only provide an advice [15]. The decision-makers' preference information plays an important role in choice of alternatives, which is missed in most research.

In light of the above observations, we summarize our main contributions as follows. First, we assume that the appropriate labels are linearly distributed around the linguistic label provided by the expert with a possibility distribution. The label provided by the expert will be called *prototype label*. And then based on the basic mass function, we can obtain the probability distribution on the linguistic labels as the aggregation result. Fuzzy modifiers [19] are also used to model some expert' confidence quantifying how he is sure of the appropriateness of other linguistic labels. Second, we propose a target-oriented ranking method incorporating decision-makers' preferences. It is well-known that human behavior should be modeled as satisficing instead of optimizing [16]. Intuitively, the satisficing approach has some appealing features because thinking of targets is quite natural in many situations.

The rest of this paper is organized as follows. Section 2 proposes a probabilistic approach to linguistic aggregation involving vague concepts. Section 3 proposes a ranking procedure based on target-oriented decision model, in which decision-makers' preferences are considered. Section 4 provides an illustrative example. Section 5 discusses the relationships between our approach and three prior approaches. Finally, Section 6 presents some concluding remarks.

2 A Probabilistic Approach to Linguistic Aggregation Involving Semantic Overlapping

In fuzzy environment, a common characteristic of the MEDM problems, is a finite set of experts, denoted by $\mathscr{E} = \{E_1, \cdots, E_k, \cdots, E_K\}$, who are asked to assess another finite set of alternatives $\mathscr{A} = \{A^1, \cdots, A^m, \cdots, A^M\}$. The linguistic assessment provided by expert E_k regarding alternative A^m is presented as $x_k^m \in \mathscr{L}$, where $\mathscr{L}$ is a finite, but totally ordered label set of linguistic variables with an odd cardinality, i.e., $\mathscr{L} = \{L_0, \cdots, L_n, \cdots, L_N\}$ with $L_n > L_l$ for $n > l$. Also, each expert is assigned a degree of importance or weight w_k, denoted as $W = [w_1, \cdots, w_k, \cdots, w_K]$.

2.1 Linguistic Aggregation Involving Vague Concepts

With the linguistic judgements for alternative A^m provided by a set of experts $\mathscr{E}$, we can obtain a linguistic judgement vector as $X^m = (x_1^m, \cdots, x_k^m, \cdots, x_K^m)$, where $x_k^m \in \mathscr{L}, k = 1, \cdots, K$. When there is no possibility of confusion, we shall drop the subscript m to simplify the notations. Our main objective is to aggregate the linguistic judgement vector X for each alternative A.

The linguistic judgement provided by one expert implies that the expert makes an assertion. It seems undeniable that humans posses some kind of mechanism for deciding whether or not to make certain assertions. Furthermore, although the underlying concepts are often vague the decision about the assertions are, at a certain level, bivalent. That is to say for an alternative A and a linguistic label L, you are willing to assert 'A is L' or not. Nonetheless, there seems to be an underlying assumption that some things can be correctly asserted while others cannot. Exactly where the dividing line between those labels are and those that are not appropriate to use may be uncertain. This is the main idea of *epistemic stance* proposed by Lawry [12].

Motivated by the *epistemic stance*, we assume that any neighboring basic linguistic labels have partial semantic overlapping in linguistic MEDM. Thus, when one expert E_k evaluates alternative A using linguistic label $x_k \in \mathscr{L}$, other linguistic labels besides x_k in $\mathscr{L}$ may also be appropriate for describing A, but which of these linguistic labels is uncertain. Here, similar with [13], the linguistic label x_k will be called *prototype label*. If experts can directly assign the appropriateness degrees of all linguistic labels, then we can obtain a possibility distribution. However, the need of experts' involvement creates the burden of decision process. Without additional information, we assume that the appropriate labels are distributed around the prototype label x_k with a linear possibility distribution. Possibility theory is convenient to represent consonant imprecise knowledge [6]. The basic notion is the possibility distribution, denoted π.

It is very rare that when all individuals in a group share the same opinion about the alternatives (options), since a diversity of opinions commonly exists [1]. With the linguistic judgement vector X for alternative A, we can define

$$L_{\min} = \min_{k=1,\cdots,K}\{x_k\}, \qquad L_{\max} = \max_{k=1,\cdots,K}\{x_k\} \tag{1}$$

where $x_k \in \mathscr{L}$, $L_{\min} < L_{\max}$, and $L_{\min}$, $L_{\max}$ are the smallest and largest linguistic labels in X, respectively. The label indices of the smallest and largest labels in judgement vector X are expressed as $\mathtt{ind}_{\min}$ and $\mathtt{ind}_{\max}$, respectively. Also, the label index of the prototype label x_k provided by expert E_k is denoted as $\mathtt{pInd}_k$.

Note that, the result of linguistic aggregation should lie between $L_{\min}$ and $L_{\max}$ (including $L_{\min}$ and $L_{\max}$). In addition, if two label indices have the same distance to the index of the prototype label x_k, we assume that they have the same appropriateness (possibility) degree. Furthermore, as Lawry [12] pointed out, "an assertability judgement between a 'speaker' and a 'hearer' concerns an assessment on the part of the speaker as to whether a particular utterance could (or is like to) mislead the hearer regarding a proposition about which it is intended to inform him." Thus if one expert is viewed as a 'speaker', then other experts will act as 'hearer'. Accordingly, we first define a parameter as

$$\Delta_k = \max\{\mathtt{pInd}_k - \mathtt{ind}_{\min}, \mathtt{ind}_{\max} - \mathtt{pInd}_k\}. \tag{2}$$

We then define a possibility distribution of around the prototype label $x_k \in \mathscr{L}$ on linguistic labels L_n as follows

$$\pi(L_n|x_k) = \begin{cases} 1 - \frac{\mathtt{pInd}_k - n}{\Delta_k + 1}, & \text{if } \mathtt{ind}_{\min} \le n < \mathtt{pInd}_k; \\ 1, & \text{if } n = \mathtt{pInd}_k; \\ 1 - \frac{n - \mathtt{pInd}_k}{\Delta_k + 1}, & \text{if } \mathtt{pInd}_k < n \le \mathtt{ind}_{\max}; \\ 0, & \text{if } n < \mathtt{ind}_{\min} \text{ or } n > \mathtt{ind}_{\max}. \end{cases} \tag{3}$$

where $n = 0, \cdots, N$. Assume that there is a set of seven linguistic labels $\mathscr{L} = \{L_0, \cdots, L_6\}$. Also, we have $L_{\min} = L_1$ and $L_{\max} = L_5$. Then for a possible prototype label x, according to Eq. (3), we obtain the possibility distribution of appropriate labels as shown in Fig. 1.

Note $\pi(L_n|x_k)$ is a possibility distribution of around prototype label x_k on the linguistic label set $\mathscr{L}$, then the possibility degrees are reordered as

$$\{\pi_1(x_k), \cdots, \pi_i(x_k), \cdots, \pi_m(x_k)\}$$

such that $1 = \pi_1(x_k) > \pi_2(x_k) > \cdots > \pi_m(x_k) \ge 0$. Then similar with [10, 11], we can derive a consonant mass assignment function $\mathbf{m}_{x_k}$ for the possibility distribution function $\pi(L_n|x_k)$, such that

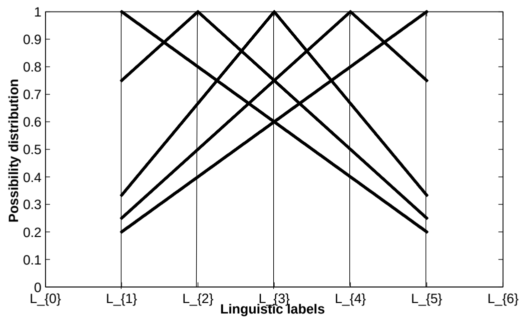

Fig. 1 Possible prototype label and its appropriate labels under $[L_1, L_5]$

$$\mathbf{m}_{x_k}(\phi) = 1 - \pi_1(x_k), \mathbf{m}_{x_k}(F_i) = \pi_i(x_k) - \pi_{i+1}(x_k), i = 1, \cdots, m-1, \mathbf{m}_{x_k}(F_m) = \pi_m(x_k) \tag{4}$$

where $F_i = \{\pi(L_n|x_k) \geq \pi_i(x_k)\}, i = 1, \cdots, m$ and $\{F_i\}_{i=1}^m$ are referred to as the focal elements of $\mathbf{m}_{x_k}$.

The notion of mass assignment suggests a means of defining probability distribution for any prototype label. Then we can obtain the least prejudiced distribution [10] of around the prototype label x_k on the linguistic label set $\mathscr{L}$ as follows:

$$p(L_n|x_k) = \sum_{F_i : L_n \in F_i} \frac{\mathbf{m}_{x_k}(F_i)}{|F_i|} \tag{5}$$

where $L_n \in \mathscr{L}$, $\mathbf{m}_{x_k}$ is the mass assignment of $\pi(x_k)$ and $\{F_i\}_i$ is the corresponding set of focal elements.

With the weighting vector $W = [w_1, \cdots, w_k, \cdots, w_K]$, we can obtain the collective probability distribution on the linguistic label set $\mathscr{L}$ as follows:

$$p_n = p(L_n) = \sum_{k=1}^{K} p(L_n|x_k) \cdot w_k \tag{6}$$

where $n = 0, \cdots, N$. We then obtain a $N+1$-tuple probability distribution on the linguistic label set $\mathscr{L}$ as follows $(p_0, \cdots, p_n, \cdots, p_N)$ for each alternative A. The probability distributions of all alternatives on the label set $\mathscr{L}$ are shown in Table 1.

2.2 Involving Expert's Attitudinal Character in Vague Concepts

Now we introduce a parameter α to model the confidence/certain degree of an expert. It quantifies to what extent the expert is sure that other linguistic labels around the prototype label are appropriate to describe an alterative. With the confidence character α, we define the possibility distribution of around prototype label $x_k \in \mathscr{L}$ on linguistic label L_n as follows:

Table 1 Probability distribution on the $N+1$ labels regarding each alternative

Alter.	Linguistic labels				
	L_0	$\cdots$	L_n	$\cdots$	L_N
A^1	p_0^1	$\cdots$	p_n^1	$\cdots$	p_N^1
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
A^m	p_0^m	$\cdots$	p_n^m	$\cdots$	p_N^m
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
A^M	p_0^M	$\cdots$	p_n^M	$\cdots$	p_N^M

$$\pi(L_n|x_k,\alpha) = \begin{cases} \left[1 - \frac{\texttt{pInd}_k - n}{\Delta_k + 1}\right]^{\alpha}, & \text{if } \texttt{ind}_{\min} \leq n < \texttt{pInd}_k; \\ 1, & \text{if } n = \texttt{pInd}_k; \\ \left[1 - \frac{n - \texttt{pInd}_k}{\Delta_k + 1}\right]^{\alpha}, & \text{if } \texttt{pInd}_k < n \leq \texttt{ind}_{\max}; \\ 0, & \text{if } n < \texttt{ind}_{\min} \text{ or } n > \texttt{ind}_{\max}. \end{cases} \tag{7}$$

where α is a linguistic modifier and $\alpha > 0$. When $\alpha > 1$ it means that the expert has an optimistic attitude (he is more sure that the prototype label is appropriate to describe an alternative); when $\alpha = 1$ it means that the expert has a neutral attitude (it is equivalent to the basic model); when $\alpha < 1$ it means that the expert has a pessimistic attitude (he is less sure that the prototype label is appropriate to describe an alternative). Without possibility of confusion, the confidence factor will be also called attitude character.

Note that each expert can assign different confidence values according to his preferences or belief. In order to better represent expert's attitude factor, we introduce another parameter β, where $\alpha = 2^{\beta}$. Although α and β have continuous forms, for purposes of simplicity, we assign β integer values distributed around 0. For example, $\beta = \{-\infty, \cdots, -3, -2, -1, 0, 1, 2, 3, \cdots, +\infty\}$, consequently we get $\alpha = \{2^{-\infty} \cdots, 1/8, 1/4, 1/2, 1, 2, 4, 8, \cdots, 2^{+\infty}\}$. In order to help experts conveniently express their confidence degree, we construct a totally ordered linguistic label set with an odd cardinality. We can define the following set of linguistic labels to represent experts' confidence degrees.

$$\begin{aligned} \mathscr{V} = \{&V_0 = \text{absolutely unsure}, V_1 = \text{very unsure}, V_2 = \text{unsure}, V_3 = \text{neutral}, \\ &V_4 = \text{sure}, V_5 = \text{very sure}, V_6 = \text{absolutely sure}\} \\ \alpha =& \{2^{-M}, 1/4, 1/2, 1, 2, 4, 2^{M}\}, \beta = \{-M, -2, -1, 0, 1, 2, M\} \end{aligned} \tag{8}$$

where M is big enough positive integer to make sure that $[\pi(L_n|x_k)]^{2^M} \to 0$ if $\texttt{ind}_{\min} \leq n < \texttt{pInd}_k$ or $\texttt{pInd}_k < n \leq \texttt{ind}_{\max}$.

And then according to the procedure mentioned in the basic model, Eqs. (4)-(6), we can infer a collective probability distribution for each alternative.

3 Ranking Based on Target-Oriented Decision Model

After linguistic aggregation, the next step of linguistic MEDM is to exploit the best option(s) using a choice function. Most MEDM process is basically aimed at reaching a “consensus”, e.g. [3, 8]. Consensus is traditionally meant as a strict and unanimous agreement of all the experts regarding all possible alternatives. The decision model presented below assumes that experts do not have to agree in order to reach a consensus. There are several explanations that allow for experts not to converge to a uniform opinion. It is well accepted that experts are not necessarily the decision-makers, but provide an advice [15]. Due to this observation, the linguistic judgements provided by the experts does not represent the decision-makers’ preferences.

The inferred probability distribution on a set of linguistic labels for each alternative, as shown in Table 1, could be viewed as a general framework of decision making under uncertainty [14], in which there are $N+1$ states of nature, whereas the probability distributions are different. Now let us consider the ranking procedure for the probability distribution on $N+1$ linguistic labels in $\mathscr{L}$, as shown in Table 1. We assume that the decision-maker has a target in his mind, denoted as T. We also assume that the target is independent on the set of M alternatives and the linguistic judgements provided by the experts. Based on target-oriented decision model [2], we define the following function

$$V(A^m) = \Pr(A^m \succeq T) = \sum_{L \in \mathscr{L}} p^m(A^m = L) \cdot \Pr(L \succeq T) = \sum_{n=0}^{N} p_n^m \cdot \Pr(L_n \succeq T) \quad (9)$$

We assume there exists a probability distribution on the uncertain target regarding each linguistic label L_n, denoted as $p_T(L_n)$, where $n = 0, \cdots, N$. Then we define the following function

$$\Pr(A^m \succeq T) = \sum_{n=0}^{N} p_n^m \cdot \left[\sum_{l=0}^{N} u(L_n, L_l) p_T(L_l) \right] \quad (10)$$

Recall that the target-oriented model has only two achievement levels, thus we can define $u(L_n, L_l) = 1$, if $L_n \geq L_l$; 0, otherwise. Then we can induce the following value function

$$\Pr(A^m \succeq T) = \sum_{n=0}^{N} p_n^m \cdot \left[\sum_{l=0}^{n} p_T(L_l) \right] \quad (11)$$

Now let us consider two special cases. Without additional information (if the decision-maker does not assign any target), we can assume that the decision-maker has a uniform probability distribution on the uncertain target T, such that

$$p_T(L_n) = \frac{1}{N+1}, n = 0, \cdots, N. \quad (12)$$

Then we can obtain the value of meeting the uniformly linguistic target as follows:

$$\Pr(A^m \succeq T) = \sum_{n=0}^{N} p_n^m \cdot \left[\sum_{l=0}^{n} p_{L_l}(T) \right] = \sum_{n=0}^{N} p_n^m \cdot \frac{n+1}{N+1} \tag{13}$$

If the decision-maker assigns a specific linguistic label L_l as his target, the probability distribution on uncertain target is expressed as

$$p_T(L_n) = \begin{cases} 1, & \text{if } L_n = L_l; \\ 0, & \text{if } L_n \neq L_l. \end{cases}$$

where $n = 0, \cdots, N$. Then the probability of meeting target is as follows:

$$\Pr(A^m \succeq L_l) = \sum_{n=0}^{N} p_n^m \cdot \Pr(L_n \succeq L_l) = \sum_{n=l}^{N} p_n^m \tag{14}$$

Having obtained the utility (probability of meeting target), the choice function for linguistic MEDM model is defined by

$$A^* = \arg \max_{A^m \in \mathscr{A}} \{V(A^m)\} \tag{15}$$

4 Illustrative Example

In this section, we demonstrate the entire process of the probabilistic model via an example borrowed from [7].

A distribution company needs to renew/upgrade its computing system, so it contracts a consulting company to carry out a survey of the different possibilities existing on the market, to decide which is the best option for its needs. The options (alternatives) are $\{A^1 : \texttt{UNIX}, A^2 : \texttt{WINDOWS-NT}, A^3 : \texttt{AS/400}, A^4 : \texttt{VMS}\}$. The consulting company has a group of four consultancy departments as $\{E_1 : \texttt{Cost anal.}, E_2 : \texttt{Syst. anal.}, E_3 : \texttt{Risk anal.}, E_4 : \texttt{Tech. anal.}\}$.

Each department in the consulting company provides an evaluation vector expressing its opinions for each alternative. These evaluations are assessed in the set $\mathscr{L}$ of seven linguistic labels as $\mathscr{L} = \{L_0 = \text{none}, L_1 = \text{very low}, L_2 = \text{low}, L_3 = \text{medium}, L_4 = \text{high}, L_5 = \text{very high}, L_6 = \text{perfect}\}$. The evaluation matrix and weighting vector are shown in Table 2.

Table 2 Linguistic MEDM problem in upgrading computing resources

Alter.	Experts			
	$E_1 : 0.25$	$E_2 : 0.25$	$E_3 : 0.25$	$E_4 : 0.25$
A^1	L_1	L_3	L_4	L_4
A^2	L_3	L_2	L_1	L_4
A^3	L_3	L_1	L_3	L_2
A^4	L_2	L_4	L_3	L_2

Now let us apply our proposed model to solve the above problem. The first step is to aggregate linguistic assessments involving vague concepts. With the linguistic evaluation matrix (Table 2), we obtain the minimum and maximum linguistic labels for each alternative according to Eq. (1) as follows:

A^1	A^2	A^3	A^4
$[L_1, L_4]$	$[L_1, L_4]$	$[L_1, L_3]$	$[L_2, L_4]$

A set of seven linguistic labels, as shown in Eq. (8), is used to represent the consultant departments's confidence degrees. Each consultant department can assign different confidence degrees according to his preference/belief. In this example, we consider two cases:

Case 1: the four departments assign *absolutely sure* as their confidence degrees.
Case 2: the four departments assign *neutral* as their confidence degrees.

According to linguistic aggregation with vague concepts, proposed in Section 2, we obtain different probability distributions for the four alternatives with respect to different cases, as shown in Table 3.

Table 3 Probability distributions on linguistic labels with respect to different cases

Cases	Alter.	Linguistic labels						
		L_0	L_1	L_2	L_3	L_4	L_5	L_6
Case 1	A^1	0.0	0.25	0.0	0.25	0.5	0.0	0.0
	A^2	0.0	0.25	0.25	0.25	0.25	0.0	0.0
	A^3	0.0	0.25	0.25	0.5	0.0	0.0	0.0
	A^4	0.0	0.0	0.5	0.25	0.25	0.0	0.0
Case 2	A^1	0.0	0.1823	0.1892	0.3038	0.3247	0.0	0.0
	A^2	0.0	0.2153	0.2847	0.2847	0.2153	0.0	0.0
	A^3	0.0	0.25	0.375	0.375	0.0	0.0	0.0
	A^4	0.0	0.0	0.375	0.375	0.25	0.0	0.0

From Table 3, it is easily seen that when the four departments assign a *absolutely sure* attitude, it means that they are absolutely sure that a label L is appropriate for describing an alternative. In this case, the group probability distribution will depend only on the weight information. For instance, for alternative A^2 under case 1, the four departments provide their judgements as $\{L_3, L_2, L_1, L_4\}$ and they have equal weight information, thus the probability distribution on the 7 labels is $(0, 0.25, 0.25, 0.25, 0.25, 0, 0)$.

Now let us rank the four alternatives according to the target-oriented ranking procedure proposed in Section 3. In this example, the four consultant departments provide their advice, but do not make decisions. The true decision-maker is the distribution company. To renew a computer system, the distribution company may simply looks for the first "satisfactory" option that meets some target. Having this in mind, we first assume that the distribution company does not assign his target, i.e., the distribution company has a uniform target T_1, which can be represented as

$(L_0 : 1/7, L_1 : 1/7, L_2 : 1/7, L_3 : 1/7, L_4 : 1/7, L_5 : 1/7, L_6 : 1/7)$. If the distribution company can provide a specific label as his target, for example, the company assigns his target as $T_2 = L_4 = high$, it means that the distribution company is satisfactory to choose an alternative as the best if its performance is at least "good" as *high*. Table 4 shows the probability of meeting those two targets assigned by the distribution company with respect to four cases of confidence degrees provided by the four consultant departments. From Table 4, option A^4 (VMS) or A^1 (UNIX) is the best choice according to the confidence degrees provided by the four departments and the targets provided by the distribution company.

Table 4 Probability of meeting targets

Cases	Targets	Alternatives			
		A^1	A^2	A^3	A^4
Case 1	T_1	**0.5714**	0.5	0.4643	0.5357
	T_2	**0.5**	0.25	0.0	0.25
Case 2	T_1	0.5387	0.5	0.4464	**0.5536**
	T_2	**0.3247**	0.2153	0.0	0.25

5 Discussions

In this section, we shall discuss the relationships between our research and three prior related approaches.

Huynh & Nakamori [9] have proposed a satisfactory-oriented approach to linguistic MEDM. In their framework, the linguistic MEDM is viewed as a decision making under uncertainty problem, where the set of experts plays the role of states of the world and the weights of experts play the role of subjective probabilities assigned to the experts. They then proposed a probabilistic choice function based on the philosophy of satisfactory-oriented principle, i.e., *it is perfectly satisfactory to select an alternative as the best if its performance is as least "good" as all the others*. In the aggregation step, such an approach does not directly take into account the underlying vagueness of the labels. The proposed linguistic aggregation somewhat generalizes the work provided in [9]. In particular, when all the experts have *absolutely sure* confidence degree, our linguistic aggregation is equivalent to that given in [9]. For example, under Case 1 of Table 3, the linguistic aggregation results with a probability distribution on the set of linguistic labels, which is dependent on the weights of experts. In the choice function step, although both our approach and that given in [9] are based on the satisfactory-oriented philosophy, we incorporate decision maker's target preference into the linguistic MEDM problems.

Ben-Arieh & Chen [1] have proposed a so-called FLOWA aggregation operation, which assigns fuzzy membership functions to all linguistic labels by linearly spreading the weights from the labels to be aggregated. The aggregating result changes from a single label to a fuzzy set with membership levels of each label. And then the fuzzy mean and standard deviation are used as two criteria to rank the aggregation results. Compared with [1], in the aggregation step, our approach provides a probabilistic formulation for the linguistic aggregation involving underlying vagueness of linguistic labels. In addition, our approach can model experts' confidence degree to quantify the appropriateness of linguistic labels. In the choice function step, our approach considers decision-maker's requirements.

Tang [17] has proposed a collective decision model based on the semantic similarities of linguistic labels [18] to deal with vague concepts and compound linguistic expressions[1]. In this approach, a similarity relation matrix $< R, \mathscr{L} >$ for a set of basic linguistic labels is defined beforehand. And then by viewing similarity distribution as possibility distribution, the collective probability distribution on the linguistic label set $\mathscr{L}$ is obtained by Eqs. (4)-(6). Finally, two methods are suggested to rank the alternatives: an expected value function and a probabilistic pairwise comparison method. The expected value function is similar to the ranking function in [1] and the pairwise comparison method is quite similar with the satisfactory-oriented principle proposed in [9]. Compared with our approach, the linguistic aggregation by [17] violates the bounded property of aggregation operation. In addition, the approach in [17] does not consider experts' confidence degrees. In the choice function step, it does not take into account decision-makers' requirements.

6 Conclusions

In this paper, we have proposed a probabilistic model for MEDM problem under linguistic assessments, which is able to deal with linguistic labels having partial semantic overlapping as well as incorporate experts's confidence degrees and decision-makers' preference information. It is well known that linguistic MEDM problems follow a common schema composed of two phases: an *aggregation phase* that combines the individual evaluations to a collective evaluations; and an *exploitation phase* that orders the collective evaluations according to a given criterion, to select the best options. For our model, our linguistic aggregation does not generate a specific linguistic label for each alternative, but a set of labels with a probability distribution, which incorporates experts' vague judgements. Moreover, experts' confidence degree is also incorporated to quantify the appropriateness of linguistic labels other than the prototype label. Having obtained the probability distributions on linguistic labels, we have proposed a target-oriented choice function to establish a ranking ordering among the alternatives. According to this choice function, the decision-maker is satisfactory to select an alternative as the best if its performance is as at least "good" as his requirements.

[1] The compound linguistic expressions is beyond the scope of our research, thus we only consider the vague concepts in linguistic MEDM problems.

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