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Semisimplicity, Amalgamation Property and Finite Embeddability Property of Residuated Lattices

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Abstract

In this thesis, we investigate semisimplicity, amalgamation property and finite embeddability property of residuated lattices. We prove semisimplicity and amalgamation property of residuated lattices which are of purely algebraic character, by using proof-theoretic methods and results of substructural logics. On the other hand, we show the finite model property for various substructural logics, including fuzzy logics as a consequence of finite embeddability property of corresponding classes of residuated lattices. Thus all of our studies are attempts at bridging gaps between algebras and logics.

The first topics of our thesis is semisimplicity of free $\mathbf{FL}_w$-algebras. An algebra is said to be semisimple if it has a subdirect representation with simple factors. Grišin proved that every free $\mathbf{CFL}_{ew}$-algebra is semisimple. To show this Grišin introduced a new sequent system which is equivalent to $\mathbf{CFL}_{ew}$ and showed that the cut elimination theorem holds for the sequent system. Later, Kowalski and Ono proved that every free $\mathbf{FL}_{ew}$-algebras is also semisimple using Grišin's idea. By using this, they proved that the variety of $\mathbf{FL}_{ew}$-algebras is generated by it finite simple members. By using the similar technique, we show that every free $\mathbf{FL}_w$-algebras is semisimple. We will introduce a new sequent system $\mathbf{FL}_w^+$ which is equivalent to $\mathbf{FL}_w$ and for which cut elimination theorem holds. Using proof-theoretic properties of $\mathbf{FL}_w^+$, we show the semisimplicity of free $\mathbf{FL}_w$-algebras.

Next, we discuss amalgamation property of commutative residuated lattices. Kowalski showed the amalgamation property (AP) for the variety $\mathcal{FL}_{ew}$ of all $\mathbf{FL}_e$-algebras. The result is obtained by the fact that (1) the logical system $\mathbf{FL}_{ew}$ has the Craig's interpolation property (CIP), and (2) the variety of $\mathcal{FL}_{ew}$ has the equational interpolation property (EIP). Wroński proved that the EIP of a variety implies the AP. Therefore the variety $\mathcal{FL}_{ew}$ has the AP. We show that Kowalski's proof of the AP works well also for the variety $\mathcal{CRL}$ of all commutative residuated lattices (CRL). To show this result, we introduce a sequent for CRL and show the CIP, and using them we prove that the variety $\mathcal{CRL}$ has the EIP. By considering filters on residuated lattices, we can show that many important subclasses of $\mathcal{CRL}$ has the AP. Moreover, we can show that if $\mathbf{L}$ is a logic which is an extension of $\mathbf{FL}_e$ with the CIP and $\mathcal{K}$ is the variety whixh is corresponding to $\mathbf{L}$ then $\mathcal{K}$ has the EIP, from which the AP of $\mathcal{K}$ follows.

Lastly, we consider finite embeddability property (FEP) of some classes of integral residuated lattices. A class of algebras has the FEP if every finite partial subalgebra of a member of the class can be embedded into a finite member of the class. Blok and van Alten showed that the class of all partially ordered biresiduated integral groupoids has the FEP. This implies that the variety of all integral residuated lattices ($\mathcal{IRL}$) has the FEP. The FEP of a given variety of $\mathcal{IRL}$ implies finite model property (FMP) for the corresponding logic. We prove the FEP for some classes of the variety $\mathcal{IRL}$. From this the FMP follows for various substructural logics including fuzzy logics.

Key Words: Residuated lattices, Semisimplicity, Amalgamation property Finite embeddability property