Optimal Control of Stochastic Hybrid system with Probabilistic Constraints

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A hybrid system is a system in which continuous dynamics are changed by discrete dynamics. Hybrid systems are widely used as mathematical models for a lot of real systems such as automated highway systems, air-traffic management system, manufacturing systems, chemical processes, robotics, real-time communication networks. So theory of hybrid systems is effective for analysis and design of distributed systems and embedded systems, and there have been a lot of studies contributed to analysis and control of hybrid systems from both control theory community and theoretical computer science community. On the other hand, recently, there have been several works on cooperative control such as obstacle avoidance of multiple vehicles and formation control of multiple airplanes. In cooperative control, it is necessary to observe an each other 's position, and follow the leader. Thus, in several situations, the behavior of systems becomes complex. Since it is necessary to model On/Off in communication networks and dynamics in airplanes, it is suitable to model dynamical systems in cooperative control as hybrid systems.

Stochastic hybrid systems are well-known as an extension of hybrid systems. Stochastic hybrid systems are in general composed of stochastic differential equations and stochastic finite automata. In this paper, we consider a special class of stochastic hybrid systems, that is, only discrete dynamics are stochastic. For example, normal/fault in mechanical systems and On/Off in communication networks can be modeled by stochastic discrete dynamics. In mechanical systems, it is appropriate to probabilistically express the transition between continuous dynamics in the normal mode and those in the fault mode. Furthermore, ON/OFF in communication networks is also probabilistic. So even if only discrete dynamics are stochastic, then there are several applications.

In the existing works on control of stochastic hybrid systems, the optimal control method has been proposed based on a mixed logical dynamical system (MLDS) representation, which is well-known as one of the typical models in hybrid systems. In the verification problem, probabilistic reachability has been discussed. Others, the external verification of the stochastic hybrid systems has been proposed, and automatic verification tools has been implemented, and it enabled to be verified efficiently by using abstraction. Moreover, in the communication network, the network has been modeled as the hybrid system by which transitions between discrete modes have probabilistic mode transitions caused such as transitions between the states of a continuous time Markov chain in this work for the purpose of realizing evasion of network confusion.

However, the optimal control problem with probabilistic constraints has been not considered so far.

In this paper, for discrete-time hybrid systems with stochastic discrete dynamics, we calculate control input sequences satisfying 1)reaches the unsafe region with the probability lower than ε. 2) is minimized a given cost function in discrete states (control locations)with realization probability higher than ρ.

The proposed solving method consists of off-line computation and on-line computation. In the off-line computation, an unsafe reachability graph is computed. The unsafe reachability graph expresses state regions, which reach a given unsafe state region. Linear inequalities with respect to the state and the control input are assigned to each edge in this graph. These inequalities give the condition to transit from some state region to other regions at one time step. Furthermore, based on the unsafe reachability graph, mode sequences that the state reaches the unsafe region with the probability lower than ε, are enumerated.

In the on-line computation, the MLDS representation is used. In discrete states with realization probability higher than ρ, for minimizing a given cost function, in the modeling using the MLDS representation, we must populate relation of a binary variable and the probability as restraint conditions.However, this restraint conditions is not transformed into MIQP problem for nonlinear form. So, we transform restraint conditions into the linear function by using logarithmic conversion. By logarithmic conversion of restraint conditions, the optimal control problem is reduced to an MIQP problem. Using the obtained MLDS representation, the finite-time optimal control problem with linear inequality constraints corresponding to enumerated mode sequences is solved. Then this problem is rewritten as an MIQP problem, and can be solved by a suitable solver. By solving this problem, we can derive the control input, which is minimized a given cost function, and satisfies the probabilistic constraints. Since the MLDS representation is the same form as constrained linear systems, we can use the model predictive control method, which is one of the efficient method in constrained systems.

In this paper, as a simple example, we consider a pump system. A pump system consists of a vessel containing an amount of water, a series of sensor, a water pump, a control system. For turning on the steam by heat, by sensing the water level by a sensor at intervals of Δ second to keep the water level constant, we control the quantity of the water which flows from a water pump. If the water level is normal, we do not need to control the quantity of the water with a water pump. However, if the water level is lower than a fixed value, we increase the water level by putting the water with a water pump. If the water level is higher than a fixed value, we decrease the water level by drying off the water in a vessel. Then, in switching a water pump to on or off, The error in which it becomes impossible to sense a water level may occur. If the error of a system continues, and the water level is higher than upper limit, or lower than lower limit, the system is stopped. In this paper, if a water level is higher than upper limit, we set an unsafe state for the state. And, we make an unsafe reachability graph, and show a numerical example in the optimal control problem.