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Exclusive covering of point set by unit disks

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Recentry, services using the electric wave communication are increasing. For example, wireless LAN, cellular phones, televisions, telecommunications, and so on. The range of the electric wave can be represented by a circle on the ground. When we use a service using the electric wave, we have to be in the circle. The situation where two circles intersect might be problematic. For example, television images will be scrambled, wireless telecommunications will be interrupted, and so on. We consider the layout problem of the base stations which send the electric wave. In this problem, any circle should not overlap another circle. Moreover, it is necessary to cover all users who receive the service using the electric wave.

We represent the users by points. We assume that the range where electric wave reaches is a unit disk. In this paper, we consider the problem "Exclusive covering of point set by unit disks". This problem was originally proposed by Inaba. This problem asks whether every point in a given point set can be covered with disjoint unit disks. Now, we think about the number of points covered with unit disks. When there is one point, a unit disk can always cover the point. When there are two points, we can cover the points with one or two unit disks. However, if a lot of points are arranged on fine grid points, we cannot cover them with any number of disjoint unit disks. Thus there is an upper bound for the number k of points that can be always covered with disjoint unit disks in any arrangement. It is known that any 10 points can be covered by some disjoint unit disks, and carefully arranged 108 points cannot be covered by any number of unit disks. Recently, Peter Winkler improved the upper bound to 60. Veit Elser improved the upper bound to 55. In this paper, we improve the upper bound 55 to less number, i.e., we show an arrangement of 54 points which cannot be covered by any arrangement of unit disks.

In this paper, we improve the upper bound using the following methods. Uehara and Asano showed that carefully arranged 108 points cannot be covered by any arrangement of unit disks. Their arrangement is based on square lattice.

In the first method, we improve their method. We show that their square lattice can be enlarged. The enlarged square lattice give the better upper bound 102.

In the second method, we use triangular lattice and hexagonal lattice instead of square lattice. Among regular polygons, it is well known that only regular triangles, squares, and regular hexagons can tile the plane. The triangular lattice and hexagonal lattice give the upper bounds 82 and 119, respectively.

In the third method, we use the notion of empty circle. In the above lattices, the radius of empty circle in each lattice is essential. We experimentally give a set of points that contain small maximum empty circle, and improve the upper bound.

Above results, all sets of points are put in a circle. But we can cut off the area and improve all the upper bound. As a result, we give a new upper bound 54 by cutting off the circle based on the triangular lattice.

For the lower bound, we try to improve the known lower bound 10, which is proved by probabilistic method. In the probabilistic method, the ratio of the covered area and non-covered area is essential. We tried to improve the lower bound from 10 to 11. More precisely, we fix one point at the origin, and arrange all other 10 points. It is not trivial, but this is equivalent to arrange 10 non-covered areas in a unit disk. We change the argument into the discrete manner, and try to check all possible positions. Unfortunately, the algorithm is too slow and we cannot improve the lower bound. This is a future work.