Temporal Logic to Represent Linguistic Features

by

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Abstract

Linear tense logics are widely accepted for structural temporal representation, where the basic K_T has two modal operators G and H, each of which represents the future and the past, respectively. Closely related to modal logics, it has been studied for a long time. We need to analyses the uses of the tense and aspect of the language in order to prepare it for the computer. This is aside from the theoretical value of studying the logical structure of time use in language. Thus far, many linguists and computer scientists have proposed the temporal relations in occurrences. Each occurrence of an event happens once and for all, and the events may have causal relations or other kinds of information flow in them. Especially, linear tense logics are well known as logics deal with time as a set of points. On the other hand, there is another approach to temporal reasoning which takes as primitive temporal intervals rather than points. These logics are well known as temporal logics which have binary relations for the relations between temporal intervals. At first, in this study, we show the logic of occurrence which has the temporal heredities for a temporal structure. We represent them by the binary relations because of a temporal heredity is associated with temporal interval logics. Thus, temporal intervals are given by a set of events, and the relationship in temporal intervals are defined also by those in events, i.e., a set of events articulates the time axis. We summarize temporal relations in occurrences, give the syntax and the semantics of the language for the occurrence logic, incorporating the concept of temporal heredity into it. And we explain the logic programming system based on the formal language.

The temporal interval relations arranged by Allen have long been the standard of natural language semantics, though it still lacks the modal-logical foundation. So, in our occurrence logic, the temporal relations between temporal intervals are utilized as the binary operators. Van Benthem proposed $\Box^{up}$ and $\Box_{down}$ in regard to the accessibility to overlapping intervals and subintervals, respectively; however, the logical feature of the modality has not well studied. So, in this study, we define the modal operators for the above temporal relations, propose a many-dimensional propositional temporal logic $K_{T\Box}$ including the conventional tense logic, together with such interval accessibility. $K_{T\Box}$ is a combined logic of conventional tense logics and temporal logics by *fusion*. For example, introducing a *precedence* relations $\prec$ and $\succ$ in two occurrences, we use a temporal operator G and H. Furthermore, introducing an *inclusion* relations $\subseteq$ and $\supseteq$ in two occurrences, we define $\Box^{\uparrow}$ and $\Box_{\downarrow}$ and use them. We summarize accessibility relations for $\succ, \prec, \subseteq$, and $\supseteq$ between possible worlds, give the syntax and the semantics of the language for $K_{T\Box}$, and we show our logic can represent the temporal heredities by the newly-formed temporal operators. Moreover, we introduce a sequent system for $K_{T\Box}$ and show a proof-search procedure. Additionally, we show the subformula property holds in our system, and thus would be able to show the decidability. Next, we show that our logic provide a formal apparatus for a precise aspectual classification. We show some formulae by our logic express some aspect classes by Vendler such as *event*, *state*, *achievement*, and so on. Furthermore, we can hypothesize the starting/ending points by assuming a certain superinterval by "$\varphi \Rightarrow \Diamond^{\uparrow}\Box_{\downarrow}\varphi$, " where $\Diamond^{\uparrow}$ is an abbreviation

of $\neg\,\Box^{\uparrow}\,\neg$ and denotes some possible world which includes the assumed starting/ending points. In the similar way, we assume a minimal interval. Especially, we can regard "$\varphi \Rightarrow \Diamond_{\downarrow}(H\neg\varphi \wedge \Box^{\uparrow}\,\varphi \wedge G\neg\varphi)$" as a representation of the *culmination* where $\Diamond_{\downarrow}$ is an abbreviation of $\neg\,\Box_{\downarrow}\,\neg$. Generally speaking, we cannot measure the *exact* size of the temporal location of an occurrence. However, by our logic, we can hypothesize the starting/ending points by assuming the certain possible worlds for a state and a event. We show such an aspectual classification and some temporal features by our logic. Furthermore, we propose $K^{+}_{T\Box}$ by adding some axioms for $K_{T\Box}$. Because of $K_{T\Box}$ is given by the simplest fusion, which is axiomatized by the union of axioms of a tense logic and a temporal logic, G, H and $\Box^{\uparrow}, \Box_{\downarrow}$ are not independence temporally, we need to introduce some axioms including these modal operators simultaneously, and show the power of temporal expression of $K^{+}_{T\Box}$.

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Chapter 1

Introduction

1.1 Backgrounds

In this study, we proposed the temporal logic to represent linguistic features which combined linear tense logic and interval logic. In the field of artificial intelligence, temporal logics are widely utilized. Temporal logic is a logic of propositions whose truth and falsity may depend on time. Closely related to modal logics, it has been studied for a long time [30, 48, 53]. We need to analyses the uses of the tense and aspect of the language in order to prepare it for the computer. This is aside from the theoretical value of studying the logical structure of time use in language. Thus far, many linguists and computer scientists have proposed the temporal relations in occurrences[2, 21, 30].

Temporal logic, as we want to describe it, is a branch of modal logic which has been studied for a long time. Modal logic deals with two propositional operators $\Box$ and $\Diamond$ interpreted as "necessarily" and "possibly". This is based on the idea that the truth of an assertion is a relative notion depending on possible worlds. A formal semantics was presented in this way by Kripke[35]. Prior[48] was the first to suggest a "temporal" interpretation of $\Box$ and $\Diamond$: "always" and "sometimes". In the sequel to this, many different systems of temporal logic were studied and an overview of these developments can be found in Rescher and Urquhart [51]. It should be noticed that in these contexts temporal logic as we want to do it is usually called tense logic whereas the term "temporal logic" is used differently. As one of the representation of a temporal modality, linear tense logic is well known which has two modal operators G and H, where G and H represent the future and the past, respectively. Lately, for a study of the multi-agent system, tense logic is utilized to represent an update of the agent's belief states[37, 64]. And also it is utilized for an aspectual studies[21].

On the other hand, temporal interval logics are studied to represent the relations between temporal intervals, by assuming the intervals of the occurrences[30, 59]. The motivation for this is partly philosophical : human being first come to experience time via extended events, and the point-based picture of non-extended temporal units seems a rather late abstraction arising out of this primary ontology. Moreover, a move toward intervals has also been advocated in linguistics, as providing intuitively and technically more appropriate "indices of valuation" for assertions in natural language[30]. Generally, temporal interval logic has some binary operators which represent the spatial-temporal relations. That is, the temporal interval logic involves a set of the continuous intervals, unlike in the case of the representation by the tense logic which regards time axis as a set

of time points.

In this study, we suppose the modal operators as substitute for these binary relations between temporal intervals, that is, temporal relations between intervals are reduced to the accessibility of possible worlds in our logic. And we clarify the logical feature of the modality.

1.2 Outline of this Thesis

Initially, we show a multi-agent system which is combined an epistemic logic and a tense logic. Epistemic logic, or logic of knowledge, have been studied in philosophy with the aim of analyzing formal properties of reasoning about knowledge and belief since the 1950s. Over the last 20 years, however, epistemic logic has found applications in various other disciplines. Here are some of them:

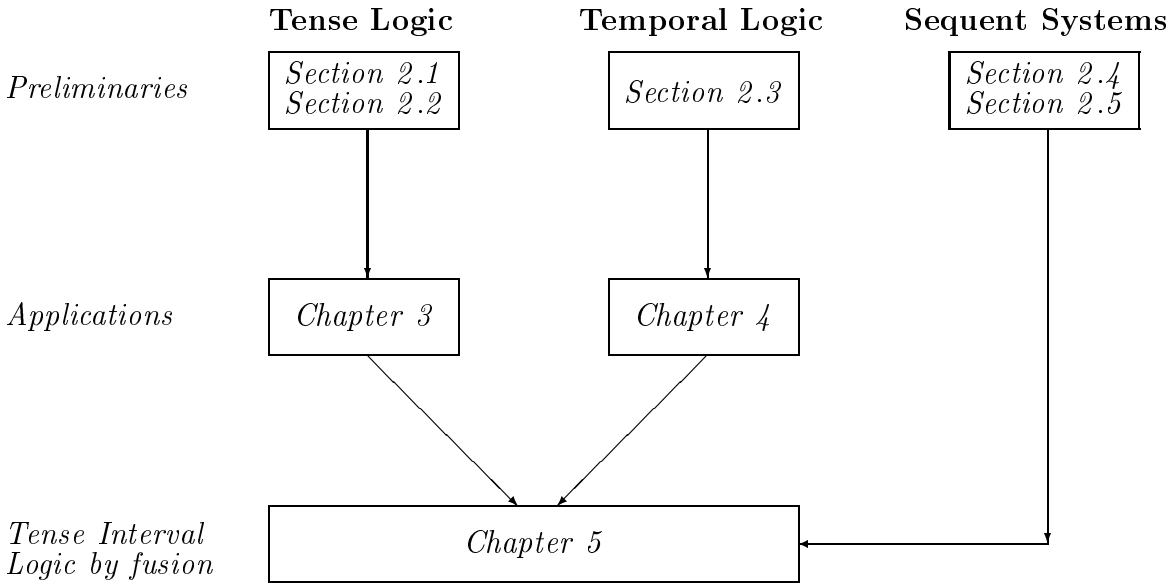
- in artificial intelligence, epistemic logic is applied in order to find out what an agent has to know to show intelligent behavior[19];
- in computer science, it is employed to analyze of multi-agent systems; (see [19] and references therein).

Temporal epistemic logic which is combined the tense logic and the epistemic logic describes the partial knowledge that different agents have as dependent on past, present, and future states of the world. And that describes protocols for communication between stations and temporal conditions for turning stations on and off. So, for a study of the multi-agent system, tense logic is utilized to represent an update of the agent's belief states. By considering the temporal epistemic logic, we show the power of an expression of the multi-agent system and tense logic.

Secondly, we show the logic of occurrence which has the temporal heredities for a temporal structure. We represent them by the binary relations because of a temporal heredity is associated with temporal interval logics. Thus, temporal intervals are given by a set of events, and the relationship in temporal intervals are defined also by those in events, as in [30], i.e., a set of events articulates the time axis. we summarize temporal relations in occurrences, give the syntax and the semantics of the language for the occurrence logic, incorporating the concept of temporal heredity into it. And we explain the logic programming system based on the formal language.

Lastly, we show our main object; temporal logic to represent linguistic features ($K_{T\Box}$). $K_{T\Box}$ is a combined logic of a conventional tense logic and temporal logic which represent the temporal modality by the modal operators. That is, we represent the temporal heredities by the newly-formed temporal operators. We propose a formalization of the temporal modality reflected with tense logic and temporal logic, and define the syntax and Kripke semantics for our $K_{T\Box}$. For a construction of tense logic and temporal logic, we use a way of fusion. Moreover, we introduce a sequent system for $K_{T\Box}$ and show a proof-search procedure and the decidability of $K_{T\Box}$. And, we show that our logic provide a formal apparatus for a precise aspectual classification. We show some formulae by our logic express some aspect classes by Vendler such as *event*, *state*, *achievement*, and so on. Generally speaking, we cannot measure the *exact* size of the temporal location of an occurrence. However, by our logic, we can hypothesize the starting/ending points by assuming the certain possible worlds for a state and a event. We show such an aspectual classification

and some temporal features by our logic. Furthermore, we propose $K^+_{T\Box}$ by adding some axioms for $K_{T\Box}$. Because of $K_{T\Box}$ is given by the simplest fusion, which is axiomatized by the union of axioms of a tense logic and a temporal logic, G, H and $\Box^{\uparrow}, \Box_{\downarrow}$ are not independence temporally, we need to introduce some axioms including these modal operators simultaneously, and show the power of temporal expression of $K^+_{T\Box}$.



Chapter 2

Preliminaries

2.1 Modal Axiomatic Systems

2.1.1 Modal Logics

Tense logic and temporal logic, as we want to describe them, are a branch of modal logic which has been studied for a long time. Modal logic deals with two propositional operators $\Box$ and $\Diamond$ interpreted as "necessarily" and "possibly". This is based on the idea that the truth of an assertion is a relative notion depending on possible worlds.

The language L of propositional modal logic consists of the following vocabulary.

propositional variables:	p, q, r,$\cdots$
logical connectives:	$\neg$, $\vee$, $\wedge$, $\Rightarrow$
modal operators:	$\Box$

We use $\varphi, \psi, \chi, \cdots$ for formulae which are constructed inductively from propositional variables, logical connectives and modal operators in the usual way. Especially, $\Box\varphi$ is a formula if φ is a formula. Modal operators $\Diamond$ is an abbreviation of $\neg\Box\neg$. A set of formulae which contains the axioms of classical propositional logic, the modal axiom

$$(K) \quad \Box(\varphi \to \psi) \to (\Box\varphi \to \Box\psi),$$

and is closed under Modus ponens, Substitution, and the rule of

$$\text{Necessitation} \qquad \text{if } \varphi \in L, \text{ then } \Box\varphi \in L$$

is called a modal logic. Every other modal logic L' can be obtained by extending this system with a set Σ of extra axioms. In this case we write

$$L' = K \oplus \Sigma.$$

If Σ can be chosen finite, then we call L' finitely axiomatizable. Here, we show that the historical names for some well-known axiom schemes are

$$(D)\ \Box\varphi \to \Diamond\varphi, \qquad (T)\ \Box\varphi \to \varphi, \qquad (4)\ \Box\varphi \to \Box\Box\varphi,$$
$$(B)\ \varphi \to \Box\Diamond\varphi, \qquad (5)\ \Diamond\varphi \to \Box\Diamond\varphi.$$

Using this notation, we can define the Lewis system **S4** and **S5** as follows:

$$(S4)\ \ K \oplus \Box\varphi \to \Box\Box\varphi \oplus \Box\varphi \to \varphi,$$
$$(S5)\ \ K \oplus \Diamond\varphi \to \Box\Diamond\varphi.$$

The $\Box$ of S5 can also be read as "I know". By accepting one or more of the axioms of S5 as properties of knowledge we can obtain new modal systems, like **KT** and **K4**:

$$(KT)\ \ K \oplus \Box\varphi \to \varphi.$$
$$(K4)\ \ K \oplus \Box\varphi \to \Box\Box\varphi.$$

The minimal deontic logic **KD** reflecting this principle is defined as

$$(KD)\ \ K \oplus \Box\varphi \to \Diamond\varphi.$$

Fifteen mutually distinct logics come out 32 ($=2^5$) ways of choice of these axioms by the following properties.

D is provable in KT	5 is provable in $K4B$
4 is provable in $K5B$	4 and B are provable in $KT5$
T is provable in $K4DB$	

Thus, we show that modal logics can be constructed from their axiom schemes as follow.

$K4 = K \oplus \{4\}$	$KD4 = K \oplus \{D, 4\}$	$KB = K \oplus \{B\}$
$K5 = K \oplus \{5\}$	$KD5 = K \oplus \{D, 5\}$	$KTB = K \oplus \{T, B\}$
$K45 = K \oplus \{4, 5\}$	$KD45 = K \oplus \{D, 4, 5\}$	$KDB = K \oplus \{D, B\}$
$KD = K \oplus \{D\}$	$S4 = K \oplus \{T, 4\}$	$KB4 = K \oplus \{B, 4\}$
$KT = K \oplus \{T\}$	$S5 = K \oplus \{T, 5\}$	

Proposition 1 *The following equivalences hold.*

1. *If either* {*T,5*},{*T,4,5*},{*D,4,B*} *or* {*D,5,B*} *is the subset of Q, then* $K \oplus Q = S5$,
2. $KB4 = K \oplus \{5,B\} = K \oplus \{4,5,B\}$,
3. $KT = K \oplus \{T,D\}$,
4. $KT4 = K \oplus \{T,D,4\}$,
5. $KTB = K \oplus \{T,D,B\}$.

By our definition, modal logics are sets of formulae, and therefore set inclusion defines a partial order in the set of all modal logics. For logics introduced above, the following inclusion relationship holds. Other typical axiom schemes are as follows:

(ax1) $\Diamond\varphi \to \Box\varphi$
(ax2) $\Diamond\varphi \leftrightarrow \Box\varphi$
(ax3) $\Box\Box\varphi \to \Box\varphi$
(ax4) $\Diamond^k\Box^l\varphi \to \Box^m\Diamond^n\varphi$
(ax5) $\Box(\varphi \wedge \Box\varphi \to \psi) \vee \Box(\varphi \wedge \Box\psi \to \varphi)$

where formulae $\Box^m\varphi$ and $\Diamond^n\varphi$ denote formulae $\Box\cdots\Box\varphi$ with m boxes and $\Diamond\cdots\Diamond\varphi$ with n diamonds.

2.1.2 Tense Logics

Tense logic has two operators "G" and "H", which means "at all future time" and "at all past time", respectively. They are really nothing more than the existential 'quantifiers', enable us to write axioms which can distinguish between different properties of the flow of time. Here, we take particular note of a conventional tense logic. Let the language L_T consists of the following vocabulary.

propositional variables:	p, q, r,$\cdots$
logical connectives:	$\neg$, $\vee$, $\wedge$, $\Rightarrow$
modal operators:	G, H

Modal operators F and P are abbreviations of $\neg G \neg$ and $\neg H \neg$, respectively. That is, modal operators are interpreted in the following.

$G\varphi$	at all future time, φ
$H\varphi$	at all past time, φ
$GF\varphi$	φ is always going to be true at some later time
$PH\varphi$	once upon a time, φ had always been the case
$F\varphi \wedge F\psi$	φ will be the case and so will ψ
$F(\varphi \wedge \psi)$	φ and ψ will be the case simultaneously
$\varphi \Rightarrow G\psi$	if φ then ψ will always be the case from now on
$G(\varphi \Rightarrow \psi)$	φ will always 'guarantee' ψ
$G(\varphi \Rightarrow F\psi)$	φ will always 'enable' ψ to become true afterwards

The weakest propositional temporal logic is the logic K_t of Arthur Prior[48]. It is presented as a Hilbert system with the following axiom schemas.

Axioms:

(i) Axiom for classical logic takes all tautologies as axiom
(ii) $G(\varphi \wedge \psi) \Leftrightarrow G\varphi \wedge G\psi$
(iii) $H(\varphi \wedge \psi) \Leftrightarrow H\varphi \wedge H\psi$
(iv) $\varphi \Rightarrow GP\varphi$
(v) $\varphi \Rightarrow HF\varphi$

Rules:

from φ and $\varphi \Rightarrow \psi$ infer ψ (Modus Ponens)
if φ is provable, then so are $G\varphi$, $H\varphi$ (Temporalization)

This system is well-known from other areas of Intensional Logic. In fact, it is rather standard bimodal calculus, be it with one peculiarity. In its most general guise, there would be two alternative relations R_G and R_H for the two operators. but the effect of the two conversion axiom to tie the two directions in time together, by making these two relations set-theoretic converses of each other. Here is an illustration of a theorem in this proof-theoretic format, that will serve as a running example.

Example 1 *(Conjunctive distribution) The following distribution principle is universally valid on tense models:*

$$G(\varphi \wedge \psi) \Leftrightarrow (G\varphi \wedge G\psi).$$

From left to right, this expresses 'monotonicity' of the universal future and tense, from right to left, its 'conjunctivity'. Here is an outline of an axiomatic derivation:

1.	$(\varphi \wedge \psi) \Rightarrow \varphi$	propositional tautology
2.	$G((\varphi \wedge \psi) \Rightarrow \varphi)$	Temporalization
3.	$G((\varphi \wedge \psi) \Rightarrow \varphi) \Leftrightarrow (G(\varphi \wedge \psi) \Rightarrow G\psi)$	Distribution
4.	$(G(\varphi \wedge \psi) \Rightarrow G\varphi)$	Modus Ponens

Additionally, we show an other axiomatic presentation of temporal logic as follows[24].

Reflexivity	$G\varphi \Rightarrow \varphi$
Transitivity	$G\varphi \Rightarrow GG\varphi$
Linearity	$G(\varphi \wedge G\varphi \Rightarrow \psi) \vee G(\psi \wedge G\psi \Rightarrow \varphi)$
density	$F\varphi \Rightarrow FF\varphi$ and $P\varphi \Rightarrow PP\varphi$
Dedekind complete	$(L(G\varphi \Rightarrow PG\varphi) \wedge G\varphi) \Rightarrow H\varphi$, $(L(H\varphi \Rightarrow FH\varphi) \wedge H\varphi) \Rightarrow G\varphi$ where $LX \equiv X \wedge GX \wedge HX$.
Irreflexive and well ordered	$F\varphi \Rightarrow F(\varphi \wedge G\neg\varphi)$ $P\varphi \Rightarrow P(\varphi \wedge H\neg\varphi)$
Irreflexive and infinite chain	$GF(\neg\bot)$ $HP(\neg\bot)$
Time is finite	$F\varphi \Rightarrow F(\varphi \wedge G\neg\varphi), P\varphi \Rightarrow P(\varphi \wedge H\neg\varphi)$
No end points	$F\top \wedge p\top$

Thus far, tense logic, which was originally developed as a logical framework in which to describe tense in natural language, is now recognized as an essential tool for reasoning about programs.

As the applications that require concurrent and distributed solutions have become more refined, so the corresponding logical tools have been extended. In representing the behavior of concurrent systems, the ability to refer to a range of possible execution paths is seen as important. Thus, there is a need for methods incorporating *branching time* tense logics [16, 17, 18]. Here, the underlying model of time is of a choice of possibilities branching into the future. Such branching time tense logics have been developed and allied to the specification of concurrent and distributed system.

It has been observed that most correctness properties of concurrent programs can be expressed in a branching time logic called Computational Tree Logic (CTL). Much of the research into the verification of concurrent and distributed systems has centered around the *model checking* technique utilizing CTL. Here the satisfiability of a CTL formula is checked with respect to a model derived from a finite state program [16]. Due to the success of this approach, together with a lack of direct applications of proof in branching time tense logics, relatively little research has been carried out on efficient decision procedures for such logics. The work that has been produced has mainly been concerned with basic tableau and automata methods for these logics [17]. However, in recent years several application of branching time tense logics requiring improved proof methods have been developed, most notably the specification and verification of multi-agent systems [50]. This has led to the requirement for more refined, and potentially more efficient, proof methods.

Let the language L_{CTL} consists of the following vocabulary.

propositional variables:	p, q, r,$\cdots$
logical connectives:	$\neg$, $\vee$, $\wedge$, $\Rightarrow$
new constant :	*start* meaning 'at the beginning of time'.
tense operators:	G, F, X, U, W
path operators:	A, E

Where X, U, and W denote 'at the next moment in time', 'until', and 'unless', respectively. And A and E denote 'on all future paths' and 'on some future path', respectively.

The set of $well-formed\ formulae$ of CTL, WFF_{CTL}, is defined as follows.

1. All propositional variables and *start* are in WFF_{CTL}.
2. If φ and ψ are in WFF_{CTL}, then so are $\varphi \wedge \psi$, $\neg\varphi$, $\varphi \vee \psi$, $\varphi \Rightarrow \psi$.
3. If φ and ψ are in WFF_{CTL}, then $G\varphi$, $F\varphi$, $E\varphi$, $\varphi U\psi$, $\varphi W\psi$ are all *path* formulae.
4. If $\mathcal{P}$ is a path formulae then $A\mathcal{P}$ and $E\mathcal{P}$ are both in WFF_{CTL}.

For example, modal operators are interpreted in the following.

$AX\varphi$:	On all future paths from current time (start), φ is true at the next moment.
$AF\varphi$:	On all future paths from current time (start), φ is true at some later time.
$EG\varphi$:	There is a some future path from current time (start) such that φ is true at all later time.
$EGF\varphi$:	There is a some future path from current time (start) such that φ is always going to be true at some later time.
$AF(\varphi \wedge \psi)$:	On all future paths from current time (start), φ and ψ will be the case simultaneously.
$AG(\varphi \Rightarrow \psi)$	On all future paths from current time (start), φ will always 'guarantee' ψ
$EG(\varphi \Rightarrow F\psi)$	There is a some future path from current time (start) such that φ will always 'enable' ψ to become true afterwards

Thus, all of WFF_{CTL} with tense operator are formed by a pair of tense operators and branching time path operators, i.e. each CTL formula has a structure where any tense operator can only be followed by a path operator or a classical operator, while any path operator can only be followed by a tense operator. As a result CTL is weaker than linear tense logic in its expressive capabilities within a path, but is more expressive in that it can quantify over paths themselves. Here, we show the axioms for CTL as follows. The following set of axioms and rules of deduction represents a complete deductive system in relation to the semantic of the logic CTL [18].

[Axiom for CTL]

(Ax1.) All axioms of propositional calculus
(Ax2.) $EF\varphi \equiv E(trueU\varphi)$
(Ax3.) $AG\varphi \equiv \neg EF\neg\varphi$
(Ax4.) $AF\varphi \equiv A(trueU\varphi)$
(Ax5.) $EG\varphi \equiv \neg AF\neg\varphi$
(Ax6.) $EX(\varphi \vee \psi) \equiv (EX\varphi \vee EX\psi)$
(Ax7.) $AX\varphi \equiv \neg EX\neg\varphi$
(Ax8.) $E(\varphi U\psi) \equiv (\psi \vee (\varphi \wedge EXE(\varphi U\psi)))$
(Ax9.) $A(\varphi U\psi) \equiv (\psi \vee (\varphi \wedge AXA(\varphi U\psi)))$
(Ax10.) $AXtrue \wedge EXtrue$
(Ax11.) $AG(\xi \Rightarrow (\neg\psi \wedge EX\xi)) \Rightarrow (\xi \Rightarrow \neg A(\varphi U\psi))$
(Ax12.) $AG(\xi \Rightarrow (\neg\psi \wedge EX\xi)) \Rightarrow (\xi \Rightarrow \neg AF\psi)$
(Ax13.) $AG(\xi \Rightarrow (\neg\psi \wedge AX\xi)) \Rightarrow (\xi \Rightarrow \neg E(\varphi U\psi))$
(Ax14.) $AG(\xi \Rightarrow (\neg\psi \wedge AX\xi)) \Rightarrow (\xi \Rightarrow \neg EF\psi)$
(Ax15.) $AG(\varphi \Rightarrow \psi) \Rightarrow (EX\varphi \Rightarrow EX\psi)$

[Rules of inference for CTL]

(R1.) from φ and $\varphi \Rightarrow \psi$ infer ψ (Modus Ponens)
(R2.) φ infers $AG\varphi$ (Generalization)

2.1.3 Epistemic Logics

Epistemic logics (logics of knowledge and those of belief) have been set to work in philosophy. We mention about the combined system with epistemic logics and computational tree logics in chapter 3. The aim of employment of their logics was to analyze formal properties of reasoning about knowledge and belief. The possible world semantics for epistemic logics originated in Hintikka[28]. Recently, for epistemic logics, their semantics using techniques developed by Kripke, that is called Kripke semantics, is often adopted. Epistemic logics are usually formulated as normal modal logics. It is natural since every agent is wanted to have minimum inferential ability.

Formalization of multi-agent systems using logical framework has served as an important bridge between communities of distributed artificial intelligence. As to former, BDI model has been studied as modal logics with three modalities for *belief*, *desire*, and *intention* [49, 66]. These logics have been studied especially as models of recognizing outside world for an agent embedded in a situation, and have been applied in *situation*(*or channel*) theory in information science [6]. If we try to build a computer systems for multi-agent systems, the most important issue would be how we formalize the change of state, how we synchronize the actions of agents on linear or branching time, and how we assure soundness and consistency of their systems. In such a situation, it will be necessary to consider logics that are obtained by combining two kinds of modal logics; epistemic logics which are logics of knowledge and belief and temporal logics which handle changes of time. In this section, we show the epistemic logic of belief.

A finite set of agent identifiers is denoted by *Agent*. In this paper, we only utilize a set of the agent's belief states, so the language of our epistemic logic consists of propositional variables, logical connectives and modal operators in the usual way. Formulae with epistemic operators are read as follow:

$B_\alpha\varphi$ agent α believes φ

The minimal logic of belief K_B is the least normal logics containing moreover following axioms:

The axioms

(1) $B_\alpha(\varphi \Rightarrow \psi) \Rightarrow (B_\alpha\varphi \Rightarrow B_\alpha\psi)$,
(2) $B_\alpha\varphi \Rightarrow \neg B_\alpha\neg\varphi$,
(3) $B_\alpha\varphi \Rightarrow B_\alpha B_\alpha\varphi$,
(4) $\neg B_\alpha\varphi \Rightarrow B_\alpha\neg B_\alpha\varphi$,

means then that

(1) agent α believes all the logical consequences of its beliefs (logical omniscience),
(2) agent α does not believe that φ is not true (consistency),
(3) agent α believes what it believes (positive introspection), and
(4) agent α believes what it does not believe (negative introspection).

Recall now that (1) is an axiom of every normal modal logic, (2) an axiom of D, (3) an axiom of 4, and (4) an axiom of 5, so (1)-(4) are axioms of $KD45$.

2.2 Possible World Semantics

2.2.1 Kripke Semantics for Normal Modal Logics

In this Section, we introduce Kripke semantics, and present semantical properties for modal logics. The provability interpretation of the necessity operator $\Box$ and its relation to intuitionism gave a strong impetus to mathematical studies in modal logic, which resulted, in establishing connections with algebra and topology, and finally led to the discovery of relational representations of modal algebras. Denoting the alternativeness (or accessibility) relation by R, we write xRy to say that y is an alternative (or possible) world of x.

Let W be a non-empty set, and $R_\Box(\subseteq W \times W)$ be a binary relation on W. Then a Kripke frame $\mathcal{F}$ is a pair $\langle W, R_\Box\rangle$. Here W is called the set of possible worlds, and $R_\Box$ is called accessibility relations. A valuation in a frame $\mathcal{F} = \langle W, R_\Box\rangle$ is a mapping $\mathcal{V}$ associating with each propositional variable p a set $\mathcal{V}(p)$ of worlds in W, which is understood as the set of those worlds where p holds true. A Kripke model for L is a pair $\mathcal{M} = \langle \mathcal{F}, \mathcal{V}\rangle$.

For a given Kripke model $\langle \mathcal{F}, \mathcal{V}\rangle$, a binary relation $\Vdash$ between $u \in W$ and formulae is defined inductively on the length of formulae as follows.

$u \Vdash p$ *iff* $p \in \mathcal{V}(p)$
$u \Vdash \varphi \wedge \psi$ *iff* $u \Vdash \varphi$ and $u \Vdash \psi$
$u \Vdash \varphi \vee \psi$ *iff* $u \Vdash \varphi$ or $u \Vdash \psi$
$u \Vdash \varphi \Rightarrow \psi$ *iff* $u \Vdash \varphi$ implies $u \Vdash \psi$
$u \Vdash \neg\varphi$ *iff* $u \nVdash \varphi$

$u \Vdash \Box\varphi$ *iff* $\forall v \in W$, $uR_{\Box}v$ implies $v \Vdash \varphi$

A formula φ is *true in model* $\mathcal{M}$=$(W, R_{\Box}, \Vdash)$, denoted by $\mathcal{M} \models \varphi$, if $u \Vdash \varphi$ for every $u \in W$.

Suppose that $\mathcal{F} = \langle W, R\rangle$ is a frame, then the binary relation R satisfies one of following conditions if and only if the axiom schemes corresponding the condition is valid in $\mathcal{F}$.

(D) :	$\forall u \exists v(uRv)$	(serial)
(T) :	$\forall u(uRu)$	(reflexive)
(4) :	$\forall u \forall v \forall w(uRv \wedge vRw \rightarrow uRw)$	(transitive)
(B) :	$\forall u \forall v(uRv \rightarrow vRu)$	(symmetric)
(5) :	$\forall u \forall v \forall w(uRv \wedge uRw \rightarrow vRw)$	(Euclidean)
(ax1) :	$\forall u \forall v(uRv \wedge uRw \rightarrow v = w)$	(partially functional)
(ax2) :	$\forall u \forall v(u = v)$	(functional)
(ax3) :	$\forall u \forall v(uRv \rightarrow \exists w(uRw \wedge wRv))$	(weakly dense)
(ax4) :	$\forall u \forall v \forall w(uR^k v \wedge uR^m w \rightarrow \exists t(vR^l t \wedge wR^n t))$	(Church-Rosser)
(ax5) :	$\forall u \forall v \forall w(uRv \wedge uRw \rightarrow vRw \vee v = w \vee wRv)$	(weakly connected)

For a logic L, every frame with the conditions corresponding to the axioms is called $L - frame$. For example, every S4-frame is reflexive and transitive. Then the following proposition 2 can be shown

Proposition 2
1. The least normal modal logic K is determined by the class of all Kripke frame; i.e. for any frame $\mathcal{F}$, $\mathcal{F} \models \varphi$ iff $\varphi \in K$

2. Let L be any of logics introduced in previous section. Then the logic L is determined by the class of all L-frames; i.e. for any L-frame $\mathcal{F}$, $\mathcal{F} \models \varphi$ iff $\varphi \in L$.

Proposition 2 is often shown by constructing their canonical models[26]. if $\varphi \in L$ then there exists a model $\mathcal{M}$, for example the canonical model of L, such that $\mathcal{M} \not\models \varphi$ by the above completeness theorem. But it would be quite useful if we could get a finite model in which a given unprovable formula is false. because a consequence of the finite model property is the decidability. A concrete finite procedure which decides to be provable or not for any formula in a system is called a *decision procedure*. If there exists a decision procedure, the system is said to be *decidable*. By the Harrop's theorem, that is "if a finitely axiomatizable logic has the finite model property, then it is decidable." A logic L has the finite model property if the following condition is satisfied.

if $\varphi \notin L$, then there is a finite L-model $\mathcal{M}$ such that $\mathcal{M} \not\models \varphi$.

2.2.2 Kripke Semantics for Tense Logics

In a similar way, we can characterize the axioms of the tense logic by Kripke semantics, introduce Kripke semantics for tense logics. A Kripke model for tense logic is a tuple $\langle W, R_T, \Vdash\rangle$, where W is a non-empty set, and R_T is a binary relation on W, and $\Vdash$ is defined inductively as follows.

$u \Vdash p \quad iff \quad p \in \mathcal{V}(p)$
$u \Vdash \varphi \wedge \psi \quad iff \quad u \Vdash \varphi$ and $u \Vdash \psi$
$u \Vdash \varphi \vee \psi \quad iff \quad u \Vdash \varphi$ or $u \Vdash \psi$
$u \Vdash \varphi \Rightarrow \psi \quad iff \quad u \Vdash \varphi$ implies $u \Vdash \psi$
$u \Vdash \neg\varphi \quad iff \quad u \nVdash \varphi$
$u \Vdash G\varphi \quad iff \quad \forall v \in W$, uR_Tv implies $v \Vdash \varphi$
$u \Vdash H\varphi \quad iff \quad \forall v \in W$, vR_Tu implies $v \Vdash \varphi$

A formula φ is *true in model* $\mathcal{M}$=$\langle W, R_T, \Vdash\rangle$, denoted by $\mathcal{M} \models \varphi$, if $u \Vdash \varphi$ for every $u \in W$. Now, the following hold.

(1) $\mathcal{M} \models G\varphi \Rightarrow GG\varphi \quad iff \quad \forall u, v, w(uR_Tv \wedge vR_Tw \rightarrow uR_Tw)$
(2) $\mathcal{M} \models H\varphi \Rightarrow HH\varphi \quad iff \quad \forall u, v, w(wR_Tv \wedge vR_Tu \rightarrow wR_Tu)$
(3) $\mathcal{M} \models \varphi \Rightarrow GP\varphi \quad iff \quad \forall u, v(uR_Tv \rightarrow vR_Tu)$
(4) $\mathcal{M} \models \varphi \Rightarrow HF\varphi \quad iff \quad \forall u, v(vR_Tu \rightarrow uR_Tv)$

An accessibility relation R_G for the future tense is a converse relation of an accessibility relation R_H for the past tense, so it is enough to take either of R_G and R_H. Hence, by taking R_T, a model for tense logics can be defined.

Additionally, we can also characterize a computational tree logic by the Kripke model. The CTL has the following features.

1. Each state can have an infinite number of successors, but should have at least one.
2. Each state belongs to some path (a sequence of states with finite past and infinite future).
3. No merging of paths is allowed − a path, once started from another one, has no more common states with any other path.

Now, we introduce Kripke semantics for CTL. The triple $\langle S_t, R, L\rangle$ is called a *Kripke model* for CTL, where S_t is a nonempty-set of states, $R(\subseteq S_t \times S_t)$ is a binary relation on S_t such that for all i there exists a j such that $\langle s_i, s_j\rangle \in R$, and L is a mapping such that $L(t) \subseteq S_t$ for each propositional variable φ, where $t \in S_t$ is a state. For an infinite sequence of states, $t_0, t_1, ...$ is called a *path* on w starting with t_0 such that $t_0R_wt_1, t_1R_wt_2, ...$ Then a binary relation '$\models$' which evaluates well-formed CTL formulae at a particular state t, in a particular model M is defined as follows:

$\langle M, t\rangle \models \varphi \iff \varphi \in L(t)$
$\langle M, t\rangle \models \varphi \wedge \psi \iff t \models \varphi$ and $t \models \psi$
$\langle M, t\rangle \models \varphi \vee \psi \iff t \models \varphi$ or $t \models \psi$
$\langle M, t\rangle \models \varphi \rightarrow \psi \iff t \models \varphi$ implies $t \models \psi$
$\langle M, t\rangle \models \neg\varphi \iff$ not $t \models \varphi$
$\langle M, t\rangle \models EX\varphi \iff \exists\sigma \in P(t), \sigma[1] \models \varphi$
$\langle M, t\rangle \models A[\varphi U\psi] \iff \forall\sigma \in P(t).\exists i \geq 0, \sigma[i] \models \psi \wedge (\forall 0 \leq j \leq i.\sigma[j] \models \varphi)$
$\langle M, t\rangle \models E[\varphi U\psi] \iff \exists\sigma \in P(t).\exists i \geq 0, \sigma[i] \models \psi \wedge (\forall 0 \leq j \leq i.\sigma[j] \models \varphi)$

Where $P(t)$ is a set of paths starting with t and $\sigma[i]$ is a i^{th} element of σ.

Definition 1 *[Satisfiability] A well-formed CTL formula φ is satisfiable if and only if it is satisfied in some possible model, i.e. $\exists M.\langle M, t\rangle \models \varphi$.*

Definition 2 *[Validity] A well-formed CTL formula φ is satisfiable if and only if it is satisfied in every possible model, i.e. $\forall M.\langle M, t\rangle \models \varphi$.*

2.2.3 Kripke Semantics for Epistemic Logics

Similar to other Kripke semantics, we can characterize the axioms of the epistemic logic by Kripke semantics.

Let *Agent* be a set with n elements. A Kripke model for logics of belief is defined as a $n+2$ tuple $\langle W, \{R_{B_\alpha} | \alpha \in Agent\}, \Vdash\rangle$, where W is a non-empty set, and $R_{B_\alpha} (\subseteq W \times W)$ are binary relations on W for $\alpha \in Agent$. Then, $\Vdash$ is defined inductively as follows.

$u \Vdash p \quad iff \quad p \in \mathcal{V}(p)$
$u \Vdash \varphi \wedge \psi \quad iff \quad u \Vdash \varphi$ and $u \Vdash \psi$
$u \Vdash \varphi \vee \psi \quad iff \quad u \Vdash \varphi$ or $u \Vdash \psi$
$u \Vdash \varphi \Rightarrow \psi \quad iff \quad u \Vdash \varphi$ implies $u \Vdash \psi$
$u \Vdash \neg\varphi \quad iff \quad u \nVdash \varphi$
$u \Vdash B_\alpha\varphi \quad iff \quad \forall v \in W, uR_{B_\alpha}v$ implies $v \Vdash \varphi$

Since the number of agents does not play an essential role in the following discussions, we consider only the case where there is a single agent in the rest of the present section. And a formula φ is *true in model* $\mathcal{M}$=$(W, R_{B_\alpha}, \Vdash)$, denoted by $\mathcal{M} \models \varphi$, if $u \Vdash \varphi$ for every $u \in W$.

Proposition 3 *(Correspondence theory) Let $\mathcal{M} = \langle W, R_{B_\alpha}, \Vdash\rangle$ be a model for epistemic logic of belief. Then the following holds.*

(1) $\mathcal{M} \models B_\alpha\varphi \Rightarrow \neg B_\alpha\neg\varphi \quad iff \quad \forall u\exists v(uB_\alpha v)$
(2) $\mathcal{M} \models B_\alpha\varphi \Rightarrow B_\alpha B_\alpha\varphi \quad iff \quad \forall u, v, w(uB_\alpha v \wedge vB_\alpha w \rightarrow uB_\alpha w)$
(3) $\mathcal{M} \models \neg B_\alpha\varphi \Rightarrow B_\alpha\neg B_\alpha\varphi \quad iff \quad \forall u, v, w(uB_\alpha v \wedge uB_\alpha w \rightarrow vB_\alpha w)$

2.3 Temporal Interval Logic

In Section2.1 and Section2.2, we considered tense logics interpreted in Kripke models the points of which are linearly ordered and represent moments of time. In this section we show another approach to temporal reasoning which takes as primitive temporal intervals rather than points. Although the picture of durationaless mathematical points has been the prevalent images of time, there have been continuing attempts at developing an alternative intuition, viewing time as consisting of extended "periods" or "intervals" as its primary stuff. The motivation for this is partly philosophical : human being first come to experience time via extended events, and the point-based picture of non-extended temporal units seems a rather late abstraction arising out of this primary ontology. Moreover, a

move toward intervals has also been advocated in linguistics, as providing intuitively and technically more appropriate "indices of valuation" for assertions in natural language[30]. For example, the following instant situation can be evaluated at single moments (points) of time:

(1) It is one o'clock now,
(2) Anna found her purse.

But there are also assertions that can be evaluated only at some interval (period) of time, for example:

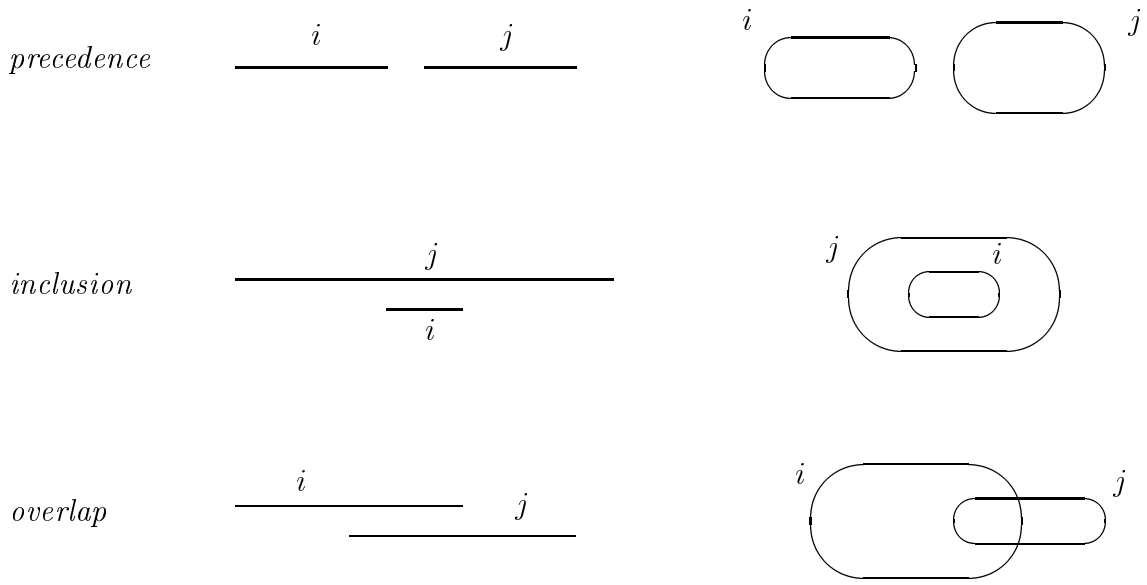
(3) I am student,
(4) Beth was sleeping.

The earlier progressive tense is more naturally understood as describing properties of intervals, rather than points in time. Interval-based temporal logics have been studied throughly in linguistical semantics, and such linguistic properties need not have any obvious reduction to distribution of corresponding "instantaneous properties" at points in time [30]. Finally, the computational literature has seen various proposals for interval-based temporal logics [59].

Therefore, we now want to introduce *interval frames* whose objects are extended temporal intervals, connected by suitable relations. As to the latter, a number of options arises, involving both temporal order and temporal occlusion, such as:

$i < j$ $\quad i$ wholly *precedes* j
$i \subseteq j$ $\quad i$ is *included* in j
$i \bigcirc j$ $\quad i$ *overlap* with j

The familiar picture that go with this intuition show intervals as linear stretches, or sometimes also as extended spatial regions:



Against this background, one can introduce more complex relations, such as one interval being the exact "sum" of two others, or corresponding interval operations, such as "union" of overlapping intervals. At present, there seems to be no uniformly accepted choice of primitive relations or operations in the field. One systematic perspective is that of representing at least all possible relative positions between bounded linear intervals. Allen[2] observed that relative positions of any two interval i and j can be described by precisely one of the following thirteen basic intervals relations: *before*(i,j), *meets*(i,j), *overlaps*(i,j), *during*(i,j), *starts*(i,j), *finishes*(i,j), their inverses, and *equal*(i,j). In the interval logic by Allen, the language whose alphabet contains these thirteen binary predicate symbols, a sufficient supply of interval variables i, j, etc., and the Booleans. Formulae of the interval logic are just Boolean combinations of the above listed atomic ones. In order to provide a semantics for formulae of interval logic, suppose that the flow of time is represented as a strict linear order $\mathcal{F} = \langle W, < \rangle$ like a dense order such as $\langle \mathbb{R}, < \rangle$. An assignment is a function a mapping the interval variables into temporal intervals, i.e. a temporal interval $a(i)$ is a non-empty subset of W such that

$$\forall x, y \in a(i) \forall z \in W (x < z < y \rightarrow z \in a(i)).$$

For example, if $u \leq v$ then the set

$$[u, v] = \{w \in W | u \leq w \leq v\}$$

is a temporal interval. Now, the truth-relations $\mathcal{F} \models^a \varphi$ for atomic formulae can be defined as follows:

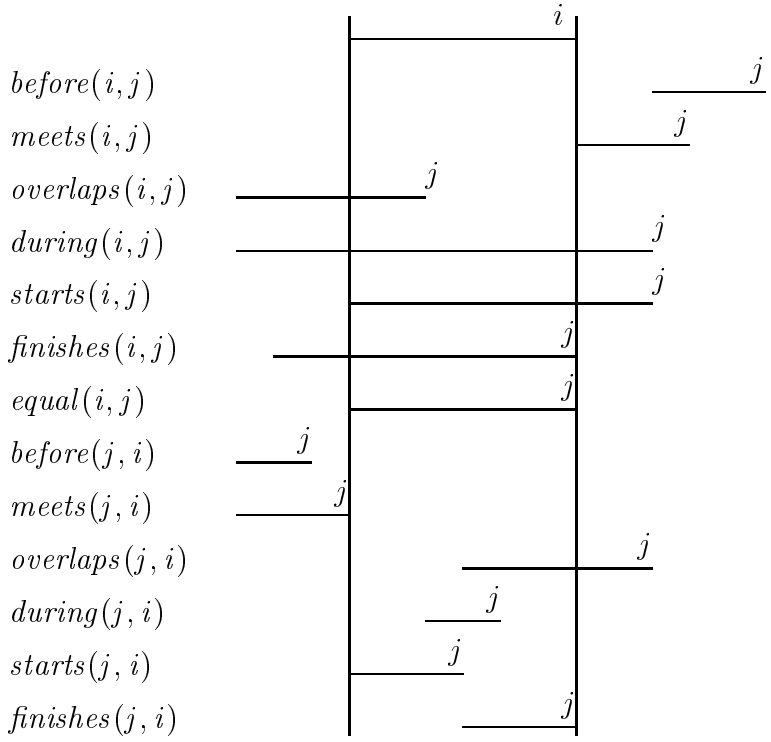


Figure 2.1: The atomic formulae of temporal interval logic by Allen

$\mathcal{F} \models^a equals(i,j)$ *iff* $a(i) = a(j)$,

$\mathcal{F} \models^a before(i,j)$ *iff* $\forall x, y(x \in a(i) \wedge y \in a(j) \rightarrow x < y \wedge \exists z(x < z < y \wedge x \notin a(i) \wedge z \notin a(j)))$,

$\mathcal{F} \models^a meets(i,j)$ *iff* $\forall x, y(x \in a(i) \wedge y \in a(j) \rightarrow x < y \wedge \forall z(x < z < y \rightarrow z \in a(i) \vee z \in a(j)))$,

$\mathcal{F} \models^a overlaps(i,j)$ *iff* $a(i) \cap a(j) \neq \emptyset \wedge \exists x, y(x < y \wedge x \in a(j) \wedge x \notin a(i) \wedge y \in a(i) \wedge y \notin a(j))$,

$\mathcal{F} \models^a starts(i,j)$ *iff* $a(i) \subseteq a(j) \wedge a(i) \neq a(j) \wedge \forall x, y(x < y \wedge x \in a(j) \wedge y \in a(i) \rightarrow x \in a(i))$,

$\mathcal{F} \models^a during(i,j)$ *iff* $\exists x, y, z(x < y < z \wedge x \in a(j) \wedge x \notin a(i) \wedge y \in a(i) \wedge z \in a(j) \wedge z \notin a(i))$,

$\mathcal{F} \models^a finishes(i,j)$ *iff* $a(i) \subseteq a(j) \wedge a(i) \neq a(j) \wedge \forall x, y(x < y \wedge y \in a(j) \wedge x \in a(i) \rightarrow y \in a(i))$.

The truth conditions for the Booleans are the same as in classical logic. We say that φ is satisfied in $\mathcal{F}$ if $\mathcal{F} \models^a \varphi$ holds for some assignment a in $\mathcal{F}$.

2.4 Gentzen Style Sequent Systems for Modal Logics

In this section, we introduce Gentzen style sequent systems for some modal logics and give a survey of their proof-theoretical properties. This Gentzen's sequent calculus is particularly interesting because it has had great impact on proof theory as well as on the development of calculi which are adequate for automated theorem proving. In a

sequent calculus the objects derived during a derivation, i.e. the members of a derivation, are so-called *sequents*. They play the role that formulae play in Frege-Hilbert calculi and that judgments play in natural deduction calculi. Gentzen style sequent systems for modal logics are obtained from Gentzen's sequent system **LK** for classical propositional logic by adding some ruled for modal operator $\Box$. Greek letters, $\Gamma, \Delta, \Pi, \Sigma, \Theta$ and Λ denote finite sets of formulae. The sequence $\Box\Gamma$ denotes $\Box\varphi_1, \Box\varphi_2, ..., \Box\varphi_n$, when Γ is $\varphi_1, \varphi_2, ..., \varphi_n$. $Sub(\Gamma)$, Γ_*, and Δ^* denote $\bigcup\{Sub(\psi)|\psi \in \Gamma\}$, $\bigwedge\{\varphi|\varphi \in \Gamma\}$, and $\bigvee\{\varphi|\varphi \in \Delta\}$, respectively, where $Sub(\psi)$ denotes a set of all subformulae of φ. Any expression of the form $\Gamma \to \Delta$ is called a *sequent*, where $\to$ denotes a *derivation relation*. The left hand side Γ of an above sequent is called *the succedent* and the right hand side Δ *theantecedent*. An *inference rule* is of the form

$$\text{either} \quad \frac{S_1}{S} \quad or \quad \frac{S_2 \quad S_3}{S},$$

where S_1, S_2, S_3, and S are sequents. In the inference, S_1, S_2, and S_3 are called the *upper sequents*, and S the *lower sequent*. The sequent system LK consists of the following initial sequent and inference rules.

[Initial sequents]
the sequents of the form $\varphi \to \varphi$

[Inference rules]
Structural rules:

$$\frac{\Gamma \to \Delta}{\varphi, \Gamma \to \Delta}\ (w \to) \qquad \frac{\varphi, \varphi, \Gamma \to \Delta}{\varphi, \Gamma \to \Delta}\ (c \to) \qquad \frac{\Gamma, \psi, \varphi, \Pi \to \Delta}{\Gamma, \varphi, \psi, \Pi \to \Delta}\ (e \to)$$

$$\frac{\Gamma \to \Delta}{\Gamma \to \Delta, \varphi}\ (\to w) \qquad \frac{\Gamma \to \Delta, \varphi, \varphi}{\Gamma \to \Delta, \varphi}\ (\to c) \qquad \frac{\Gamma \to \Delta, \psi, \varphi, \Sigma}{\Gamma \to \Delta, \varphi, \psi, \Sigma}\ (\to e)$$

$$\frac{\Gamma \to \Delta, \varphi \quad \varphi, \Pi \to \Sigma}{\Gamma, \Pi \to \Delta, \Sigma}\ (cut)$$

Logical rules:

$$\frac{\varphi, \Gamma \to \Delta}{\varphi \wedge \psi, \Gamma \to \Delta}\ (\wedge \to) \qquad \frac{\varphi, \Gamma \to \Delta}{\psi \wedge \psi, \Gamma \to \Delta}\ (\wedge \to) \qquad \frac{\Gamma \to \Delta, \varphi \quad \Gamma \to \Delta, \psi}{\Gamma \to \Delta, \varphi \wedge \psi}\ (\to \wedge)$$

$$\frac{\Gamma \to \Delta, \varphi}{\Gamma \to \Delta, \varphi \vee \psi}\ (\to \vee) \qquad \frac{\Gamma \to \Delta, \psi}{\Gamma \to \Delta, \varphi \vee \psi}\ (\to \vee) \qquad \frac{\varphi, \Gamma \to \Delta \quad \psi, \Gamma \to \Delta}{\varphi \vee \psi, \Gamma \to \Delta}\ (\vee \to)$$

$$\frac{\Gamma \to \Delta, \varphi \quad \psi, \Pi \to \Sigma}{\varphi \Rightarrow \psi, \Gamma, \Pi \to \Delta, \Sigma}\ (\Rightarrow \to) \qquad \frac{\varphi, \Gamma \to \Delta, \psi}{\Gamma \to \Delta, \varphi \Rightarrow \psi}\ (\to \Rightarrow)$$

$$\frac{\Gamma \to \Delta, \varphi}{\neg\varphi, \Gamma \to \Delta}\ (\neg \to) \qquad \frac{\varphi, \Gamma \to \Delta}{\Gamma \to \Delta, \neg\varphi}\ (\to \neg)$$

Inference rules $(w \rightarrow)$ and $(\rightarrow w)$ are called *weakning rules*, $(c \rightarrow)$ and $(\rightarrow c)$ *contraction rules*, and $(e \rightarrow)$ and $(\rightarrow e)$ *exchange rules*. Weakening, contraction and exchange rules are called *weak inferences*. The formula φ in cut rules is called the *cut furmula* of the rule.

In a sequent system $\mathcal{G}$, *proofs* of $\mathcal{G}$ and *end sequents* are defined inductively as follows:

1. Each initial sequent is a proof of $\mathcal{G}$, and the end sequent of the proof is itself,
2. Let P_1 and P_2 are proofs of $\mathcal{G}$ with the end sequent S_1 and S_2, respectively. If

$$\text{either} \quad \frac{S_1}{S} \quad or \quad \frac{S_2 \quad S_3}{S},$$

is one of the inferences in the system of $\mathcal{G}$, then

$$\text{either} \quad \frac{P_1}{S} \quad or \quad \frac{P_2 \quad P_3}{S},$$

is a proof of $\mathcal{G}$, and the end sequent is S. A sequent S is provable in $\mathcal{G}$ if there exists a proof of $\mathcal{G}$ whose end sequent is S.

If a sequent S is provable in a system $\mathcal{G}$, then it is often denoted by $\mathcal{G} \vdash S$. For a formula φ, if the sequent $\rightarrow \varphi$ is provable in a sequent system, then it is often said the formula φ is provable in the system.

The major reason that the sequent calculus is of great interest for proof theory as well as for automated theorem proving is the fact that it is complete also without the cut rule (cut elimination theorem). We then speak of the $cut-free\ sequent\ calculus$. The rules of the cut-free sequent calculus have the *subformula property*, i.e. every formula occurring in a premise of a rule instance is a subformula of a formula occurring in the conclusion of this rule instance. In proof theory this property is used to prove the consistency of sequent calculi. Cut-elimination theorem for a given sequent system $\mathcal{G}$ says that any sequent S which is provable in $\mathcal{G}$ has a proof of S containing no applications of cut rule. Such a proof is called a cut-free proof. When cut-elimination theorem holds for $\mathcal{G}$, sometimes we say that $\mathcal{G}$ has the cut-elimination property. Then the following holds.

Thorem 1 *[Cut-elimination theorem for LK] The system LK has the cut-elimination property. In fact, every proof in LK can be transformed, without changing the end-sequent, into cut-free one.*

As a corollary of Theorem 1, the following can be shown by checking all inference rules except cut rules.

Corollary 1 *[Subformula property] For any provable sequent in LK, proof of it can be consist only of subformulae of formulae in the sequent.*

One of most important consequences of the cut-elimination theorem is the decidability. In general, the decision procedure by using cut-elimination theorem goes as follows:

1. First, we show cut-elimination theorem.

2. Then, we derive subformula property. In many cases, subformula property follows from cut-elimination theorem. This is shown by checking that in each inference rule except cut rule every formula in an upper sequent of the rule is a subformula of some formulae in the lower sequent.

3. We show the fitness of proof-search procedure. That is, for a given sequent $\Gamma \to \Delta$, we show that the number of "candidates" of proof of $\Gamma \to \Delta$ is finite. If we succeed to show this, we make an exhaustive search of these candidates and check whether some of them are "correct proofs" of $\Gamma \to \Delta$ or not. This gives us a decision procedure. To show the finiteness of proof-search procedure, a standard strategy is as follows;

   (a) restriction to reduced sequents:
   We show that it is enough to consider sequents of a special form. For example, in LK we need to consider only sequents such that each formula occurs at most three times in the antecedent and the succedent.

   (b) restriction to proofs without repetitions:
   Apparently, if a proof contains the same sequent in a different place of one of its branches, this proof is redundant, and hence such a repetition can be eliminated.

If we succeed to show both (1) and (2), we can also the finiteness of proof-search procedure.

In the following, we will consider sequent systems for modal logics with some of axioms T, D, 4, 5, and B. Their sequent system are obtained from LK by adding the following rules for the modal operator $\Box$.

$$\frac{\Gamma \to \Theta}{\Box\Gamma \to \Box\Theta}\ (SR1) \qquad \frac{\varphi, \Gamma \to \Delta}{\Box\varphi, \Gamma \to \Delta}\ (SR2) \qquad \frac{\Box\Gamma, \Gamma \to \Theta}{\Box\Gamma \to \Box\Theta}\ (SR3)$$

$$\frac{\Gamma \to \Box\Delta, \Theta}{\Box\Gamma \to \Box\Delta, \Box\Theta}\ (SR4) \qquad \frac{\Box\Gamma, \Gamma \to \Box\Delta, \Theta}{\Box\Gamma \to \Box\Delta, \Box\Theta}\ (SR5)$$

$$\frac{\Gamma \to \Box\Pi, \Theta}{\Box\Gamma \to \Pi, \Box\Theta}\ (SR6) \qquad \frac{\Box\Gamma, \Gamma \to \Box\Delta, \Box\Omega, \varphi}{\Box\Gamma \to \Box\Delta, \Omega, \Box\varphi}\ (SR7)$$

In rules $(SR6)$ and $(SR7)$, $\Box\Pi \subseteq Sub(\Gamma \cup \{\varphi\})$ and $\Box\Omega \subseteq Sub(\Box\Gamma \cup \Delta \cup \{\varphi\})$, respectively. Also we assume that Θ consists of a single formula. When we relax this condition on Θ and assume that Θ consists of *at most one formula*, we will add the superscript D to these rules, like $(SR1)^D$. This relaxation is necessary when a modal logic under consideration includes the axiom D. Also, when a rule (SRi) is assumed for a particular modal operator $\Box$, we write it as $(SRi)_\Box$, if necessary. In the following, the Gentzen style sequent system for a modal logic L is denoted by $\mathcal{G}(L)$. Here we will introduce sequent systems for some of well-known modal logics.

$\mathcal{G}(K) : (SR1)$	$\mathcal{G}(KT) : (SR1),(SR2)$	$\mathcal{G}(KB) : (SR6)$
$\mathcal{G}(K4) : (SR3)$	$\mathcal{G}(S4) : (SR2),(SR3)$	$\mathcal{G}(KTB) : (SR2), (SR6)$
$\mathcal{G}(K5) : (SR4)$	$\mathcal{G}(KD4) : (SR3)^D$	$\mathcal{G}(KDB) : (SR6)^D$
$\mathcal{G}(K45) : (SR5)$	$\mathcal{G}(KD5) : (SR4)^D$	$\mathcal{G}(KB4) : (SR7)$
$\mathcal{G}(KD) : (SR1)^D$	$\mathcal{G}(KD45) : (SR5)^D$	$\mathcal{G}(S5) : (SR2),(SR5)$

2.5 Fusion

Most of studies of modal logics until recent years are concerned with monomodal logics, i.e. modal logics with a single modal operators. We have already had many of strong and general results on monomodal logics. On the other hands, it is quite natural and necessary to introduce modal logics with many modal operators when we want to use modal logics as frameworks for describing problems in philosophy, linguistics and computer science. Such modal logics, called many-dimensional modal logics (or combined modal logics), have been one of most important topics of modal logics.

So far we have been considering modal formalisms intended for reasoning about time, knowledge, beliefs, space independently each other. There is an many applications from the fact that in reality all these entities exists in closed interaction such as knowledge, beliefs and spatial regions can change over time, agents in a multi-agent system may have their own knowledge bases, and so on. In this section, we show fusion which are able to capture such interactions.

The formation of *fusions*, or *independent joins*, is the simplest and perhaps most frequently used way of combining logics. Now, let L_1 and L_2 be two modal logics. If L_1 is axiomatized by a set of axioms Ax_1 and L_2 is axiomatized by Ax_2, then the fusion $L_1 + L_2$ [1] of L_1 and L_2 is axiomatized by the union $Ax_1 \cup Ax_2$.

And, fusion have a natural semantical interpretation as well, at least for logics which are Kripke complete. Suppose two classes $\mathcal{C}_1$ and $\mathcal{C}_2$ of m-frames and n-frames, respectively, are closed under disjoint union. The fusion $\mathcal{C}_1 + \mathcal{C}_2$ of $\mathcal{C}_1$ and $\mathcal{C}_2$ is the class of all $m + n$-frames of the form

$$\langle W, R_1, R_2, ..., R_m, R'_1, R'_2, ..., R'_n \rangle$$

such that $\langle W, R_1, R_2, ..., R_m \rangle \in \mathcal{C}_1$ and $\langle W, R'_1, R'_2, ..., R'_n \rangle \in \mathcal{C}_2$. Thus, $\mathcal{C}_1 + \mathcal{C}_2$ consists of arbitrary combinations of frames from $\mathcal{C}_1$ and $\mathcal{C}_2$ sharing the same set of worlds.

And we discuss a proof-theoretical properties of Gentzen type sequent systems for fusion of well-known monomodal logics. From the point of view of constructing theorem provers and implementing them, proof-theoretical approach to fusions will be also desirable. In the following, we will consider fusions of modal logics with some of axioms T, D, 4, 5, or B by Gentzen type sequent systems. Their sequent systems can be obtained from the sequent system LK for classical propositional logic simply by adding both of the rules for components of the fusion. For example, the system $\mathcal{G}$(S4) consists of rules of LK,

$$\frac{\varphi, \Gamma \to \Delta}{\Box\varphi, \Gamma \to \Delta} \quad \text{, and} \quad \frac{\Box\Gamma, \Gamma \to \Theta}{\Box\Gamma \to \Box\Theta} \ ,$$

and the system $\mathcal{G}$(S5) consists of rules of LK,

[1] The fusion can be also denoted as $L_1 \otimes L_2$.

$$\frac{\varphi, \Gamma \to \Delta}{\Box\varphi, \Gamma \to \Delta} \quad \text{, and} \quad \frac{\Box\Gamma, \Gamma \to \Box\Delta, \Theta}{\Box\Gamma \to \Box\Delta, \Box\Theta} \ .$$

Other sequent systems for fusions can be also constructed in the similar way. We can easily show the following proposition 4.

Proposition 4 *Let L_1 and L_2 be of the form $K \oplus Q$ where $Q \subseteq \{T, D, 4, 5, B\}$. Then $L_1 + L_2 \vdash \varphi$ iff $\mathcal{G}(L_1 + L_2 \vdash\Rightarrow \varphi)$.*

The rest of this section presents the proof-theoretical property for fusion. Then the following holds.

Thorem 2 *(cut elimination theorem for fusion) Let $L_1 + L_2$ be any of K, K4, K45, KT, S4, KD, KD4, and KD45. Then the sequent system $\mathcal{G}(L_1 + L_2)$ for each fusion of their logics has the cut elimination property.*

This theorem can be proved by Gentzen's method for any logics of them, and similarly to the case with the each logic since each modality is independent of another one. It is difficult to prove a cut-elimination property when a logic contains the axiom 5, however, it for K45 and KD45 is proved in [54]. The sequent system $G(S5)$ defined above lacks the cut elimination property, but in [52], a cut-free Gentzen type sequent system for the modal logic S5 is introduced by means of eliminating the visible cut one by one. As to the logics K5 and KD5, cut-free systems are known yet.

An important consequence of cut elimination theorem for a sequent system $\mathcal{G}$ is the subformula property. That is, is a sequent $\Gamma \to \Delta$ is provable in $\mathcal{G}$ then it is has a proof which consists only of sequents containing subformulae of formulae in $\Gamma \cup \Delta$. In fact, most of outcomes of cut elimination theorem, including decidability, can be derived from the subformula property. In standard sequent systems, only cut rule violates the subformula property, i.e. the cut formula in a given application of cut rule may not appear in the lower sequent. In other words, if we can restrict any application of cut rule to the following way,

$$\frac{\Gamma \to \Delta, \varphi \quad \varphi, \Pi \to \Sigma}{\Gamma, \Pi \to \Delta, \Sigma}, \quad \text{where } \varphi \in \mathit{Sub}(\Gamma \cup \Pi \cup \Delta \cup \Sigma),$$

then we can obtain the subformula property. We call such cut rule, *acceptable cut*. Takano succeeded to show that including KB, KTB, KDB, KB4, S5, every provable sequent has a proof in which every rule is acceptable [55, 56]. In [56], he show the cut restriction property for K5 and KD5 by using extension of the acceptable cut rule. It is called that the sequent system $\mathcal{G}$ has the *cut restriction property*, if every proof in $\mathcal{G}(L)$ can be transformed, without changing the end sequent, into the proof in which every cut rule applied in it is acceptable. Note that every system with the cut elimination property has the cut restriction property. By means of some method of derivation of the cut restriction property for KB, KTB, KDB, KB4, S5, K5, and KD5 which was shown by Takano, the following cut restriction theorem can be shown.

Thorem 3 *(cut-restriction theorem for fusion) Let L_1 and L_2 be of the form $K + Q$ where $Q \subseteq \{T, D, 4, 5, B\}$. Then the sequent system $\mathcal{G}(L_1 + L_2)$ for each fusion of these logics has the cut restriction property, i.e. has the subformula property.*

This theorem can be shown through Kripke semantics; i.e. for logics L_1 and L_2 which are of the form $K + Q$ where $Q \subseteq \{T, D, 4, 5, B\}$, the restricted system for $L_1 + L_2$ is determined by $L_1 + L_2$-frame, and similarly to the case with the each logic since each modality is independent of another one.

Chapter 3

Multi-agent System with Tense Logic

3.1 Introduction

In this chapter, we show an application for the tense logic. Lately, for a study of the multi-agent system, tense logic is utilized to represent an update of the agent's belief states. An agent is an autonomous computer system that perceives information from surrounding environments and takes relevant actions. Such an agent has been formalized in terms of logic as a rational agent [43, 49, 50, 66, 68], especially in temporal epistemic logic [37, 40, 64, 67]. BDI (belief-desire-intention) logic is a result of such effort, though it mainly treats an epistemic state of an isolated single agent; thus, it is rather clumsy to handle interaction of epistemic states in multiple agents.

One of the most important issues in multi-agent system is interaction, or communication, that may directly affect their epistemic states [11, 29]. Thus far, several models which include the notion of communication have rather naïvely rendered that agents are always communicable, i.e., that channels between them are omnipresent. However, in practical cases, communication is not free. We should consider that reliable channels exist only between certain agents at certain time.

The purpose of this chapter is to show a multi-agent system with tense logic and introduce a logic to treat epistemic states of multiple agents where channels are unevenly distributed. We show that in this logic we can decide whether an agent would come to know a formula of certain information after iterated communications through channels, updating her epistemic state.

In this chapter, at first, we formalize communication between agents, based on *inform* in ACL (Agent Communication Language) defined by FIPA (Foundations of Intelligent Physical Agents) [20, 38]. In this definition, its pre-condition and post-condition only concern an agent's epistemic states. Then, we revise the idea so as to declare a channel explicitly between agents in the pre-condition.

Secondly, we introduce a temporal epistemic logic system CB_{CTL} for reasoning agent's epistemic states, based on CTL (Computational Tree Logic) [25]. The logic has the branching time, so that each agent may have different epistemic state in future. Although CB_{CTL} may be regarded as a variant of BDI logic, we handle only beliefs of multiple agents. The reasoning system evaluates the truth value of formulae in logical model basically in Kripke semantics. However, the system also evaluates the future possibility for the epistemic state. For such a future belief, CB_{CTL} may add a state that corresponds to a time in a possible world, which is spliced on some branch, as a result of a communication.

Thus, the following inference would be feasible.

> Agent α has a belief φ at time t, Agent β does not have a belief about φ, and these two agents can communicate each other. When we assume the presence of a channel between α and β (since α can inform β of φ), the sentence "β can hold belief φ at the next time of t" is true at time t.

Thirdly, we implement the above reasoning system on a computer in Prolog, and show that the system would decide the veridicality of such belief update in finite time, presenting examples.

In the following section, we propose a formalization of communication for our logic CB_{CTL}, and define the syntax and Kripke semantics. In Section 3.3, we show the inference system and its decidability. In Section 3.4, we show a prover for our logic and some examples by our computer system. In the final section, we discuss some branching points of our theory and summarize our contribution of this chapter.

3.2 Logic of Agent's Epistemic State with Communication Channel

3.2.1 Preliminaries

In this section, we introduce a temporal epistemic logic with communication channel CB_{CTL}, based on computational tree logic (CTL) [16, 17, 18]. Generally, when we consider multi-agent models, it is appropriate to include the branching time.

The language of CTL consists of propositional temporal operator EX, AU and EU. All are formed by a pair of symbols. The first symbols (A or E) are quantifiers and the second pair X and U mean 'next' and 'until', respectively. We define the abbreviated notations as follows:

$$\begin{array}{lllllllllll} AX\varphi & \equiv & \neg EX\neg\varphi & & EF\varphi & \equiv & E(\mathrm{true}U\varphi) & & AF\varphi & \equiv & A(\mathrm{true}U\varphi) \\ EG\varphi & \equiv & \neg AF\neg\varphi & & AG\varphi & \equiv & \neg EF\neg\varphi & & & & \end{array}$$

Where the second pair G and F mean "some future state" and "all future states."

We introduce Kripke semantics for CTL. The triple (S_t, R, L) is called a *Kripke model* for CTL, where S_t is a nonempty-set of states, $R(\subseteq S_t \times S_t)$ is a binary relation on S_t, and L is a mapping such that $L(t) \subseteq S_t$ for each propositional variable φ, where $t \in S_t$ is a state. For an infinite sequence of states, $t_0, t_1, ...$ is called a *path* on w starting with t_0 such that $t_0 R_w t_1, t_1 R_w t_2, ...$ Then a binary relation '$\models$' is defined as follows:

$$\begin{array}{lll} t \models \varphi & \Longleftrightarrow & \varphi \in L(t) \\ t \models \varphi \wedge \psi & \Longleftrightarrow & t \models \varphi \text{ and } t \models \psi \\ t \models \varphi \vee \psi & \Longleftrightarrow & t \models \varphi \text{ or } t \models \psi \\ t \models \varphi \rightarrow \psi & \Longleftrightarrow & t \models \varphi \text{ implies } t \models \psi \\ t \models \neg\varphi & \Longleftrightarrow & \text{not } t \models \varphi \\ t \models EX\varphi & \Longleftrightarrow & \exists\sigma \in P(t), \sigma[1] \models \varphi \\ t \models A[\varphi U\psi] & \Longleftrightarrow & \forall\sigma \in P(t).\exists i \geq 0, \sigma[i] \models \psi \wedge (\forall 0 \leq j \leq i.\sigma[j] \models \varphi) \\ t \models E[\varphi U\psi] & \Longleftrightarrow & \exists\sigma \in P(t).\exists i \geq 0, \sigma[i] \models \psi \wedge (\forall 0 \leq j \leq i.\sigma[j] \models \varphi) \end{array}$$

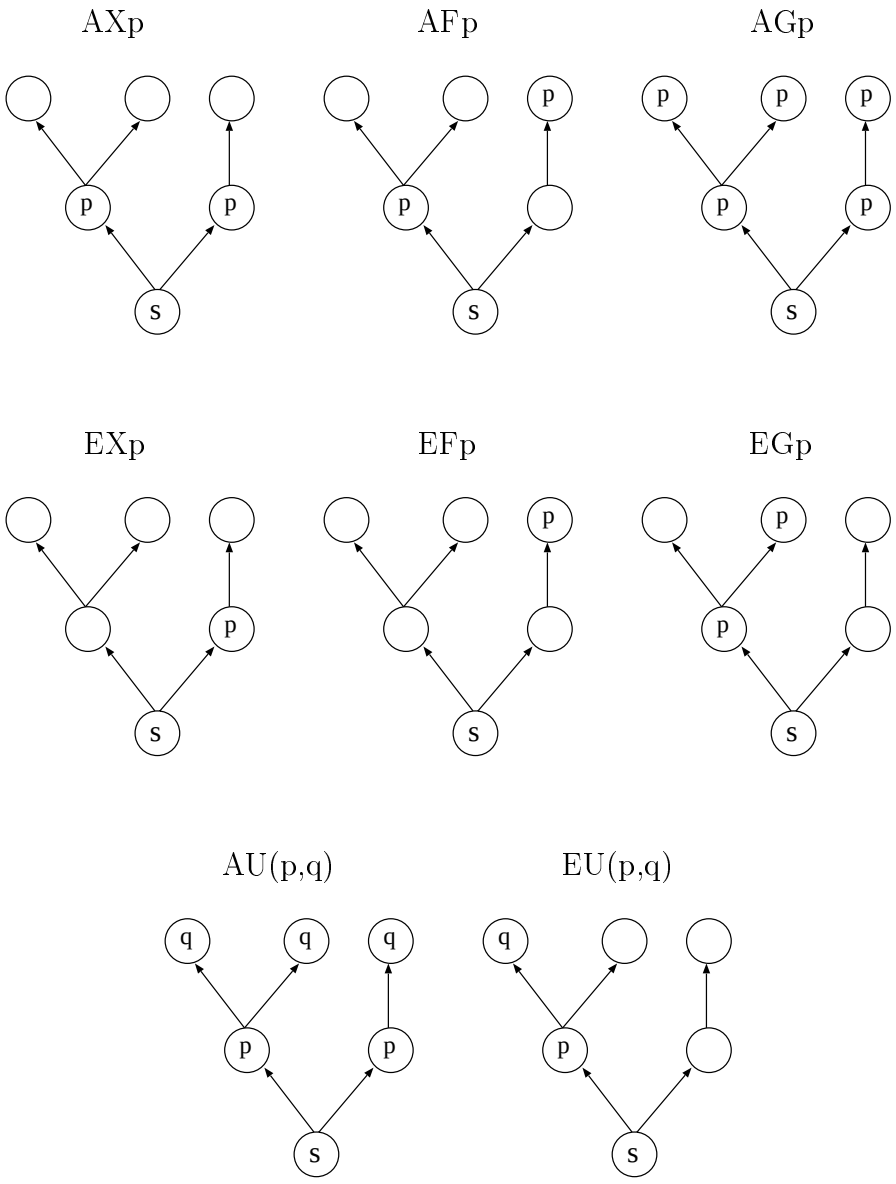


Figure 3.1: Example of the Computational Tree Logic

Where $P(t)$ is a set of paths starting with t and $\sigma[i]$ is a i^{th} element of σ.

3.2.2 Revision of *inform**

At first, we formalize the communication between agents. The *inform* of ACL/ FIPA is well known as an existing formalization of communication between agents [20, 43]. A definition of this *inform* is given as follows:

Definition 3 [*inform*]
$\langle \alpha,$ *inform*$(\beta,\varphi)\rangle$
feasibility pre-condition: $B_\alpha\varphi \wedge \neg B_\alpha(Bif_\beta\varphi \vee Uif_\beta\varphi)$
rational effect: $B_\beta\varphi$

where $Bif_\beta\varphi$ and $Uif_\beta\varphi$ are abbreviation of '$B_\beta\varphi \vee B_\beta\neg\varphi$' and '$U_\beta\varphi \vee U_\beta\neg\varphi$', respectivly. a formula $B_\beta\varphi$ is read as "Agent β believes φ," and a formula $U_\beta\varphi$ is read as "Agent β is uncertain about φ, but thinks that φ is more likely than $\neg\varphi$."

We add the concept of a communication pathway, or *channel* [6], and a time progress in the above definition, revising its pre-condition and post-condition.

In this study, we exclude the epistemic operator U because there is no sound formalization of the U, though the later extensibility is preserved as we will discuss in Section 3.5. At this stage, we use only an epistemic operator B_α. Then we define our extended inform *inform** as follows:

Definition 4 [*inform**]
$\langle \alpha,$ *inform*$^*(\beta,\varphi)\rangle$
feasibility pre-condition: $B_\alpha\varphi \wedge \neg B_\alpha(Bif_\beta\varphi) \wedge C_{\alpha\beta}$
rational effect: $X(B_\beta\varphi)$

Here, X is a temporal operator meaning "next state." $C_{\alpha\beta}$ is a member of propositional variables, let us read $C_{\alpha\beta}$ to mean *"there is a communication channel from Agent α to Agent β."* That is, $C_{\alpha\beta}$ is not equivalent to $C_{\beta\alpha}$. On the research of multi-agent finite state machines (MAFSM) [8], a network (a communication protocol) of FIPA is defined as a propositional atom[1] . In contrast, on the research of the social commitment of multi-agent system, ALBATROSS (Agent language Base on the Treatment of Social Semantics) [12, 13] is proposed, which includes the modal operator $C_{\alpha\beta}$ as *commitment* that is defined in advance. In this paper, we define the Kripke model with communication based on CTL. If we define a communication protocol as a modal operator, it is necessary to define its conditions for all the states of all the possible worlds. Avoiding such messy complications of modalities, we define a communication channel as a proposition and *inform** as an action. That is, we deal with a communication protocol as a knowledge included in agent's epistemic states. We will discuss other options in Section 3.5. We give the example of the telephone game of a communication in a row as follows:

[1] As in [8], $Do(put\text{-}msg(inform(\alpha,\beta,\varphi)))$ is a propositional atom of the language of α which is true only if α is in a state in which it has just performed the action "put-msg(inform(α,β,φ))," where (inform(α,β,φ))$\equiv\langle\alpha,$ inform$(\beta,\varphi)\rangle$.

Example 2 [*telephone game*]
$\langle \alpha_1,\ inform^*(\alpha_n, \varphi)\rangle$

feasibility pre-condition: $\bigwedge_{2\le m\le n} B_{\alpha_{m-1}}\varphi \wedge \neg B_{\alpha_{m-1}}(Bif_{\alpha_m}\varphi) \wedge C_{\alpha_{m-1}\alpha_m}$

rational effect: $\bigwedge_{2\le m\le n} X^{m-1}(B_{\alpha_m}\varphi)$

where $n \geq 2$ *and* X^m *stands for repeating* X m *times.*

3.2.3 Syntax

We introduce a temporal epistemic logic system CB_{CTL} for reasoning agent's epistemic states with communications. In this logic, an agent's epistemic state is modified by one time step per a communication. Therefore, the temporal operator is restricted only to the *next* operator X.

Definition 5 (Signature) *The language* L_{CB} *consists of the following vocabulary.*

P	*a set of propositional variables*
C	*a set of communication channels*
$Agent$	*a set of agents*

In addition, the following symbols are used.

$\neg, \vee$	*logical connectives*
EX	*propositional temporal operator*
B_α	*propositional epistemic operator where* $\alpha \in Agent$

Parentheses and punctuation are added if necessary.

We use $\varphi, \psi, \chi, \cdots$ for propositional variables and $\alpha, \beta, \cdots$ for agents.

Definition 6 (Formula) *Formulae, denoted by* $\varphi, \psi, ...$, *are constructed in the usual way from propositional variables, logical connectives and operators. In particular,* $EX\varphi$ *and* $B_\alpha\varphi$ *are formulae when* φ *is a formula. And we treat* C *the same as* P.

And we define the abbreviated notations as follows:

- $\varphi \wedge \psi \equiv \neg(\neg\varphi \vee \neg\psi)$
- $\varphi \rightarrow \psi \equiv \neg\varphi \vee \psi$
- $\varphi \Leftrightarrow \psi \equiv (\varphi \rightarrow \psi) \wedge (\psi \rightarrow \varphi)$
- $AX\varphi \equiv \neg EX\neg\varphi$

3.2.4 Semantics of CB_{CTL}

Similar to other Kripke semantics, we give a Kripke model to CB_{CTL}. A model M is such a tuple that $M = \langle W, St_w, R_w, B_\alpha, V\rangle$, where W is a set of possible worlds, St_w is a set of states for each $w \in W$, R_w is a se of temporal relations for each $w \in W$, and B_α is a set of the accessbility relations; if $(w, t, w') \in B_\alpha$ and $t \in St_w$ then $t \in St_{w'}$. And V is a *valuation* such that $V(w,t) = L(w,t) \cup CL(w,t)$, where L is a valuation for propositional variable such that $L(w,t) \subseteq P$ for each $w \in W, t \in St_w$, and CL is a valuation for communication channels such that $CL(w,t) \subseteq C$ for all $w \in W, t \in St_w$. A binary relation '$\models$' is defined inductively as follows:

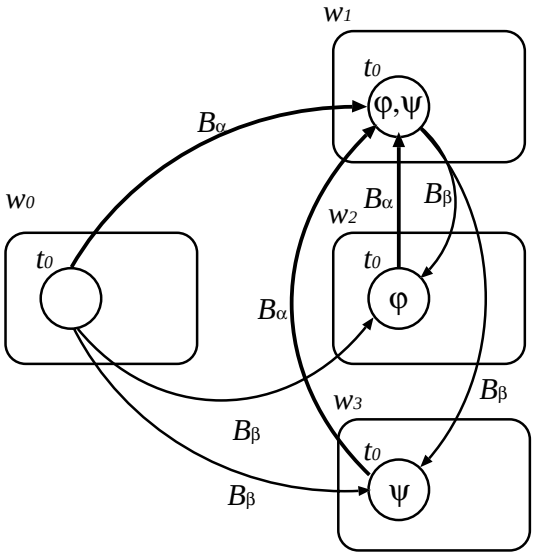


Figure 3.2: Example of a Kripke model

$$
\begin{array}{lcl}
(M,w,t) \models \varphi & \Longleftrightarrow & \varphi \in V(w,t) \\
(M,w,t) \models \neg\varphi & \Longleftrightarrow & \text{not } (M,w,t) \models \varphi \\
(M,w,t) \models \varphi \vee \psi & \Longleftrightarrow & (M,w,t) \models \varphi \text{ or } (M,w,t) \models \psi \\
(M,w,t) \models B_\alpha\varphi & \Longleftrightarrow & \forall w'\{(w,t,w') \in B_\alpha \rightarrow (M,w',t) \models \varphi\} \\
(M,w,t) \models EX\varphi & \Longleftrightarrow & \exists t' \text{ such that } (t,t') \in R_w \text{ and } (M,w',t) \models \varphi
\end{array}
$$

Here, we show an example of Kripke model.

Example 3 *Let* $M = \langle W, St_w, R_w, B_\alpha, V\rangle$ *be given as follows:*

(1) $W = \{w_0, w_1, w_2, w_3\}$
(2) $\forall w \in W,\ St_w = \{t_0\}$
(3) $\forall w \in W,\ R_w = \emptyset$
(4) $B_\alpha = \{(w_0, t_0, w_1), (w_2, t_0, w_1), (w_3, t_0, w_1)\}$
(5) $B_\beta = \{(w_0, t_0, w_2), (w_0, t_0, w_3), (w_1, t_0, w_2), (w_1, t_0, w_3)\}$
(6) $L = \{(w_1, t_0, \varphi), (w_1, t_0, \psi), (w_2, t_0, \varphi), (w_3, t_0, \psi)\}$
(7) $C = \{(w_0, t_0, C_{\alpha\beta}), (w_1, t_0, C_{\alpha\beta}), (w_2, t_0, C_{\alpha\beta}), (w_3, t_0, C_{\alpha\beta})\}$

Then, we show this model in Fig.3.2. Here, $(M, w_0, t_0) \models B_\alpha(\varphi \wedge \psi)$ *and* $(M, w_0, t_0) \models \neg B_\beta(\varphi \vee \psi)$ *are true.*

3.3 Reasoning System with *inform**

We propose a reasoning system for CB_{CTL}. This reasoning system evaluates truth values of logical formulae in Kripke semantics. Since the communication is included in the reasoning process, the result would differ from that of usual evaluation in the model. That is, we need to add a new state in each world, that is a progress of one unit time, as a result of a communication. In the new state, newly validated formulae are included as well as existent ones.

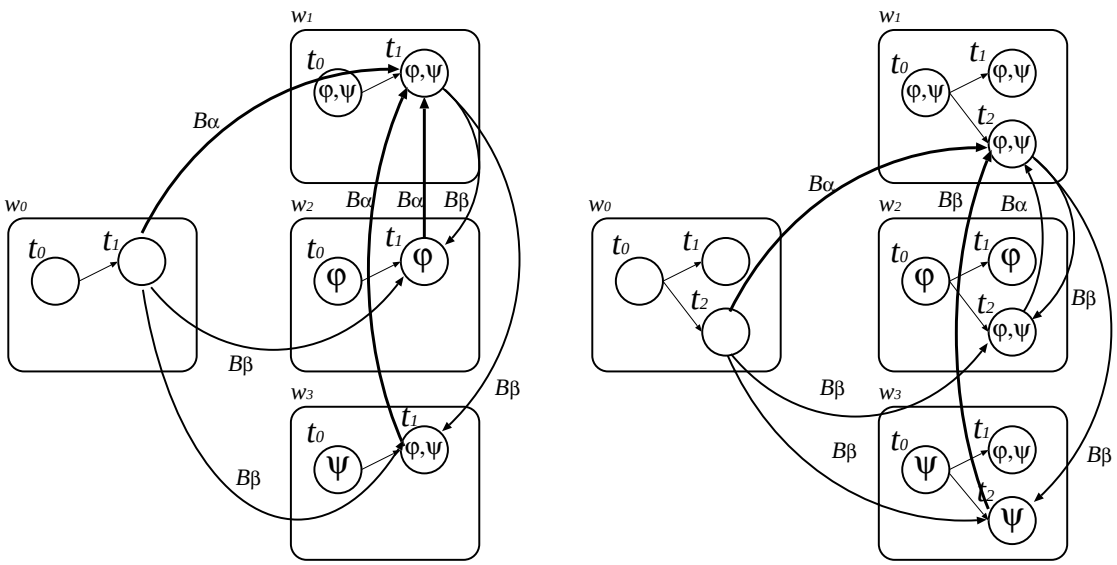


Figure 3.3: Example of communication from α to β

3.3.1 Example of Communication and Reasoning

In this section, we show an example of a communication in which the epistemic states of agents change by the communication. For example, in Fig.3.2, because $(M, w_0, t_0) \models B_\alpha \varphi \wedge \neg B_\alpha(Bif_\beta \varphi) \wedge C_{\alpha\beta}$ and $(M, w_0, t_0) \models B_\alpha \psi \wedge \neg B_\alpha(Bif_\beta \psi) \wedge C_{\alpha\beta}$, it is possible to communicate φ from α to β and also be ψ. So, we can evaluate whether $(M, w_0, t_0) \models EXB_\beta \varphi$ and $(M, w_0, t_0) \models EXB_\beta \psi$. We show the result of reasoning for φ and ψ in Fig.3.3.

3.3.2 Rules of the Reasoning System

We define some rules of model as actions. Each rule functions as a user command on the reasoning system on a computer.

Rule 1 ($inform^*$)
$\langle inform^*(w, t, \alpha, \beta, \varphi) \rangle$
feasibility pre-condition: $B_\alpha \varphi \wedge \neg B_\alpha(Bif_\beta \varphi) \wedge C_{\alpha\beta}$, *current time* $= t$
rational effect: $AXB_\beta \varphi$, *current time* $= t'$

where $w \in W$, $t \in St_w$, $\alpha, \beta \in Agent$ and $\varphi \in P$. For Rule 1, the system executes the following procedure:

1: $\forall w \in W$, add new state t' and new relation R_w such that $tR_w t'$.
2: $\forall w \in W$, delete all state $t_0, t_1, ...$ such that $tR_w t_0, t_0 R_w t_1, ...$
3: $\forall w \in W$, $L(v, t') := L(v, t)$
4: $\forall w' \in W$ such that $(w, t, w') \in B_\beta$, add a valuation to $L(w', t')$.
5: current time $:= t'$

By the rule $inform^*$, some agent sends information to other agent.

Rule 2 (*Add a communication channel*)
$\langle add\_cc(t, \alpha, \beta)\rangle$
feasibility pre-condition: $\neg C_{\alpha\beta}$, *current time* $= t$
rational effect: $AXC_{\alpha\beta}$, *current time* $= t'$

For Rule 2, the system executes the procedure as follows:

1-3: i.q. Rule1
4: $\forall w \in W$, add a valuation to $CL(w, t')$ for $C_{\alpha\beta}$.
5: i.q. Rule1

The rule *add_cc* adds a new communication channel.

Rule 3 (*Delete a communication channel*)
$\langle del\_cc(t, \alpha, \beta)\rangle$
feasibility pre-condition: $C_{\alpha\beta}$, *current time* $= t$
rational effect: $AX\neg C_{\alpha\beta}$, *current time* $= t'$

For Rule 3, the system executes the procedure as follows:

1-3: i.q. Rule1
4: $\forall w \in W$, delete a valuation to $CL(w, t')$ for $C_{\alpha\beta}$.
5: i.q. Rule1

The rule *del_cc* deletes some current communication channel.

We can use the above rules for given L_{CB}, Kripke model, and current time(state). Here, we show an example for these rules.

Example 4 *Let Kripke model* $M = \langle W, St_w, R_w, B_\alpha, V\rangle$ *and current time be given as follows:*

(1) $W = \{w_0, w_1, w_2, w_3\}$
(2) $\forall w \in W$, $St_w = \{0, 1\}$
(3) $\forall w \in W$, $R_w = \{(0, 1)\}$
(4) $B_\alpha = \{(w_0, 0, w_1), (w_2, 0, w_1), (w_3, 0, w_1)\}$
(5) $B_\beta = \{(w_0, 0, w_2), (w_0, 0, w_3), (w_1, 0, w_2), (w_1, 0, w_3)\}$
(6) $L = \{(w_1, 0, \varphi), (w_2, 0, \varphi)\}$
(7) $C = \{(w_0, 0, C_{\alpha\beta}), (w_1, 0, C_{\alpha\beta}), (w_2, 0, C_{\alpha\beta}), (w_3, 0, C_{\alpha\beta}),$
$(w_0, 1, C_{\alpha\beta}), (w_1, 1, C_{\alpha\beta}), (w_2, 1, C_{\alpha\beta}), (w_3, 1, C_{\alpha\beta})\}$
(8) *current time = 0*

We show this situation in the left-hand side of Fig.3.4. For (M,w_0,0), *inform**(w_0,0,α,β,φ) updates the situation to be shown in the right-hand side of Fig.3.4. That is, both of α and β come to believe φ by a communication, because the formula φ and communication channel $C_{\alpha\beta}$ are included in the state 2 of the possible worlds w_1, w_2 and w_3. In particular, the epistemic state of β is updated by *inform**. For (M,w_0,2) in the right-hand side of Fig.3.4, by *del_cc*(1,α,β), The situation is updated as is shown in the left-hand side of Fig.3.5. That is, in the state 3, the communication channel is deleted between α and β. For (M,w_3,3) in the left-hand side of Fig.3.5, by *add_cc*(2,α,β), the situation changes as is shown in the right-hand side of Fig.3.5. That is, in the state 4, a communication channel is added between α and β.

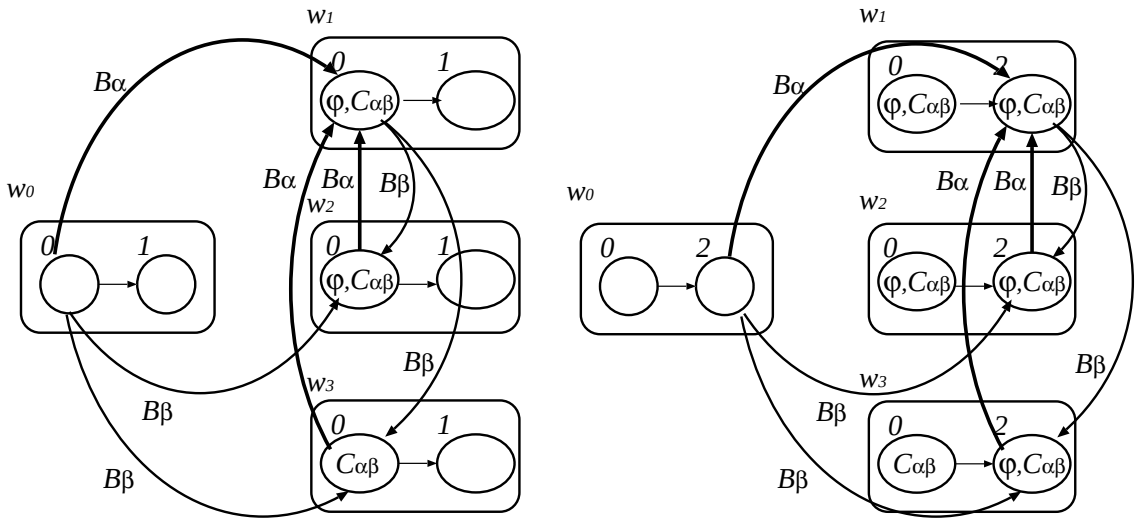


Figure 3.4: Example of the rule $inform^*$

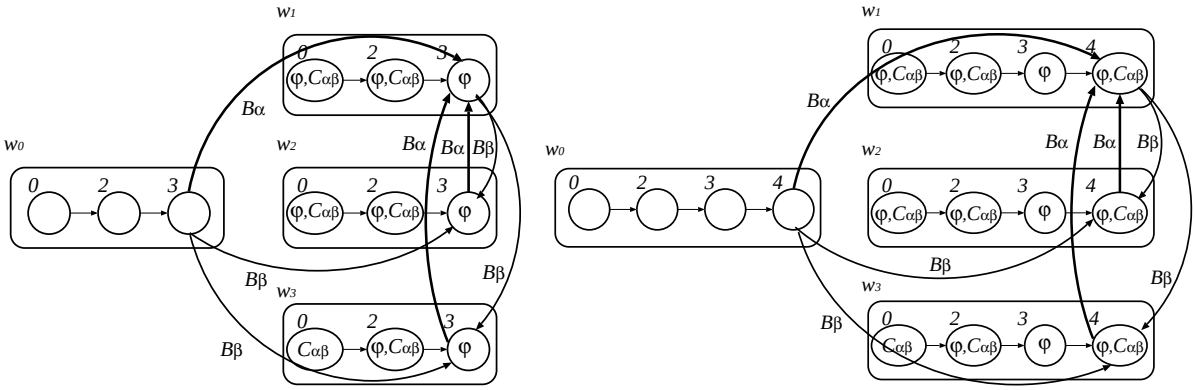


Figure 3.5: Example of the rule del_cc and the rule add_cc

3.3.3 Syntax-sensitive Rules

Now, we give rules for the following formulae:

(a) $EXB_\alpha(\varphi \vee \psi)$
(b) $EXB_\alpha B_\beta(\varphi)$
(c) $EXB_\alpha EX\varphi$
(d) $EX(B\alpha\varphi \vee B\beta\psi)$

For (a),(b),(c), and (d), we apply the following Rule 4, Rule 5, Rule 6, and Rule 7 to the model, respectively.

For the above case (a), we need to classify multiple valuations at the same state, that is, for tuple (φ,ψ), we need to prepare the evaluations (true,true), (true,false), and (false,true) at the same time for each possible world. However, these multiple states

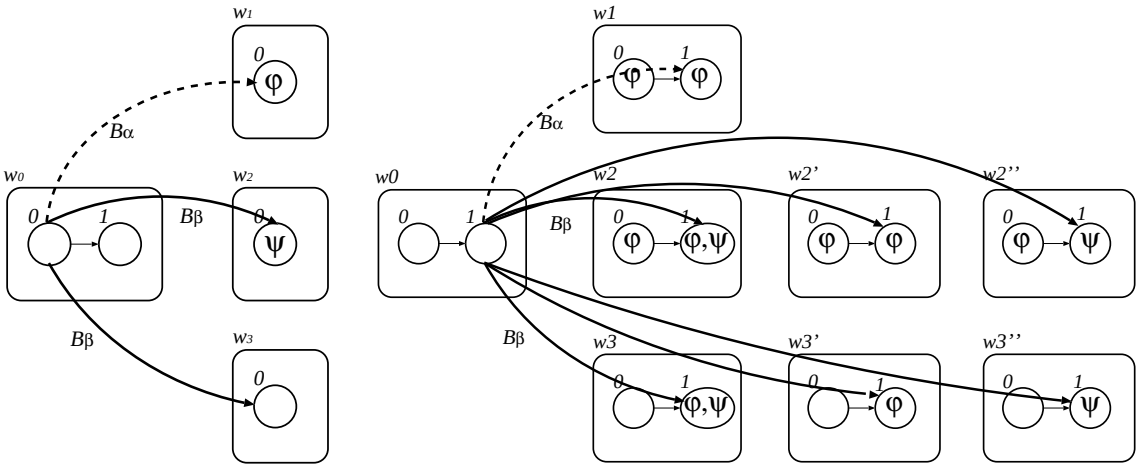


Figure 3.6: Example of the communication with logical disjunction

cannot coexist at the same time in each possible world. To deal with this case, we supply new possible worlds by an increment of state. We define the following rule for this case.

Rule 4 (*Communication with disjunction*)
(1): $\forall w \in W$*, add a new a state* t' *and* R_w *such that* tR_wt'
(2): $\forall w_n$ *such that* $(w_0, t, w_n) \in B_\alpha$*, add* $w_{n'}$ *and* $w_{n''}$*, equalize states, temporal relation and valuation in* w_n*,* $w_{n'}$ *and* $w_{n''}$*.*
(3): Define $(w_0, t', w_{n'}) \in B_\alpha$ *and* $(w_0, t', w_{n''}) \in B_\alpha$*, and for each* w_m *such that* $(w_n, t, w_m) \in B_\alpha$*, define* $(w_{n'}, t, w_m) \in B_\alpha$ *and* $(w_{n''}, t, w_m) \in B_\alpha$*.*
(4): Update a valuation such that $(w_n, t') \models \varphi \wedge (w_n, t') \models \psi$*,* $(w_{n'}, t') \models \varphi \wedge (w_{n'}, t') \models \neg\psi$*, and* $(w_{n''}, t') \models \neg\varphi \wedge (w_{n''}, t') \models \psi$*.*

This rule is applied when a message includes a logical disjunction. The system evaluates the above formula (a) at state t in a possible world w_0 and performs the above procedures. For example, for a given model as in the left-hand side of Fig.3.6, a result of sending a message $\varphi \vee \psi$ from α to β is shown in the right-hand side of Fig.3.6. In this case, we need to provide extra possible worlds $w_2', w_2'', ...$, and so on.

Rule 5 (*Transfer of accsessbility relation*)
For $EXB_\alpha B_\beta(\varphi)$*, A valuation is changed to satisfy that* φ *is consisted in the possible worlds of* B_β*-accessible world from the possible worlds of* B_α*-accessible world.*

Rule 6 (*Identification of structure*)
In our reasoning system, for all possible worlds, we identify the tree structure of states with a temporal relation. Therefore, $EXB_\alpha\varphi \Leftrightarrow B_\alpha EX\varphi$*.*

Rule 7 (*Distributive principle of modality*)
In accordance with the semantics given in Section 3.2.4,

$$(M, w, t) \models EX(\varphi \vee \psi) \Leftrightarrow (M, w, t) \models EX\varphi \vee EX\psi.$$

Note that through Rule 4-7, all the formulae are reduced to the following form:

$$\underbrace{EX \cdots EX}_{n \ times} B_\alpha \varphi.$$

Finally, we summarize the rule for those with the negative connective. As for $EXB_\alpha \neg\varphi$, we just need to change the truth value of φ in each newly-added state. In case $EX\neg B_\alpha\varphi$, we can prove if φ is true in every B_α-accessible world after the communication.

3.3.4 Decidability

In case there is no communication in agents, the proof of the veridicality of a formula is same as the usual process. As a formula is decomposed into a finite number of subformulae and is reduced to a finite number of atomic propositions with logical connectives, all these subformulae could be given truth values in finite steps.

In case a subformula includes a communication, that is, the subformula may be headed by multiple EX's in front of $B_\alpha\varphi$. First, within the precondition of $inform^*$ there is no communication. Thus, as far as the number of EX is finite, the veridicality of the precondition is judged in finite steps. Because the number of addition of new states in each world is equal to or less than the number of EX's, if the number of possible worlds is finite then such addition of new states necessarily halts. Note that a new state is added according to the progress of time, i.e., the occurrence of communication in each world, so that there is no loop in each world.

3.4 A Model Checker for CB_{CTL}

The emulator of L_{CB} was implemented in Prolog (SWI-Prolog [63]) on Solaris 5.7, on SUN^{TM} Sparc station Ultra 5-10. In this section, we show several results of a model cheking.

A formula $EX \cdots EXB_\alpha\varphi$ is evaluated false at a current time t, but would be also evaluated true at a current time $t'(> t)$. Our system, for an above case, evaluates a truth value and outputs a result in the future. Based on the disjunction in Section 3.3.4, our system assesses inductively by using the following criteria.

Rule 8 (*Model checking*)
$(M, w, t) \models EX \cdots EXB_\alpha\varphi$ *if each one of the channels* $C_{\alpha_1\alpha_2}$,$C_{\alpha_2\alpha_3}$, $\cdots$, $C_{\alpha_n\alpha}$, *is true at* t, $B_{\alpha_1}\varphi$ *is true, and* $\neg B_{\alpha_1}(Bif_{\alpha_2}\varphi)\wedge \cdots \wedge \neg B_{\alpha_n}(Bif_\alpha\varphi)$ *is true.*

Here, we show an example by using this rule for telephone game.

Example 5 *Let* $\alpha, \beta, \gamma \in Agent$, α *and* β *has a communication channel* $C_{\alpha\beta}$, *and also* β *and* γ. α *has a belief* φ. *Then, will* γ *have a belief* φ *in the future?*

To describe the above situation, we declare the following model:

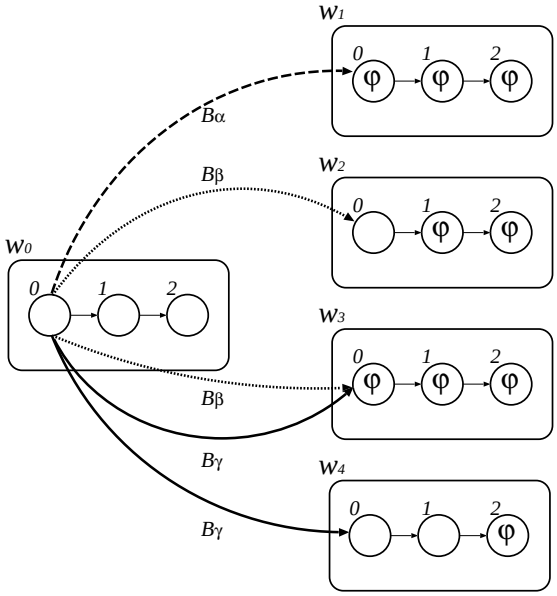


Figure 3.7: Example of a telephone game

(1) $W = \{w_0, w_1, w_2, w_3, w_4\}$
(2) $\forall w \in W,\ St_w = \{0\}$
(3) $\forall w \in W,\ R_w = \emptyset$
(4) $B_\alpha = \{(w_0, 0, w_1), (w_2, 0, w_1), (w_3, 0, w_1), (w_4, 0, w_1)\}$
(5) $B_\beta = \{(w_0, 0, w_2), (w_0, 0, w_3), (w_1, 0, w_2), (w_1, 0, w_3), (w_4, 0, w_2), (w_4, 0, w_3)\}$
(6) $B_\gamma = \{(w_0, 0, w_3), (w_0, 0, w_4), (w_1, 0, w_3), (w_1, 0, w_4), (w_2, 0, w_3), (w_2, 0, w_4)\}$
(7) $L = \{(w_1, 0, \varphi), (w_3, 0, \varphi)\}$
(8) $C = \{(w_0, 0, C_{\alpha\beta}), (w_1, 0, C_{\alpha\beta}), (w_2, 0, C_{\alpha\beta}), (w_3, 0, C_{\alpha\beta}), (w_4, 0, C_{\alpha\beta}), (w_0, 0, C_{\beta\gamma}), (w_1, 0, C_{\beta\gamma}), (w_2, 0, C_{\beta\gamma}), (w_3, 0, C_{\beta\gamma}), (w_4, 0, C_{\beta\gamma})\}$
(8) *current time = 0*

In this situation, we get an answer that $(M, w_0, 0) \not\models EXB_\gamma\varphi$ *and* $(M, w_0, 0) \models EXEXB_\gamma\varphi$. *And this model is updated as follows:*

(2′) *For all* $w \in W,\ St_w = \{0, 1, 2\}$
(3′) *For all* $w \in W,\ R_w = \{(0, 1), (1, 2)\}$
(7′) $L = \{(w_1, 0, \varphi), (w_3, 0, \varphi), (w_1, 1, \varphi), (w_2, 1, \varphi), (w_3, 1, \varphi), (w_1, 2, \varphi), (w_2, 2, \varphi), (w_3, 2, \varphi), (w_4, 2, \varphi)\}$

We show this situation in Fig.3.7.

Next, we show an example of the exploration activity.

Example 6 *Let* $\alpha, \beta, \gamma \in Agent$, *there are communication channels between every agents. Now,* α *is a researcher on the earth,* β *and* γ *are probes on the Mars. Then, a communication channel between* α *and* β *is disconnected. So, a researcher sends an order "self-repair"*

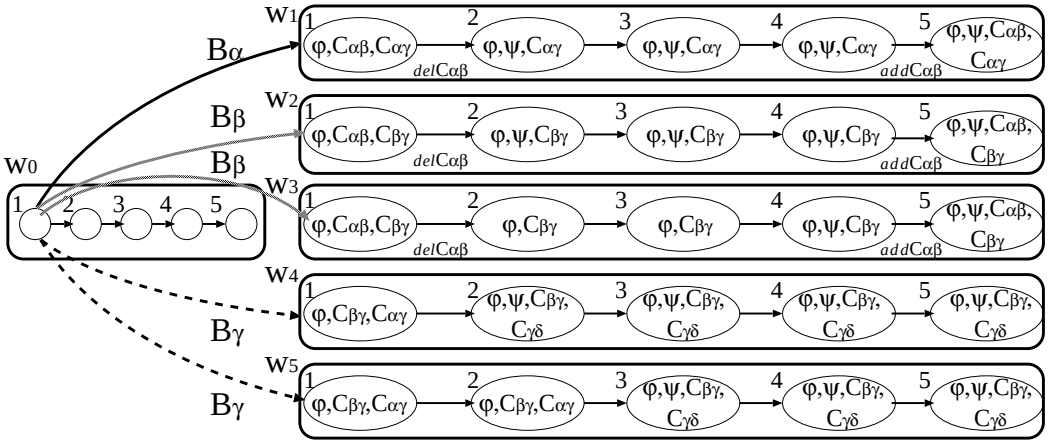


Figure 3.8: Example of a self-repair and a connection recovery

to γ because γ is connected with β. To repair the connection, how many steps does it take?

To describe the above situation, we declare the following model:

(1) $W = \{w_0, w_1, w_2, w_3, w_4, w_5\}$
(2) $\forall w \in W,\ St_w = \{0\}$
(3) $\forall w \in W,\ R_w = \emptyset$
(4) $B_\alpha = \{(w_0, 0, w_1), (w_2, 0, w_1), (w_3, 0, w_1), (w_4, 0, w_1), (w_5, 0, w_1)\}$
(5) $B_\beta = \{(w_0, 0, w_2), (w_0, 0, w_3), (w_1, 0, w_2), (w_1, 0, w_3), (w_4, 0, w_2),$
$(w_4, 0, w_3), (w_5, 0, w_2), (w_5, 0, w_3), \}$
(6) $B_\gamma = \{(w_0, 0, w_4), (w_0, 0, w_5), (w_1, 0, w_4), (w_1, 0, w_5), (w_2, 0, w_4),$
$(w_2, 0, w_5), (w_3, 0, w_4), (w_3, 0, w_5), \}$
(7) $L = \{(w_1, 0, \varphi), (w_2, 0, \varphi)(w_3, 0, \varphi), (w_4, 0, \varphi), (w_5, 0, \varphi), \}$
(8) $C = \{(w_0, 0, C_{\alpha\beta}), (w_1, 0, C_{\alpha\beta}), (w_2, 0, C_{\alpha\beta}), (w_3, 0, C_{\alpha\beta}),$
$(w_4, 0, C_{\alpha\beta}), (w_5, 0, C_{\alpha\beta}), (w_0, 0, C_{\beta\gamma}), (w_1, 0, C_{\beta\gamma}),$
$(w_2, 0, C_{\beta\gamma}), (w_3, 0, C_{\beta\gamma}), (w_4, 0, C_{\beta\gamma}), (w_5, 0, C_{\beta\gamma})$
(8) current time = 0

We show the result for the above question in Fig.3.8. In this Fig.3.8, φ is a "Mars exploration" and ψ is a "self-repair".

Next, we show an example of the computer network.

Example 7 *Let $\alpha, \beta, \gamma, \delta \in Agent$, each agent is linked with some other agents by the computer network communications circuit. We show this network composition in Fig.3.9. Then, agent α hopes to send a message φ to all the other agents, however a circuit from α to β is disconnected. To send to all the other agents, how many steps does it take?*

We show the result for the above question in Fig.3.10. Since the disconnection of the circuit from α to β, in the possible world w_3, φ is not included at the state 2. However, φ is included at the state 4 because it is sent from γ to β by $C_{\gamma\beta}$.

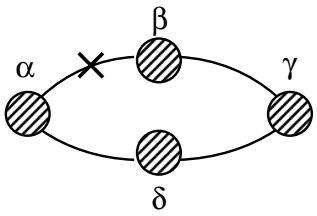


Figure 3.9: Network communications circuit

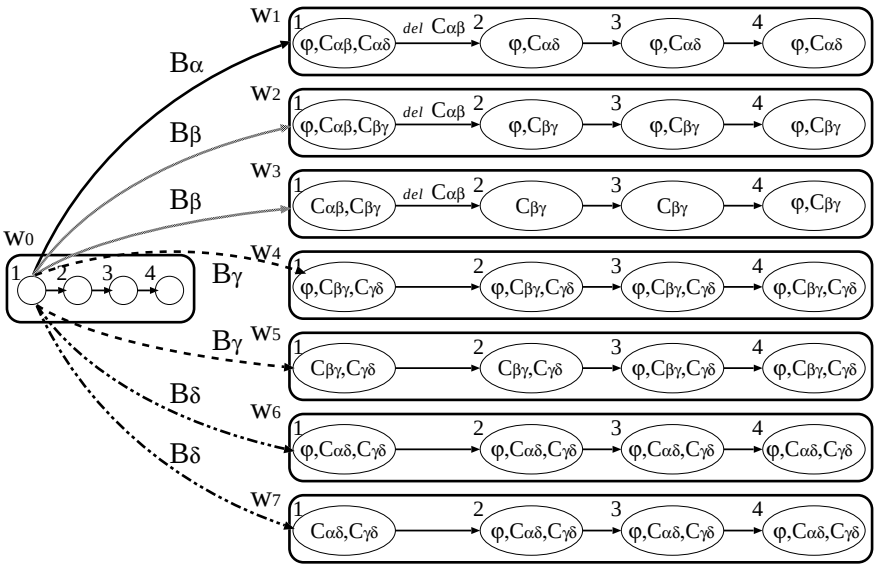


Figure 3.10: Example of a computer network

In this situation, γ sends φ to β no matter if α sends it to β. For a representation of the communication between agents, we need to explicitly introduce other operators such as D and I in BDI logics.

3.5 Discussion

We introduced CB_{CTL} and the reasoning system for it, based on temporal epistemic logic CTL. Because there has been no sound formalization of the modality U in the definition of *inform* in ACL/FIPA thus far, we did not include the modality in our logic in order to avoid fruitless complication. However, we can simply add U to our $inform^*$ later when it is adequately introduced. With regard to the existence of a reliable channel, we defined it as a proposition in the precondition of $inform^*$. Actually, the channel could be defined in such other ways as a modal operator, a higher-order meta-predicate, a background condition of inference, and so on. However, the definition by proposition seemed simplest as far as it did not affect belief operator, accessibility, and branching time of the logic; so that we adopted the current scheme.

We have used the term *update* of epistemic state when we splice a new state to a branching time path. Furthermore, in case of communication of $\varphi \vee \psi$, new possible worlds are provided. This view is based on the practical reason, that is, that the epistemic state could be changeable so as to satisfy the formula given as a query, accompanying a series of communications. Actually, what the prover does is to detect such a satisfiable path that is not explicitly mentioned at the time of query. Namely, one extreme view is that a new Kripke frame is given for each time step. However, in the strict view of modal logic, all the possible branching time can be regarded to be given *a priori* immediately when a user declared a set of worlds, accessibility in them, and a set of communication channels.

We showed its decidability of the logic and implemented a model checker; if it is directly provable or if it could be validated through the chains of communications, the system returns the proof.

As we have mentioned above, there are several future subjects: (i) the inclusion of U and (ii) communication channel with other definitions. In addition, (iii) the variety of channels, the temporary channel with limited period, the iteration of a channel $C^*_{\alpha\beta}$, and so on, may contribute to the further development of the theory of communication.

Chapter 4

Occurrence Logic

4.1 Introduction

In this chapter, we mention about the temporal aspectual studies. In order to do this analysis, we utilize aspectual information is called heredity for each occurrences by using a concept of a temporal interval. So, as an application of the temporal logic, we propose a logic of occurrence which is based on order-sorted logic.

A predicate of logic generically represents a static feature, viz. a *property* of their arguments. Our objective in this paper is to present a logic of *occurrence*, as the opposing concept to property. Each occurrence of an event happens once and for all, and the events may have causal relations or other kinds of information flow [6] in them. In order to present a logic of occurrence, we need to identify each occurrence of an event. Because each occurrence is *situated* [4], we can give an identity marker to the situation itself, called a *token* [5]. Especially in this paper, we pay attention to *temporal locations* of situations, viz., we interpret the semantics of a token as a temporal location. For example, introducing a *precedence* relation $\prec$ in two tokens x and y, we can represent a temporal order:

$$x{:}\, run \Rightarrow y{:}\, muscle\_pain \quad \text{iff} \quad x \prec y.$$

That is, "(someone) *runs*" on a certain time x caused a *muscle_pain* in the later time y. Furthermore, introducing an *inclusion* relation $\lhd$ in two tokens, we can represent temporal heredity.

$$x{:}\, dream \Rightarrow y{:}\, asleep \quad \text{iff} \quad x \lhd y,$$

that is, "(someone) dreams while sleeping." In this paper, we do not deal with tense operators; instead, we compare the above temporal relations in tokens.

In the following section, we summarize temporal relations in occurrences. In Section 4.3, we propose a language that includes a sort hierarchy, and we give the syntax and the semantics of the language, incorporating the concept of temporal heredity into it. In Section 4.4, we explain the logic programming system based on the formal language, and show the inference examples by our computer system. In the final section, we discuss some branching points of our theory and summarize our contribution.

4.2 Temporal Relations and Heredities

In this section, we define relations between temporal locations, that is, upward, downward, rightward, and leftward heredities in temporal extents.

4.2.1 Open/closed-end

We presuppose that there is a temporal interval which corresponds to each event. Thus, temporal intervals are given by a set of events, and the relationship in temporal intervals are defined also by those in events, as in [30], i.e., a set of events articulates the time axis. Because this formalism is based on the interval semantics [59], time points are introduced as a secondary concept; still we can assume a time point if (i) a starting time or a finishing time are explicitly specified in the context, or (ii) there are precedence/inclusion relations in two events. Hereafter, in case an interval *begins* or *terminates* at some fixed point, we show it by placing a short vertical bar in the following figures.

4.2.2 Upward/ downward Heredity

Suppose that there are two temporal intervals, one including the other. Then, an affair that occurs on a restricted temporal duration, that we call an *event*, has the following *upward heredity*: "Anna found a purse between 4PM and 5PM" implies "Anna found a purse between 3:30PM and 5:30PM." On the contrary, an affair that occurs on a certain time duration, that we call a *state*, has the following *downward heredity*: "Beth was sleeping between 3:30PM and 5:30PM" implies "Beth was sleeping between 4PM and 5PM." This reversion of implication has been discussed as temporal well-/ill-foundedness [15] or as solid/liquid occurrences [53].

Thus far, many linguists and computer scientists have proposed the classification of event types ([3, 31, 41, 58], and so on). However, the objective of this paper is not to implement a precise classification of aspect system, but to realize a practical logic system. For this purpose, we restrict our attention to the distinction of upward hereditary occurrences (event) and downward hereditary ones (state). A class 'event' does not refer to the inside of the occurrence [14], so that an event is viewed topologically as a point on the time axis. If the point (segment) is within a certain temporal location l, then it is also included in such l' that $l' \supset l$ (the left-hand side of Figure 4.1). On the contrary, a class 'state' gives the *imperfective* aspect on an occurrence and it is viewed as a certain temporal duration. If the occurrence persists in a certain interval l', then it also persists during such l that $l \subset l'$ (the right-hand side of Figure 4.1).

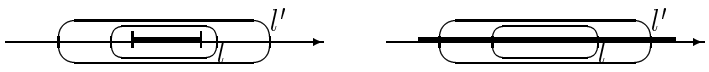


Figure 4.1: Upward/ downward heredity

Generally speaking, we cannot measure the *exact* size of the temporal location of an occurrence. However, for an upward hereditary situation, if a statement is valid on some temporal location, then the location could be narrower on the time axis. On the contrary,

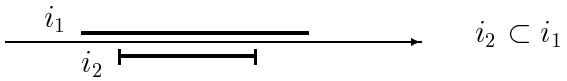


Figure 4.2: Inclusion relation

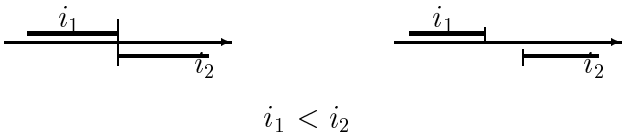


Figure 4.3: Precedence relation

for a downward hereditary situation, the *exact* location is wider on the current temporal location.

We define a temporal relation '$\lhd$' between two tokens; when $e_2 \lhd e_1$, the temporal locations i_1 of the token e_1 properly includes the temporal extent i_2 of the token e_2 as in Figure 4.2. That is, $i_2 \subset i_1$.

4.2.3 Rightward/ leftward Heredity

The second relation between temporal extents is the *precedence* relation; that simply mentions that if a statement is valid on a certain temporal location, it would be also valid in the preceding or succeeding temporal locations. We define a temporal relation '$\prec$' as in Figure 4.3, where the temporal interval i_1 of the token e_1 does not persist after the start of the temporal interval i_2 of the token e_2 when $e_1 \prec e_2$, that is, the two intervals have the separated precedence relation. Now, We denote by $i_1 < i_2$ any two members of the set of intervals for the relation.

The left-hand one in Figure 4.4 represents all those temporal extents that do not precede i. Similarly, the right-hand one in Figure 4.4 represents all those which do not persist after i. If a statement is valid in any time after its occurrence, it is called rightward hereditary. Ditto for the leftward heredity.

Considering all those temporal relations above, we can describe a complicated temporal situation as we show in Section 4.4.3. We propose a logic for these temporal relations, in the following section.

Figure 4.4: Rightward/ leftward hereditary

4.3 Logic of Occurrence and Heredity

In this section, we define the syntax and semantics of a logic of occurrence, by basing on order-sorted logic [1, 32] where every object is classified into a *sort*. Because predicates are used for occurrences, we need to have another way to represent static properties. This is the reason we integrate the order-sort logic into our occurrence logic. Beierle [7] has proposed a knowledge representation system in which a sort symbol can be expressed as a unary predicate in causal forms. Since a sort and a unary predicate have the same expressive power, we can regard a subsort declaration $s_1 \sqsubseteq s_2$ as the following generic implication as $s_1(x)$ implies $s_2(x)$.

4.3.1 Syntax

Definition 7 (Signature) *The language OL_1 consists of the following vocabulary.*

O_c	*a set of object constants*	O_v	*a set of object variables*
T_c	*a set of token constants*	T_v	*a set of token variables*
S	*a set of sort symbols*	P	*a set of predicate symbols*

In addition, the following symbols are used.

:	*token delimiter*
/	*sort substitution*
$\sqsubseteq$	*subsumption relation in sorts*
$\lhd$	*inclusion relation in tokens*
$\prec$	*precedence relation in tokens*
$\neg, \vee, \wedge, \Rightarrow$	*logical connectives*
$^{\vee}, ^{\wedge}, ^{>}, ^{<}$	*heredity markers*

Parentheses and punctuation are added if necessary.

We use $a, b, c, \cdots$ for object constants, $z_1, z_2, \cdots$ for object variables, $s_1, s_2, \cdots$ for sorts, $e_1, e_2, \cdots$ for token constants, and $x_1, x_2, \cdots$ for token variables.

Definition 8 (Subsumption) *There are partial orders in sorts: for $s_1 \sqsubseteq s_2$, we say s_2 subsumes s_1. '$\sqsubseteq$' is reflexive and transitive.*

Next, we define the hierarchy in sorts.

Definition 9 (Sort hierarchy) *For any two members s_i and s_j of the set of sorts S, we can define $s_i \sqcap s_j$ and $s_i \sqcup s_j$, each of which is called 'meet' and 'join' of s_i and s_j; especially, we call 'top' for $\top = \sqcup S$, and 'bottom' for $\bot = \sqcap S$, and thus, S forms a lattice.*

We assume that each predicate has its own argument set.

Definition 10 (Argument set) *For $p \in P$, the function A_{rg} retrieves the argument set of the predicate, that consists of sorts. $A_{rg}(p) \subseteq S$. A tuple $\langle p, A_{rg}(p) \rangle$ is called a predicate declaration.*

Definition 11 (Predicate expression) *A predicate expression consists of a predicate symbol with possibly multiple arguments. Each argument is a substituted sort.*

$$p(o_1/s_1, o_2/s_2, \cdots, o_n/s_n),$$

where p is a predicate symbol, $o_1, o_2, \cdots, o_n$ are objects (either constants and variables), and $s_1, s_2, \cdots, s_n$ are sorts, respectively.

We use Greek letters $\phi, \psi, \cdots$ for predicate expressions.

Example 8 Let P_d and R_s be predicate declarations and sort subsumption relations, respectively. Given:

$$\begin{aligned} O_c &= \{a, b\}, \quad O_v = \{z\}, \\ S &= \{man, woman, human, animal, \top, \bot\}, \\ P_d &= \{\langle love, \{human, human\}\rangle, \langle walk, \{animal\}\rangle, \langle run, \{animal\}\rangle\}, \\ R_s &= \{man \sqsubseteq human, woman \sqsubseteq human, human \sqsubseteq animal\}, \end{aligned}$$

we can write an expression '$love(a/man, b/woman)$,' describing that "a man a loves a woman b." Because $man \sqsubseteq human$ and $human \sqsubseteq animal$, '$walk(z/man)$' ("an anonymous man z walks") is a well-formed expression with regard to its predicate declaration, while '$walk(z/fish)$' is not.

Besides the subsumption relation in sorts, we introduce two relations in tokens.

Definition 12 (Inclusion and Precedence) *There are partial orders in tokens. For $e_1 \prec e_2$, we say e_1* precedes *e_2, and for $e_1 \lhd e_2$, we say e_2 includes e_1. Both of '$\prec$' and '$\lhd$' are transitive.*

Example 9 Let R_i be inclusion relations. Given:

$$\begin{aligned} O_c &= \{john, mary, ball\}, \quad T_c = \{e_1, e_2\} \\ S &= \{human, obj\}, \quad R_i = \{e_1 \lhd e_2\} \\ P_d &= \{\langle kick, \{human, obj\}\rangle, \langle sleep, \{human\}\rangle\} \end{aligned}$$

then, as in Section 4.2.2,

$$\begin{aligned} e_1\!: kick(john/human, ball/obj) &\Rightarrow e_2\!: kick(john/human, ball/obj), \\ e_2\!: sleep(mary/human) &\Rightarrow e_1\!: sleep(mary/human), \end{aligned}$$

when we interpret '$\lhd$' as the temporal inclusion.

Definition 13 (hereditary relations) *For hereditary relations, we define the following forms:*

$$\begin{aligned} e^{\wedge}\!:\phi \quad & \textit{iff} \quad \forall\ e'\ (\rhd e)\ [e'\!:\phi] \\ e^{\vee}\!:\phi \quad & \textit{iff} \quad \forall\ e'\ (\lhd e)\ [e'\!:\phi] \\ e^{>}\!:\phi \quad & \textit{iff} \quad \forall e'\ (\succ e \ \textit{or}\ \exists e_3, e_4(e_3 \lhd e, e_3 \lhd e', e_4 \lhd e, e_4 \not\lhd e, e_4 \prec e_3))\ [e'\!:\phi] \\ e^{<}\!:\phi \quad & \textit{iff} \quad \forall e'\ (\prec e \ \textit{or}\ \exists e_3, e_4(e_3 \lhd e, e_3 \lhd e', e_4 \lhd e, e_4 \not\lhd e, e_3 \prec e_4))\ [e'\!:\phi] \end{aligned}$$

The heredity markers: $^{\wedge}, ^{\vee}, ^{>}$, and $^{<}$ are read as *up, down, right*, and *left*, respectively; meanings of which are given in the following subsection. We denote by T_v^* the set $\{x_i^* \mid x_i \in T_v\}$ of token variables with a heredity maker $*$. The set of the all token variables is denoted by $T_v^+ = T_v \cup \bigcup_{* \in \{\wedge,\vee,>,<\}} T_v^*$.

Definition 14 (Interval and Atomic Formula) *Suppose that e is a token and ϕ is a predicate expression, then $e{:}\,\phi$ is an atomic formula. Also, the following forms: $e^{\wedge}{:}\,\phi$, $e^{\vee}{:}\,\phi$, $e^{>}{:}\,\phi$, and $e^{<}{:}\,\phi$ are atomic formulae of the language OL_1. For each of the above formulae, e could be replaced for a token variable.*

4.3.2 Semantics of Occurrence

Similar to other formal semantics, we give a universe U_o, each member of which corresponds to an individual by the function $[\![\ ]\!]$ to interpret OL_1. As for tokens, if there occurred an event then there must be a spatio-temporal location for the occurrence; thus, we reduce the meaning of a token of an occurrence to a temporal extent. Avoiding the general discussion as to what is the component of time [59], we only assume that there is a set of extents of time and those temporal extents are partially ordered by *precedence* or by *inclusion*.

We introduce a set of temporal extents U_t in our semantics, so that each token is interpreted as a member of U_t. If there occurred an event, then it is anchored to the corresponding temporal extent.

Definition 15 (Structure) *A structure M (for OL_1) is a tuple $\langle U, \{R_{p_i}\}, [\![\ ]\!]\rangle$ such that:*

1. *$U = U_o \cup U_t$: a non-empty set (i.e. the universe of M) where U_o is a set of individuals and U_t is a set of temporal extents such that $U_o \cap U_t = \emptyset$,*
2. *$\{R_{p_i}\}$: a set of relations $R_{p_i} \subseteq U_t \otimes [\![s_1]\!] \otimes \cdots \otimes [\![s_n]\!]$ where $p_i \in P$ and $A_{rg}(p_i) = \{s_1, s_2, \cdots, s_n\}$*
3. *$[\![\ ]\!]$: an interpretation function such that*
   - *(a) $[\![c]\!] \in U_o$ for $c \in O_c$,*
   - *(b) $[\![s]\!] \subseteq U_o$ for $s \in S$,*
   - *(c) $[\![s_1]\!] \subseteq [\![s_2]\!]$ for $s_1 \sqsubseteq s_2$ $(\in R_s)$,*
   - *(d) $[\![e]\!]$ is an element of U_t (i.e. $[\![e]\!] = i$ with $i \in U_t$) for $e \in E_c$,*
     - *i. $[\![e^{\wedge}]\!] = \{[\![e']\!] \mid [\![e]\!] \subset [\![e']\!]\}$*
     - *ii. $[\![e^{\vee}]\!] = \{[\![e']\!] \mid [\![e']\!] \subset [\![e]\!]\}$*
     - *iii. $[\![e^{>}]\!] = \{[\![e']\!] \mid [\![e]\!] < [\![e']\!]\}$*
     - *iv. $[\![e^{<}]\!] = \{[\![e']\!] \mid [\![e']\!] < [\![e]\!]\}$*
   - *(e) $[\![\lhd]\!] = \{([\![e]\!], [\![e']\!]) \mid [\![e]\!] \subset [\![e']\!]\}$, $[\![\prec]\!] = \{([\![e]\!], [\![e']\!]) \mid [\![e]\!] < [\![e']\!]\}$,*
   - *(f) $[\![p]\!] \subseteq [\![s_1]\!] \otimes \cdots \otimes [\![s_n]\!]$ for $p \in P$ and $A_{rg}(p) = \{s_1, s_2, \cdots, s_n\}$*

   where '$\otimes$' is the Cartesian product.

Note that '$<$' is a precedence relation in intervals (See Section 4.2.3). We introduce two types of variable assignments in M in order to interpret object and token variables. An object variable assignment (simply an object assignment) is a function $\alpha_o: O_v \rightarrow U_o$. A token variable assignment (simply a token assignment) is a function $\alpha_t: T_v^+ \rightarrow U_t$ such that $\alpha_t(x) \in U_t$, $\alpha_t(x^\wedge) = \{y \in U_t \mid (y, \alpha_t(x)) \in [\![\lhd]\!]\}$, $\alpha_t(x^\vee) = \{y \in U_t \mid (\alpha_t(x), y) \in [\![\lhd]\!]\}$, $\alpha_t(x^<) = \{y \in U_t \mid (y, \alpha_t(x)) \in [\![\prec]\!]\}$ and $\alpha_t(x^>) = \{y \in U_t \mid (\alpha_t(x), y) \in [\![\prec]\!]\}$, where $y(\in U_t)$ represents an interval variable. Using these assignments, we extend the interpretation function to include object and token variables as follows.

Definition 16 (Interpretation) *Let M be a structure, α_o be an object assignment and α_t be a token assignment. An interpretation $\mathcal{I}$ is a tuple $\langle M, \alpha_o, \alpha_t \rangle$. The denotation $[\![\ ]\!]_{(\alpha_o, \alpha_t)}$ is defined by the following.*

1. $[\![c/s]\!]_{(\alpha_o,\alpha_t)} = [\![c]\!]$ *with* $[\![c]\!] \in [\![s]\!]$
2. $[\![z/s]\!]_{(\alpha_o,\alpha_t)} = \alpha_o(z)$ *with* $\alpha_o(z) \in [\![s]\!]$
3. $[\![e^\wedge]\!]_{(\alpha_o,\alpha_t)} = [\![e^\wedge]\!]$, $[\![e^\vee]\!]_{(\alpha_o,\alpha_t)} = [\![e^\vee]\!]$, $[\![e^>]\!]_{(\alpha_o,\alpha_t)} = [\![e^>]\!]$, $[\![e^<]\!]_{(\alpha_o,\alpha_t)} = [\![e^<]\!]$
4. $[\![e^\wedge]\!]_{(\alpha_o,\alpha_t)} = \{\alpha_t(x^\wedge)\}$, $[\![e^\vee]\!]_{(\alpha_o,\alpha_t)} = \{\alpha_t(x^\vee)\}$,
   $[\![e^>]\!]_{(\alpha_o,\alpha_t)} = \{\alpha_t(x^>)\}$, $[\![e^<]\!]_{(\alpha_o,\alpha_t)} = \{\alpha_t(x^<)\}$

Definition 17 (Satisfiability) *Let $\mathcal{I} = \langle M, \alpha_o, \alpha_t \rangle$ be an interpretation and F be a formula. The satisfiability $\mathcal{I} \models F$ is defined by:*

1. $\mathcal{I} \models e{:}\, p(o_1/s_1, \cdots, o_n/s_n)$ *iff* $\langle rel \rangle \in [\![p]\!]$ *and, for* $i = [\![e]\!]_{(\alpha_o,\alpha_t)}$, $\langle i, rel \rangle \in R_p$ *where rel is the sequence* $[\![o_1/s_1]\!]_{(\alpha_o,\alpha_t)}, \cdots, [\![o_n/s_n]\!]_{(\alpha_o,\alpha_t)}$
2. $\mathcal{I} \models e^*{:}\, p(o_1/s_1, \cdots, o_n/s_n)$ *iff* $\langle rel \rangle \in [\![p]\!]$ *and, for all* $i \in [\![e^*]\!]_{(\alpha_o,\alpha_t)}$, $\langle i, rel \rangle \in R_p$ *where* $*$ *is the heredity marker*
3. $\mathcal{I} \models \neg A$ *iff* $\mathcal{I} \not\models A$,
4. $\mathcal{I} \models A \vee B$ *iff* $\mathcal{I} \models A$ *or* $\mathcal{I} \models B$,
5. $\mathcal{I} \models (\forall z/s)A$ *iff for all* $d \in [\![s]\!]$, $\mathcal{I}' \models A$ *where* $\mathcal{I}' = \langle M, \alpha_o[z \mapsto d], \alpha_t \rangle$,
6. $\mathcal{I} \models (\forall x)A$ *iff for all* $y \in U_t$, $\mathcal{I}' \models A$ *where* $\mathcal{I}' = \langle M, \alpha_o, \alpha_t[x \mapsto y] \rangle$.

The truth values for an atomic formulae $e{:}\,\phi$, $e^\wedge{:}\,\phi$, $e^\vee{:}\,\phi$, $e^>{:}\,\phi$, and $e^<{:}\,\phi$, are inferred operationally from a given set of true clauses, called a *program*. We will explain this inference rules in the following section.

4.4 Logic Programming System

We propose a logic inference system on occurrences, based on the standard Horn clause calculus. During the past 30 years, logic programming has grown from a new discipline to a mature field. Logic programming is a direct outgrowth of work that started in automated theorem proving. In this case a *literal* is an atomic formula of Section 4.3.1, and a *clause* is a disjunction of multiple literals, where a positive literal may appear at most once.

4.4.1 Objects and Sorts

In the following rules, G is a sequence of literals, and a unifier is shown with square brackets '[ ]' where 'A/B' represents that 'B' is replaced by 'A.'

Rule 9 (Object Instantiation)

$$\frac{\texttt{?-}\ G', e{:}\, p(z/s) \qquad e{:}\, p(c/s) \Leftarrow G}{\texttt{?-}\ (G', G)[c/z]} \qquad \frac{\texttt{?-}\ G', e{:}\, p(c/s) \qquad e{:}\, p(z/s) \Leftarrow G}{\texttt{?-}\ (G', G)[c/z]}$$

Rule 9 is used in a sequence of logical deduction when object variables are instantiated.

Example 10 The following example shows an *instantiation*.

$$\frac{\texttt{?-}\ e{:}\, fly(z/bird) \qquad e{:}\, fly(tweety/bird) \Leftarrow}{\Box},$$

where '$\Box$' is an *empty* clause. In the ordinary style of logic programming, the right-hand side of the upper deck is the fact:

$$e{:}\, fly(tweety/bird).$$

and the left-hand side of the upper deck corresponds to a query:

$$e{:}\, fly(z/bird).$$

The answer should be:

$$e{:}\, fly(tweety/bird), \quad \text{yes}.$$

Rule 10 (Sort specialization)

$$\frac{\texttt{?-}\ G', e{:}\, p(c/s_2) \qquad e{:}\, p(c/s_1) \Leftarrow G}{\texttt{?-}\ (G', G)[s_1/s_2]}\ (s_1 \sqsubseteq s_2).$$

Rule 10 concerns the sort hierarchy. if $s_1 \sqsubseteq s_2$, then s_1 has more narrow scope than s_2. This means that the expression with s_1 has richer information than that with s_2. Thus, if the head part contains a more narrow sort than the one in the body part of another rule, then the body part is replaced by the more narrow sort.

Example 11 The following example is a sort specialization, under $man \sqsubseteq human$.

$$\frac{\texttt{?-}\ e{:}\, fly(a/human) \qquad e{:}\, fly(a/man) \Leftarrow}{\Box}$$

fact	$e{:}\, fly(a/human)$.
query	?- $e : fly(a/human)$.
answer	$e{:}\, fly(a/man)$, yes.

4.4.2 Token Rules

Hereafter, we consider the specification of inferences on occurrence. Prior to that, we write down several examples that adequately represent the meaning of temporal heredity.

(i) The simple unification:
fact $e^*\!:\phi, \quad e' * e$
query ?-$e'\!:\phi$
answer yes.
where the heredity marker in e^* and the relation '$e' * e$' must be *synchronized* as in Table 4.1.

$[\![e']\!] \in [\![e^*]\!]$	$e' * e$
$[\![e']\!] \in [\![e^\wedge]\!]$	$e' \rhd e$
$[\![e']\!] \in [\![e^\vee]\!]$	$e' \lhd e$
$[\![e']\!] \in [\![e^<]\!]$	$e' \prec e$
$[\![e']\!] \in [\![e^>]\!]$	$e' \succ e$

Table 4.1: '$*$'-synchronization

(ii) Because each heredity marker is reflexive,
fact $e^<\!:\phi$.
query ?- $e\!:\phi$.
answer yes.

(iii) We adopt *Close World Assumption*; thus, what are not mentioned are implicitly negated.
fact $e_1\!:\phi, \quad e_1 \prec e_2$.
query ?- $e_1^>\!:\phi$.
answer no,
because we cannot infer $e_2\!:\phi$ unless all the necessary information were explicitly written.

According to the above specification, we set up the general rule of token specialization.

Rule 11 (Token specialization)

$$(i) \quad \frac{\texttt{?-}\ G', e'\!:\phi \qquad e^*\!:\phi \Leftarrow G}{\texttt{?-}\ (G', G)[e^*/e']}\ (e' * e)$$

where '$*$' synchronizes as in Table 4.1.

$$(ii) \quad \frac{\texttt{?-}\ G', e\!:\phi \qquad e^*\!:\phi \Leftarrow G}{\texttt{?-}\ (G', G)}$$

$$(iii) \quad \frac{\texttt{?-}\ G', e^*\!:\phi \qquad e\!:\phi \Leftarrow G}{\blacksquare}$$

where '$\blacksquare$' means that the further resolution is impossible.

Example 12 Suppose $e_1 \lhd e_2 \lhd e_3$.

$$\frac{\texttt{?-}\ e_1\colon find(c/purse) \qquad e_2^{\wedge}\colon find(c/purse) \Leftarrow}{\square}$$

fact	$e_2^{\wedge}\colon find(c/purse)$
query1	?- $e_3\colon find(c/purse)$
answer1	$e_3\colon find(c/purse)$, yes.
query2	?- $e_1\colon find(c/purse)$.
answer2	no.

$$\frac{\texttt{?-}\ e_3\colon sleep(a/agt) \qquad e_2^{\vee}\colon sleep(a/agt) \Leftarrow}{\square}$$

fact	$e_2^{\vee}\colon sleep(a/agt, classroom/place)$
query1	?- $e_1\colon sleep(z/agt)$
answer1	$e_1\colon sleep(a/agt, classroom/place)$, yes.
query2	?- $e_3\colon sleep(z/agt)$
answer2	no.

Then, we consider the unification of token variables. In the similar way, we first give examples.

(i) The simple unification:

fact	$e\colon \phi$.	fact	$x\colon \phi$.
query	?- $x\colon \phi$.	query	?- $e\colon \phi$.
answer	$x = e$, yes.	answer	$x = e$, yes.

(ii) The token with a heredity marker in the head part is also simply replaced by a token constant:

fact	$x^{<}\colon \phi$.
query	?- $e\colon \phi$.
answer	$x = e$, yes.

(iii) Close World Assumption:

fact	$e_1\colon \phi, \quad e_1 \prec e_2$.
query	?- $x^{<}\colon \phi$.
answer	no.

because we cannot infer $e_2\colon \phi$ unless all the necessary information are explicitly written.

Rule 12 (Token instantiation)

$$(i) \quad \frac{\texttt{?-}\ G', x\colon \phi \qquad e\colon \phi \Leftarrow G}{\texttt{?-}\ (G', G)[e/x]} \qquad \frac{\texttt{?-}\ G', e\colon \phi \qquad x\colon \phi \Leftarrow G}{\texttt{?-}\ (G', G)[e/x]}$$

$$\frac{\texttt{?-}\ G', x_1\colon \phi \qquad x_2\colon \phi \Leftarrow G}{\texttt{?-}\ (G', G)[x_1/x_2]}$$

$$(ii) \quad \frac{\texttt{?-}\ G', e\colon \phi \qquad x^*\colon \phi \Leftarrow G}{\texttt{?-}\ (G', G)[e'/x]}\ (e' * e)$$

where '$*$' synchronizes as in Table 4.1.

$$(iii) \quad \frac{\texttt{?-}\ G', e^*\colon \phi \qquad x\colon \phi \Leftarrow G}{\blacksquare}$$

4.4.3 Examples

The emulator of OL_1 was implemented in JAVA (JDK 1.2) on Solaris 5.7, on SUN^{TM} Sparc station Ultra 5-10. In this section, we show several results of our inference system. In the following examples, we omit the argument part, in brief.

Example 13 "After the water in the pot bubbled, boil the chicken. During which, frizzle garlics until the scent comes out."

First, we define **until** and **while** constraints as follows:

$$\begin{cases} x^{\wedge}\!: boil \Leftarrow x\!: frizzle. & \text{; while boiling, frizzle.} \\ x^{<}\!: frizzle \Leftarrow x\!: scent. & \text{; frizzle, until the scent comes out.} \end{cases}$$

To describe the situation, we declare the following facts and relations:

$$\begin{cases} e_1\!: bubble \Leftarrow . \\ e_2\!: boil \Leftarrow . \\ e_3\!: frizzle \Leftarrow . \\ e_4\!: scent \Leftarrow . \\ e_3 \lhd e_2,\ e_1 \prec e_2 \end{cases}$$

In this situation, we ask whether 'boiling' occurs also in e_4.

$$\dfrac{\dfrac{\dfrac{}{\texttt{?-}\ e_4\!: boil \quad x_1^{\wedge}\!: boil \Leftarrow x_1\!: frizzle}}{\texttt{?-}\ e_4\!: frizzle \qquad x_2^{<}\!: frizzle \Leftarrow x_2\!: scent}}{\texttt{?-}\ e_4\!: scent \qquad e_4\!: scent \Leftarrow}$$

$\square$

Thus, the *boiling* property is inherited downward to e_4.

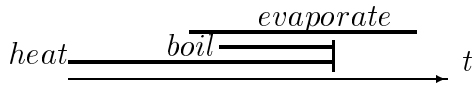
Example 14 "Water was heated up, and boiled. Then, in the process, some water must have been evaporated."

We define the following constraints:

$$\begin{cases} x^{>}\!: boil \Leftarrow x\!: heat. & \text{; boiling occurs as a result of heating.} \\ x^{\wedge}\!: evaporate \Leftarrow x\!: boil. & \text{; evaporating around the time of boiling.} \end{cases}$$

In addition, we define the following facts and token relations.

$$\begin{cases} e_1\!: heat \Leftarrow . \\ e_2\!: boil \Leftarrow . \\ e_3\!: evaporate \Leftarrow . \\ e_2 \lhd e_1,\ e_2 \lhd e_3 \end{cases}$$



We try the following query, that is, "evaporating and heating somewhere?"

$$\dfrac{\dfrac{\dfrac{}{\texttt{?-}\ x_1\!: heat, x_1\!: evap \quad x_2^{\wedge}\!: evap \Leftarrow x_2\!: boil}}{\texttt{?-}\ x_1\!: heat, x_1\!: boil \qquad x_3^{>}\!: boil \Leftarrow x_3\!: heat}}{\texttt{?-}\ x_1\!: heat \qquad e_1\!: heat \Leftarrow}$$

$\blacksquare$

In the first step, x_1 must be a member of $x_2^{\wedge}$; then in the following step, x_1 must be also a member of $x_3^{>}$. The prover can find a candidate e_3 so far, because of the following proof.

$$\dfrac{\dfrac{\texttt{?-}\ x_1\!:\!evap \quad e_3\!:\!evap \Leftarrow e_2\!:\!boil}{\texttt{?-}\ e_2\!:\!boil} \qquad e_2\!:\!boil \Leftarrow e_1\!:\!heat}{\square}$$

Unfortunately, '$\texttt{?-}\ e_3\!:\!heat$' and '$e_1\!:\!heat \Leftarrow$' are not resolved. However at this stage, we can find that if e_1 and e_3 has common time $e_1 \cap e_2$, that is the solution of the initial question.

Example 15 "The star was twinkling" implies "the star has twinkled," whereas "Carol was crossing the road" does not imply "Carol has crossed the road" (*imperfective paradox* [47, 10, 57]). Actually, the class 'accomplishment' [14, 31] is not downward hereditary. These sentences are formalized as follows.

$$\begin{cases} x^{\vee}\!:\!twinkling \Leftarrow x\!:\!twinkle. \\ x^{\vee}\!:\!crossing \Leftarrow x\!:\!cross. \\ x^{>}\!:\!twinkled \Leftarrow x\!:\!twinkling. \\ x^{>}\!:\!crossed \not\Leftarrow x\!:\!crossing. \end{cases}$$

Note that the perfect aspect is intrinsically rightward hereditary. Then, we ask "when the star has twinkled?"

$$\dfrac{\dfrac{\texttt{?-}\ x_1\!:\!twinkled \quad x_2^{>}\!:\!twinkled \Leftarrow x_2\!:\!twinkling}{\texttt{?-}\ x_2\!:\!twinkling} \qquad e\!:\!twinkling \Leftarrow}{\square}$$

Thus, $x = e$, meaning that the star has *twinkled*, just at the temporal extent of *twinkling*.

As is shown in the above figure, *twinkling* and *twinkled* can share the common time.

4.5 Concluding Remarks

We summarize our contribution as follows. (i) We proposed a logic of occurrence, extending the universe of structure from conventional U to U_o and U_t, where U_t is a set of temporal extents. (ii) In addition, giving inclusion relations in these temporal extents, we could distinguish upward-hereditary events from downward-hereditary states. Also, we introduced precedence relations in temporal extents, so as to represent rightward and leftward heredities in occurrences. (iii) Lastly, we showed a Horn clause calculus that enabled us the inference on occurrences, and implemented a computer system.

Chapter 5

Temporal logic to represent linguistic features

In chapter 4, we presented a logic of occurrence. In the above logic and temporal logic, the temporal relations between temporal intervals are utilized as the binary operators or a predicate. In this chapter, we define the modal operators for the above temporal relations, suppose a propositional temporal logic. And, by combining the results of the above chapters, we present a many-dimensional modal logic named temporal logic to represent linguistic features ($K_{T\Box}$) which is including a conventional tense logic, together with such temporal interval logic. And We show that our logic provided a formal apparatus for a precise aspectual classification. Moreover, we introduce a sequent system for our logic and show its decidability.

5.1 Introduction

Linear tense logics are widely accepted for structural temporal representation, where the basic K_T has two modal operators G and H, each of which represents the future and the past, respectively. On the other hand, the temporal interval relations arranged by Allen have long been the standard of natural language semantics, though it still lacks the modal-logical foundation. Van Benthem [60] proposed $\Box^{up}$ and $\Box_{down}$ in regard to the accessibility to overlapping intervals and subintervals, respectively; however, the logical feature of the modality has not well studied. In this study, we propose a many-dimensional logic including the conventional tense logic, together with such interval accessibility, and show its decidability.

In the following section, we propose a formalization of the temporal relations and define the syntax and Kripke semantics for our tense interval logic. In Section 5.3, we show the study of aspectual classification by our logic. In Section 5.4, we introduce a sequent system for our logic and show a proof-search procedure. In the final section, we discuss some branching points of our theory and summarize our contribution.

5.2 $K_{T\Box}$

The prime distinction of states of affairs, that is, *event* and *state*, is explained by the following *upward/downward* heredity [53]. "Anna found her purse between 4PM and 5PM"

implies "Anna found it between 3:30PM and 5:30PM." Thus, if an instantaneous *event* is mentioned in an interval, then so is also in overlapping intervals; that is upward hereditary. On the contrary, "Beth was sleeping between 3:30PM and 5:30PM" implies "Beth was sleeping between 4PM and 5PM." Therefore, if a durative *state* is valid all through the interval, then so is also in its subintervals. This is said to be downward hereditary. We define the *inclusion* relation '$\subseteq$' between temporal extents, as well as the conventional *precedence* relation '$\prec$', and propose a many-dimensional logic with these two different accessibilities, regarding a temporal extent as a possible world.

5.2.1 Syntax

Definition 18 (Signature) *The language L_1 consists of the following vocabulary.*

propositional variables:	$p,\ q,\ r, \cdots$
logical connectives:	$\neg,\ \vee,\ \wedge,\ \Rightarrow$
modal operators:	$G,\ H,\ \Box^{\uparrow},\ \Box_{\downarrow}$

Parentheses and punctuation are added if necessary.

We use $\varphi, \psi, \chi, \cdots$ for formulae which are constructed inductively from propositional variables, logical connectives and modal operators in the usual way. Modal operators F, P, $\Diamond^{\uparrow}$, and $\Diamond_{\downarrow}$ are abbreviations of $\neg G \neg$, $\neg H \neg$, $\neg \Box^{\uparrow} \neg$, and $\neg \Box_{\downarrow} \neg$, respectively. Modal operators are interpreted in the following.

$G\varphi$	at all future time, φ
$H\varphi$	at all past time, φ
$\Box^{\uparrow}\varphi$	in all superintervals, φ
$\Box_{\downarrow}\varphi$	in all subintervals, φ

As is often the case with mathematical logic, we identify a given logic $\mathcal{L}$ with the set of all formulae which are provable in $\mathcal{L}$. Following this identification, if a satisfies the following (A1), (A2), (R1), and (R2) for a modal operator, the $\mathcal{L}$ is called a normal modal logic.

(A1)	$\{\varphi \mid \varphi \text{ is a tautology}\} \subseteq \mathcal{L}$
(A2)	$L(\varphi \Rightarrow \psi) \Rightarrow (L\varphi \Rightarrow L\psi) \in \mathcal{L}$
(R1)	If $\varphi \in \mathcal{L}$ and $\varphi \Rightarrow \psi \in \mathcal{L}$, then $\psi \in \mathcal{L}$
(R2)	If $\varphi \in \mathcal{L}$, then $L\varphi \in \mathcal{L}$

It is natural to assume that tense and temporal logics are normal. Hereafter, we abbreviate $\Box^{\uparrow}$ and $\Box_{\downarrow}$ to $\Box$ if they are commonly treated. Each of the minimal tense logic K_T and $K_{\Box}$ is defined to be the least *normal modal logic* containing each of the following axioms, respectively.

	K_T		$K_{\Box}$
(A_T1)	$G\varphi \Rightarrow GG\varphi$	$(A_{\Box}1)$	$\Box\varphi \Rightarrow \Box\Box\varphi$
(A_T2)	$H\varphi \Rightarrow HH\varphi$	$(A_{\Box}2)$	$\Box\varphi \Rightarrow \varphi$
(A_T3)	$\varphi \Rightarrow GP\varphi$		
(A_T4)	$\varphi \Rightarrow HF\varphi$		

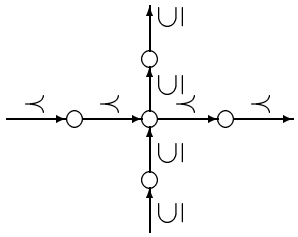


Figure 5.1: An Accessibility relations between possible worlds

Optionally, the seriality of the interval ordering would be reflected by

$$(A_{\Box}3) \quad \Box\varphi \Rightarrow \neg\Box\neg\varphi.$$

For above axioms, observe that $(A2)$ and $(A_{\Box}2)$ are axiom scheme K and T, hence $(A_{\Box}3)$ is provable. In this study, $K_{\Box}$ is axiomatized by the axioms $(A_{\Box}1)$, $(A_{\Box}2)$, and $(A_{\Box}3)$. That is, we regard $K_{\Box}$ as the modal logic **S4**. Our logic $K_T + K_{\Box}$ is obtained by *fusion* of K_T and $K_{\Box}$. Now, let L_1 and L_2 be two modal logics. If L_1 is axiomatized by a set of axioms Ax_1 and L_2 is axiomatized by Ax_2, then the fusion $L_1 + L_2$ [1] of L_1 and L_2 is axiomatized by the union $Ax_1 \cup Ax_2$ [22]. Here, we define the heredities for a state and an event by modal operators as follows:

Definition 19 *A state and an event satisfy*

$$\varphi \Rightarrow \Box_{\downarrow}\varphi \quad \textit{and} \quad \varphi \Rightarrow \Box^{\uparrow}\varphi,$$

respectively for the upward/downward heredity.

Our logic bears a resemblance to the conventional interval logics. Each possible world shows a feature of the temporal interval, however, unlike the conventional interval logic, our logic represents the *discrete* [2] temporal relations by the accessibility of possible worlds.

5.2.2 Kripke Semantics

We introduce Kripke semantics for $K_{T\Box}$. We show an example of accessibility relations of $\succ, \prec, \subseteq, and \supseteq$ in Figure 5.1.

A Kripke model for our logic is a tuple $(W, R_T, R_{\diamond}, \Vdash)$, where W is a non-empty set, and R_T and $R_{\diamond}$ are binary relations on W, and $\Vdash$ is defined inductively as follows.

$(M1)$ $u \Vdash \varphi \wedge \psi$ *iff* $u \Vdash \varphi$ and $u \Vdash \psi$
$(M2)$ $u \Vdash \varphi \vee \psi$ *iff* $u \Vdash \varphi$ or $u \Vdash \psi$
$(M3)$ $u \Vdash \varphi \Rightarrow \psi$ *iff* $u \Vdash \varphi$ implies $u \Vdash \psi$

[1] The fusion can be also denoted as $L_1 \otimes L_2$.

[2] If the time axis is the *dense*, we can represent it by adding the axioms '$GG\varphi \Rightarrow G\varphi$' and '$HH\varphi \Rightarrow H\varphi$'[53]. And, we regard our logic represents Dedekind complete time[23].

$(M4)$ $u \Vdash \neg\varphi$ *iff* $u \nVdash \varphi$
$(M5)$ $u \Vdash G\varphi$ *iff* $\forall v \in W$, uR_Tv implies $v \Vdash \varphi$
$(M6)$ $u \Vdash H\varphi$ *iff* $\forall v \in W$, vR_Tu implies $v \Vdash \varphi$
$(M7)$ $u \Vdash \Box^{\uparrow}\varphi$ *iff* $\forall v \in W$, $uR_{\diamond}v$ implies $v \Vdash \varphi$
$(M8)$ $u \Vdash \Box_{\downarrow}\varphi$ *iff* $\forall v \in W$, $vR_{\diamond}u$ implies $v \Vdash \varphi$

A formula φ is *true in model* $\mathcal{M}=(W, R_T, R_{\diamond}, \Vdash)$, denoted by $\mathcal{M} \models \varphi$, if $u \Vdash \varphi$ for every $u \in W$. Now, the following hold.

(1) $\mathcal{M} \models G\varphi \Rightarrow GG\varphi$ *iff* $\forall u, v, w(uR_Tv \wedge vR_Tw \rightarrow uR_Tw)$
(2) $\mathcal{M} \models H\varphi \Rightarrow HH\varphi$ *iff* $\forall u, v, w(wR_Tv \wedge vR_Tu \rightarrow wR_Tu)$
(3) $\mathcal{M} \models \varphi \Rightarrow GP\varphi$ *iff* $\forall u, v(uR_Tv \rightarrow vR_Tu)$
(4) $\mathcal{M} \models \varphi \Rightarrow HF\varphi$ *iff* $\forall u, v(vR_Tu \rightarrow uR_Tv)$
(5) $\mathcal{M} \models \Box\varphi \Rightarrow \neg\Box\neg\varphi$ *iff* $\forall u \exists v(uR_{\diamond}v)$
(6) $\mathcal{M} \models \Box\varphi \Rightarrow \Box\Box\varphi$ *iff* $\forall u, v, w(uR_{\diamond}v \wedge vR_{\diamond}w \rightarrow uR_{\diamond}w)$
(7) $\mathcal{M} \models \Box\varphi \Rightarrow \varphi$ *iff* $\forall u(uR_{\diamond}u)$

If R_T and $R_{\diamond}$ satisfy all of conditions from (1) to (7) for $\mathcal{M}$, $\mathcal{M}$ is called a $K_T + K_{\Box}$-model.

Now, we have the following proposition, constructing the canonical model[26] of our logic.

Proposition 5 *Let* $\mathcal{L}$ *be* $K_T + K_{\Box}$. $\forall\varphi$, $\varphi \notin \mathcal{L}$ *iff there exists* $\mathcal{L}-$*model* $\mathcal{M}$ *such that* $\mathcal{M} \not\models \varphi$.

The tense operators F and P represent an relation between separated intervals, thus we define the following definitions.

$$\forall w_1, w_2 \in W,\ w_1R_Fw_2 \equiv \forall t \in w_1 \forall t' \in w_2 : t \prec t'$$
$$\forall w_1, w_2 \in W,\ w_1R_Pw_2 \equiv \forall t \in w_1 \forall t' \in w_2 : t \succ t'$$

5.3 Aspectual Classification

Generally, a relation between two temporal interval is represented by the binary operators. Allen[2] observed that relative positions of any two intervals i and j can be described by the following thirteen interval relations: *before*(i, j), *meets*(i, j), *overlaps*(i, j), *during*(i, j), *starts*(i, j), *finishes*(i, j), their inverses, and *equal*(i, j). We can regard these interval relations as a representation with tense operator F and P by assuming the bunching possible worlds as is shown in the figure 5.2. In the figure 5.2, let the current time point is t with φ and ψ, i.e. $(\varphi \wedge \psi)$. And $\varphi \wedge \neg\psi$ consists before a certain past time point, $\neg\varphi \wedge \psi$ consists after a certain future time point. So, an overlap relation is represented by a formula

$$PH(\varphi \wedge \neg\psi) \wedge (\varphi \wedge \psi) \wedge FG(\neg\varphi \wedge \neg\psi)$$

Blackburn showed a substitution from all thirteen relation in the interval logic into the point-based tense logic in [9].

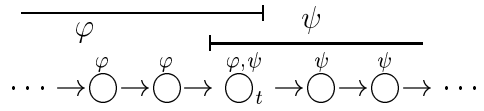


Figure 5.2: Substitution from interval logic into point-based tense logic

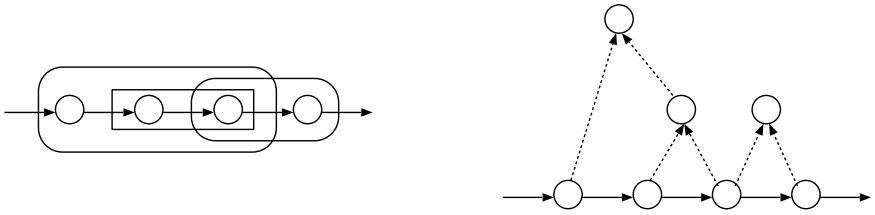


Figure 5.3: Example of accessibility relations

In our logic, we note only accessibility relations between possible worlds. We show a relation between possible worlds in Fig.5.3. In Fig.5.3, for an inclusion and overlapping relations of the left hand side, we reduce to a certain possible world with some possible worlds by $\Box^{\uparrow}$ and $\Box_{\downarrow}$. In Fig.5.3, a solid line and a dotted line stand for an accessibility relation of $\succ, \prec$ and $\subseteq, \supseteq$, respectively.

Here, we consider about the aspectual classification. A state and an event satisfy

$$\varphi \Rightarrow \Box_{\downarrow} \varphi \quad \text{and} \quad \varphi \Rightarrow \Box^{\uparrow} \varphi,$$

respectively by the definition 19. For a state, we may hypothesize the starting/ending points by assuming a certain superinterval. Then,

$$\varphi \Rightarrow \Diamond^{\uparrow} \Box_{\downarrow} \varphi,$$

where $\Diamond^{\uparrow}$ denotes some possible world which includes the assumed starting/ending points. If we could clearly specify the starting point and the ending point, then we can claim

$$\varphi \Rightarrow \Diamond^{\uparrow}(H\neg\varphi \wedge \Box_{\downarrow} \varphi) \quad \text{and} \quad \varphi \Rightarrow \Diamond^{\uparrow}(\Box_{\downarrow} \varphi \wedge G\neg\varphi),$$

respectively. In the similar way, if we assume a minimal interval of an event,

$$\varphi \Rightarrow \Diamond_{\downarrow} \Box^{\uparrow} \varphi,$$

where $\Diamond_{\downarrow}$ denotes the subinterval including the instance of the event. As, $\varphi \Rightarrow \Diamond_{\downarrow}(H\neg\varphi \wedge \Box^{\uparrow} \varphi \wedge G\neg\varphi)$ represents the *achievement* of the aspect class by Vendler[61], we can regard this formula as a representation of the *culmination*. We show some relations as in the figure 5.4.

On the contrary, Shoham [53] defined the upward/downward heredities in a fundamentally different way. In Shoham's definition, upward/downward heredities is defined as follows:

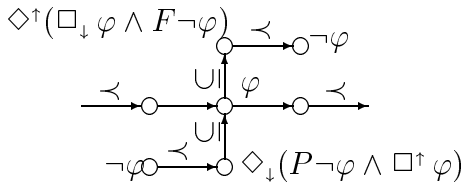


Figure 5.4: Relations between possible worlds

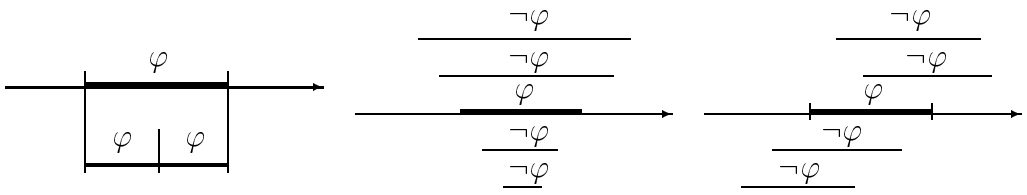


Figure 5.5: Concatenable, gestalt, and solid of shoham's categorization

Definition 20 [*Shoham's downward-hereditary*] *A proposition-type x is downward-hereditary if, whenever it holds over an interval, it holds over all of its subintervals, possibly excluding the two end points.*

Definition 21 [*Shoham's upward-hereditary*] *A proposition-type x is upward-hereditary if, whenever it holds for all proper subintervals of some nonpoint interval (except possibly at its end points), it also holds over the nonpoint interval itself.*

Then, Shoham presented some categories for temporal relations as follows:

Definition 22 [*liquid*] *A proposition-type is liquid if it is both upward-hereditary and downward-hereditary.*

Definition 23 [*concatenable*] *A proposition-type is concatenable if whenever it holds over two consecutive intervals it holds also over their union.*

We show this concatenable relation as in the left-hand side of Figure 5.5.

Definition 24 [*gestalt*] *A proposition-type is gestalt if it never holds over two intervals one of which properly contains the other.*

We show this gestalt relation as in the middle of Figure 5.5.

Definition 25 [*solid*] *A proposition-type is solid if it never holds over two properly overlapping intervals.*

We show this solid relation as in the right-hand side of Figure 5.5.

Now, we consider these shoham's catogories for our logic. We already showed that our logic can represent some possible worlds which includes the assumed starting/ending points. For a certain superinterval of a state satisfies $\varphi \Rightarrow \Diamond^{\uparrow}\Box_{\downarrow}\varphi$, every $\Box_{\downarrow}$-accessible

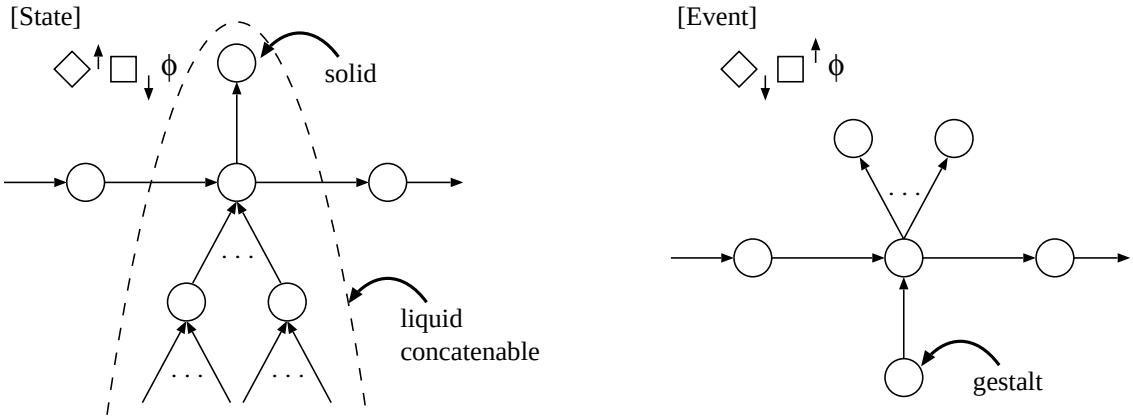


Figure 5.6: Logical expression for Shoham's categorization

worlds have a property of the *liquid* and the *concatenable*. Especcially, the possible world indicated by $\Diamond^{\uparrow}\Box_{\downarrow}\varphi$ has a property of the *solid*. On the contrary, for a event, we can claim that the possible world indicated by $\Diamond_{\downarrow}\Box^{\uparrow}\varphi$ has a property of the *gestalt*. We show these features of our logic in the Figure 5.6. That is, our logic which is included $\Box$ operator contributes to an implication of the shoham's category.

Moreover, we show a relevant study, aspectual calculus proposed by Galton[21]. Independent intuition abound in the temporal system of natural languages. In the past decade, interesting logical systems have appeared taking more cues from the latter field. One noticeable example is the *aspectual calculus* of Galton[21]. The ontological picture behind natural language is lush, unlike the Spartan spirit behind most logical formalisms. We are living in a rich common sense world populated not just by individuals and events, but also 'processes', 'state', and so on. In Galton's formalism, states and events appear on a par as basic temporal entities. In [21], Galton introduce the category of *event-radicals*. An event-radical is a complete expression that is neither a proposition nor a proper name. It 'denotes' an event. In the resulting two-level system, the earlier tense F, P become operators from states to states, whereas the Progressive (PROG) as well as the Perfect (PERF) change events into states. So he introduced three aspect operators are called the *perfect*, *progress* and *prospective* operators denoted by the symbols $Perf$, $Prog$, and $Pros$, respectively. But there are also operators changing state into events, such as INGR ('begin to') or PO ('spend a while'). By using these aspectual operators and a concept of the event radical, he represent the class of state and progressive form of event, and perfect form of event, with tense operators. And he present some interesting results;

$$
\begin{aligned}
&ProsE \rightarrow HProsE,\\
&PerfE \rightarrow GPerfE,\\
&ProsE \rightarrow FPerfE,\\
&PerfE \rightarrow PProsE,\\
&ProgE \rightarrow FPerfE, \text{ and so on.}
\end{aligned}
$$

Our main object does not show the syntactic distinction in natural language, however, we might represent some of them, because of we can regard that the representation '$\Diamond^{\uparrow}\Box_{\downarrow}$'

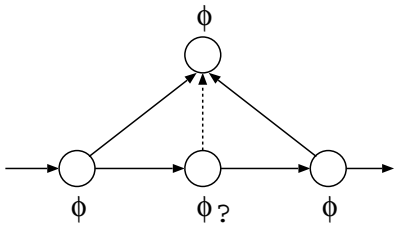


Figure 5.7: A continuousness of occurrences

has the property of progressive form.

5.3.1 Additional axioms for a structural relation

Now, note the differences in conditions between following formulae. The relation $\prec$ and the relation $\succ$ in section 5.2 denote that there is not an intersectional temporal interval (or possible worlds) between two occurrences. That is, the interpretation of the following formulae is differently.

(1) $F\,\Box^{\uparrow}\,\varphi$
(2) $\Box^{\uparrow}\,F\varphi$

The formula (1) means "at some future, φ is true in all superintervals," (2) "in all superintervals, φ is true at some future." So, it remains to be seen whether the possible worlds held up (1) include a current time (or possible world), however, the possible worlds held up (2) include a current time (or possible world). Ditto F to P. On the contrary, neither the following formulae (3) nor (4) include a current time.

(3) $F\,\Box_{\downarrow}\,\varphi$
(4) $\Box_{\downarrow}\,F\varphi$

Thus, because of both R_T and $R_{\diamond}$ represent temporal relation, it is necessary to prepare the following axioms.

$(Ax1)\ \ \Diamond^{\uparrow}\,F\varphi \Rightarrow F\varphi$ $\qquad$ $(Ax3)\ \ F\,\Diamond_{\downarrow}\,\varphi \Rightarrow F\varphi$
$(Ax2)\ \ \Diamond^{\uparrow}\,P\varphi \Rightarrow P\varphi$ $\qquad$ $(Ax4)\ \ P\,\Diamond_{\downarrow}\,\varphi \Rightarrow P\varphi$

Additionally, for operators $\Box^{\uparrow}$ and $\Box_{\downarrow}$, it is necessary to prepare the following axioms.

$(Ax\Box1)\ \ \varphi \Rightarrow \Box^{\uparrow}\,\Diamond_{\downarrow}\,\varphi$ $\qquad$ $(Ax\Box2)\ \ \varphi \Rightarrow \Box_{\downarrow}\,\Diamond^{\uparrow}\,\varphi$

We describe the axiomatic system for $K_{T\Box}$ with the axiom $(Ax\Box1)$, the axiom $(Ax\Box2)$, and the axioms from $(Ax1)$ to $(Ax4)$ as $K^{+}_{T\Box}$.

Furthermore, I show the necessary axioms to represent time. The current axiomatic system $K^{+}_{T\Box}$ is not included a continuousness of occurrences. We show this in Figure 5.7. So, we need to add the following axiom:

$$(Ax5)\quad FF\Diamond^{\uparrow}\varphi \wedge \Diamond^{\uparrow}\varphi \Rightarrow F\Diamond^{\uparrow}\varphi$$

But, we can claim $K^{+}_{T\Box}$ has nondiscrete set by contraries. That is, we can represent a certain possible world which consists φ, some possible worlds for a subinterval of this consist φ and the other possible worlds do not. So, a reiteration of the occurrences is expressed by $K^{+}_{T\Box}$, and also $K_{T\Box}$.

There is a difference in structure between $K_{T\Box}$ and $K^{+}_{T\Box}$, however, a power of the linguistic representation of $K^{+}_{T\Box}$ does not turn into a power of the linguistic representation of $K_{T\Box}$. $\Box^{\uparrow}$ and $\Box_{\downarrow}$ can not express the linguistical dynamic/static feature of action, that is the operators in our logic can not represent a difference between continuity of activity and a consequence of action. We focus attention on a minimal normal modal logic $K_{T\Box}$ and show its decidability in the following section.

5.4 Decidability

In this section, we introduce a sequent system for the $K_{T\Box}$. We show the subformula property holds in our system, and thus are able to show the decidability. In the following, uppercase Greek letters, $\Gamma, \Delta, \Pi, \Sigma, \Theta$ and Λ denote finite sets of formulae. And $\Box\Gamma$ denotes $\{\Box\varphi | \varphi \in \Gamma\}$ for $\Box \in \{\Box^{\uparrow}, \Box_{\downarrow}\}$. $Sub(\Gamma)$, Γ_{*}, and Δ^{*} denote $\bigcup\{Sub(\psi)|\psi \in \Gamma\}$, $\bigwedge\{\varphi|\varphi \in \Gamma\}$, and $\bigvee\{\varphi|\varphi \in \Delta\}$, respectively, where $Sub(\psi)$ denotes a set of all subformulae of φ. Any expression of the form $\Gamma \to \Delta$ is called a *sequent*, where $\to$ denotes a *derivation relation*. An *inference rule* is of the form

$$\frac{S_1}{S} \quad or \quad \frac{S_2 \quad S_3}{S},$$

where S_1, S_2, S_3, and S are sequents. In the inference, S_1, S_2, and S_3 are called the *upper sequents*, and S the *lower sequent*.[3]

5.4.1 Sequent System for $K_{T\Box}$

The sequent system $G(K_T + K_{\Box})$ is obtained from the sequent system **LK** for the classical propositional logic by adding the following four rules.

$$(\Box 1)\quad \frac{\varphi, \Sigma \to \Lambda}{\Box\varphi, \Sigma \to \Lambda}$$

$$(\Box 2)\quad \frac{\Box\Sigma, \Sigma \to \Theta}{\Box\Sigma \to \Box\Theta}$$

$$(T1)\quad \frac{G\Sigma, \Sigma \to H\Lambda, H\Theta, \varphi}{G\Sigma \to H\Lambda, \Theta, G\varphi}$$

$$(T2)\quad \frac{H\Sigma, \Sigma \to G\Lambda, G\Theta, \varphi}{H\Sigma \to G\Lambda, \Theta, H\varphi}$$

Rule $(T1)$ and $(T2)$ are introduced by Nishimura in [44]. Since $K_{\Box}$ is the modal logic **S4**, we introduced Rule $(\Box 1)$ and $(\Box 2)$.

[3] If a sequent S is provable in a system G, then it is often denoted by $G \vdash S$.

Proposition 6
$\Gamma_* \Rightarrow \Delta^* \in K_T + K_\Box \quad \Leftrightarrow \quad G(K_T + K_\Box) \vdash \Gamma \to \Delta.$

The sequent system $G(K_T + K_\Box)$, however, lacks the cut-elimination property, so that subformula property does not hold. For example, a sequent $\varphi \to G\neg H\neg\varphi$ is not provable without the cut rule [4] . Takano[55] introduced the restricted cut rule (AC) and Maruyama et. al.[39] introduced the following restricted rules $(T1)'$ and $(T2)'$ for $(T1)$ and $(T2)$, respectively. They showed that those with the subformula property become provable in the revised sequent system. So, for $G(K_T + K_\Box)$, by applying the restricted rules, we introduce $G^-(K_T + K_\Box)$ as follows.

$$(AC) \quad \frac{\Sigma \to \Lambda, \varphi \qquad \varphi, \Pi \to \Theta}{\Sigma, \Pi \to \Lambda, \Theta}$$
where $\varphi \in Sub(\Sigma \cup \Lambda \cup \Pi \cup \Theta)$

$$(T1)' \quad \frac{G\Sigma, \Sigma \to H\Lambda, H\Theta, \varphi}{G\Sigma \to H\Lambda, \Theta, G\varphi}$$
where $H\Theta \subseteq Sub(\Sigma \cup \Lambda \cup \{\varphi\})$

$$(T2)' \quad \frac{H\Sigma, \Sigma \to G\Lambda, G\Theta, \varphi}{H\Sigma \to G\Lambda, \Theta, H\varphi}$$
where $G\Theta \subseteq Sub(\Sigma \cup \Lambda \cup \{\varphi\})$

In (AC), $(T1)'$ and $(T2)'$, we can easily see that every formula occurring in the upper sequents consists of subformulae of formulae in the lower sequent. Maruyama et al. showed the completeness theorem of the restricted system in [39], and thus we can prove the completeness theorem of our system in the same way. That is, we have the following theorem.

Thorem 4 *If a sequent $\Gamma \to \Delta$ is not provable in $G^-(K_T + K_\Box)$, then there is a finite $K_T + K_\Box$-model $\mathcal{M}$ such that $\mathcal{M} \not\models \Gamma_* \Rightarrow \Delta^*$.*

Here, we summarize the above results as follows:

$\Gamma_* \Rightarrow \Delta^* \notin \mathcal{L}$	$\Leftrightarrow$	$M_\mathcal{L} \not\models \Gamma_* \Rightarrow \Delta^*$
$\Updownarrow$Prop.6	Prop.5	
$G(\mathcal{L}) \nvdash \Gamma \Rightarrow \Delta$		$\Uparrow$
$\Downarrow$[55]	Theorem4	
$G^-(\mathcal{L}) \nvdash \Gamma \Rightarrow \Delta$	$\Rightarrow$	$FM_\mathcal{L} \not\models \Gamma_* \Rightarrow \Delta^*$

where $\mathcal{L}$, $M_\mathcal{L}$, and $FM_\mathcal{L}$ denote $K_T + K_\Box$, a model of $\mathcal{L}$, and a finite model of $\mathcal{L}$, respectively. That is, the restricted systems $G^-(\mathcal{L})$ is equivalent to $G(\mathcal{L})$.

Corollary 2 *If $\varphi \notin K_T + K_\Box$, then there exists a finite $K_T + K_\Box$-model $\mathcal{M}$ such that $\mathcal{M} \not\models \varphi$.*

[4] For the initial sequent '$\varphi \to \varphi$', we apply the rule ¬-left to it, get the sequent '$\neg\varphi, \varphi \to$.' And for the initial sequent '$H\neg\varphi \to H\neg\varphi$', we apply rule ¬-right and rule $T1$ to it, get the sequent '$\to \neg\varphi, G\neg H\neg\varphi$.' For these two sequents, by applying the cut rule, we get a proof figure of the sequent '$\varphi \to G\neg H\neg\varphi$.' That is, a sequent '$\varphi \to G\neg H\neg\varphi$' is provable, however, the sequent is not provable without applying the cut rule.

The decidability of $K_T + K_\Box$ follows Harrop's theorem[26].

Thorem 5 *[Harrop] If a finitely axiomatizable logic has the finite model property, then it is decidable.*

In this thesis, we took particular note of a sequent system for $K_{T\Box}$, however, for $K^+_{T\Box}$, it is only necessary to add the following rules.

$$(R1) \quad \frac{\Sigma \to \Box_\downarrow \Theta, \varphi}{\Box^\uparrow \Sigma \to \Theta, \Box^\uparrow \varphi}.$$

$$(R2) \quad \frac{\Sigma \to \Box^\uparrow \Theta, \varphi}{\Box_\downarrow \Sigma \to \Theta, \Box_\downarrow \varphi}.$$

$$(R3) \quad \frac{G\Sigma, \Sigma \to \Lambda, \Theta}{G\Sigma \to G\Lambda, \Box_\downarrow \Theta}.$$

$$(R4) \quad \frac{H\Sigma, \Sigma \to \Lambda, \Theta}{H\Sigma \to H\Lambda, \Box_\downarrow \Theta}.$$

Where (R1), (R2), (R3) and (R4) are reflected in the axioms $(Ax\Box 1)$, $(Ax\Box 2)$, $(Ax1)$ and $(Ax3)$, and $(Ax2)$ and $(Ax4)$, respectively. To show the decidability of $K^+_{T\Box}$, then we must restrict the rule $(R1)$ and $(R2)$.

By [34], the fusions of modal logics with finite model property have the finite model property. Besides that, for complete modal logics L_1 and L_2 not containing $\bot$, the fusion $L_1 + L_2$ is decidable if both components L_1 and L_2 are decidable. The decision procedure by Harrop's theorem is extremely inefficient, and not feasible practically [5] . But, if a logic has the finite model property, there exists a model which invalidates unprovable formulae. Therefore, we can expect that such formulae are found in the model efficiently. The decision procedure by using the restricted sequent system is more efficient procedure. We show a proof-search procedure for $K_T + K_\Box$ in the following section.

5.4.2 Proof-search Procedure

A decision procedure for $K_T + K_\Box$ is a concrete finite procedure which decides whether a given formula is provable or not in a logic $K_T + K_\Box$. For $\Gamma \Rightarrow \Delta$, when the same formula appears only once in each of Γ and Δ, we call it *1-reduce*. If $\Gamma \Rightarrow \Delta$ is not 1-reduce, then we obtain a 1-reduce sequent by using contraction and exchange rules. So, it is enough to search a proof for 1-reduce sequents. Here, a reduced sequent which consists of formulae in $Sub(\Gamma \cup \Delta)$ is called a *suitable sequent*. Then, it is enough to search a proof which consists only of suitable sequents. Every proof can be transformed into the proof without any repetition of sequents. Here we call 'partially constructed proofs', *inference figure*. In the inference figure, each rule must be applied in a correct way, but the uppermost

[5] Aside from this, by using a concept of a partial valuation, which means limiting the size of the tree for Kripke model, we can prove a decidability of the modal logic S4.

sequents are not necessarily the initial sequents. For each i, let $\mathcal{G}_i$ be the set of all the inference figures in which inference rules are applied at most $i-1$ times. Paying attention to these things, we can obtain the following procedure.

1. $\mathcal{G}_1$ is the singleton set consists only of $\Gamma \Rightarrow \Delta$. The figure of such a set is an inference figure.
2. Suppose that $\mathcal{G}_i$ is already defined.
   2.1 $\mathcal{G}_{i+1} := \mathcal{G}_i$.
   2.2 $\forall \mathcal{F} \in \mathcal{G}_i$, If $\exists \mathcal{I}$ such that $\mathcal{F}^\wedge = \mathcal{I}^\vee$ and $\mathcal{F}^\wedge \notin \{\mathcal{F}^\vee\}$, then $\mathcal{G}_{i+1} := \mathcal{G}_{i+1} \cup \mathcal{F}'$ such that $\mathcal{F}' - \mathcal{F} = \mathcal{I}^\wedge$.
3. If $\mathcal{G}_{i+1} = \mathcal{G}_i$, then output "$\Gamma \Rightarrow \Delta$ is not provable," and terminates.
4. If $\exists \mathcal{F}$ such that $\mathcal{F} \in \mathcal{G}_{i+1} - \mathcal{G}_i$ and $\mathcal{F}^\wedge = IS$, then output "$\Gamma \Rightarrow \Delta$ is provable" and terminates. Otherwise, go to step 2.

Where $\mathcal{F}^\wedge$, $\mathcal{F}^\vee$, $\mathcal{I}$, and IS denotes the uppermost sequent of $\mathcal{F}$, the lower sequents of $\mathcal{F}$, the inference rules, and the initial sequents, respectively. The above procedure demands the backtracking for the loop-checking by 2.2. So, a sequent system proves a bottom-up manner. Since the set of all the non-repetition inference figures which consist only of acceptable sequents is finite, and so there must exist a natural number j such that $\mathcal{G}_{j+1} = \mathcal{G}_j$. Therefore, the above procedure eventually terminates.

Example 16 *We will consider the following formula.*

$$\neg(\Box^\uparrow(G\varphi \wedge \varphi) \wedge (G\neg\varphi \wedge \neg\varphi))$$

Suppose that if P_1 *is* $\rightarrow \neg(\Box^\uparrow(G\varphi \wedge \varphi) \wedge (G\neg\varphi \wedge \neg\varphi))$, *then* $P_1 \in \mathcal{G}_1$.

If P_2 *is* $\dfrac{\Box^\uparrow(G\varphi \wedge \varphi) \wedge G\neg\varphi \wedge \neg\varphi \rightarrow}{P_1}$, *then* $P_2 \in \mathcal{G}_2$.

$$\vdots$$

If P_8 *is* $\dfrac{\Box^\uparrow(G\varphi \wedge \varphi), G\neg\varphi, \neg\varphi \rightarrow}{P_5}$, *then* $P_6 \in \mathcal{G}_6$.

If P_9 *is* $\dfrac{G\varphi \wedge \varphi, G\neg\varphi, \neg\varphi \rightarrow}{P_6}$, *then* $P_7 \in \mathcal{G}_7$.

Thus, we have the following proof.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\varphi \rightarrow \varphi}{\neg\varphi, \varphi \rightarrow}\,(\neg \Rightarrow)}{\varphi, \neg\varphi \rightarrow}\,(exchange)}{G\varphi \wedge \varphi, \neg\varphi \rightarrow}\,(\wedge \Rightarrow)}{G\varphi \wedge \varphi, G\neg\varphi, \neg\varphi \rightarrow}\,(weakning)}{\Box^\uparrow(G\varphi \wedge \varphi), G\neg\varphi, \neg\varphi \rightarrow}\,(\Box 1)}{\Box^\uparrow(G\varphi \wedge \varphi) \wedge (G\neg\varphi \wedge \neg\varphi) \rightarrow}}{\rightarrow \neg(\Box^\uparrow(G\varphi \wedge \varphi) \wedge (G\neg\varphi \wedge \neg\varphi))}\,(\neg \Leftarrow)$$

The double line in the above represents that applications of $\wedge \Rightarrow$*-rule and contraction-rule are omitted.*

Example 17 *Next, we will consider the following formula.*

$$H\neg G\Box\varphi \vee \varphi$$

Suppose that if P_1 *is* $H\neg G\Box\varphi \vee \varphi$ *then* $P_1 \in \mathcal{G}_1$.

$$\vdots$$

If P_4 *is* $\dfrac{\to H\neg G\Box\varphi, \varphi}{P_3}$, *then* $P_4 \in \mathcal{G}_4$.

If P_5 *is* $\dfrac{\to \Box\varphi, H\neg G\Box\varphi \qquad \Box\varphi \to \varphi}{P_4}$, *then* $P_5 \in \mathcal{G}_5$.

If $P_{5.1}$ *and* $P_{5.2}$ *are* $\to \Box\varphi, H\neg G\Box\varphi$ *and* $\Box\varphi \to \varphi$, *respectively,*

then $\dfrac{\dfrac{\to G\Box\varphi, \neg G\Box\varphi}{P_{5.1}} \qquad \dfrac{\varphi \to \varphi}{P_{5.2}}}{P_5} \in \mathcal{G}_6$.

If P_6 *is* $\to G\Box\varphi, \neg G\Box\varphi$, *then* $\dfrac{\dfrac{\dfrac{G\Box\varphi \to G\Box\varphi}{P_6}}{P_{5.1}} \qquad \dfrac{\varphi \to \varphi}{P_{5.2}}}{P_5} \in \mathcal{G}_7$.

Thus, we have the following proof.

$$\dfrac{\dfrac{\dfrac{\dfrac{G\Box\varphi \to G\Box\varphi}{\to G\Box\varphi, \neg G\Box\varphi}}{\to \Box\varphi, H\neg G\Box\varphi} \qquad \dfrac{\varphi \to \varphi}{\Box\varphi \to \varphi}}{\to H\neg G\Box\varphi, \varphi}}{\to H\neg G\Box\varphi \vee \varphi}$$

5.5 Concluding Remarks

We proposed the $K_{T\Box}$ which combined linear tense logic and interval logic. In our logic, temporal relations between intervals are reduced to the accessibility of possible worlds, given inclusion relations. We showed that our logic provided a formal apparatus for a precise aspectual classification. Lastly, we introduced a sequent system for $K_{T\Box}$ and showed its decidability. As a future subject, we show a decidability of $K^{+}_{T\Box}$ and consider an expanded system of $K^{+}_{T\Box}$ such as $K^{++}_{T\Box}$.

Chapter 6

Conclusion

In this study, we proposed the temporal logic to represent linguistic features ($K_{T\Box}$) which combined linear tense logic and interval logic. First, we took particular note of a conventional tense logic and temporal logic, we showed an application of the tense logic as a multi-agent system with tense logic. Especially, we note the agent's communication, by using a concept of communication channel, we defined an update of the agent's belief states. We introduced CB_{CTL} and the reasoning system for it, based on temporal epistemic logic CTL. Because there has been no sound formalization of the modality U in the definition of *inform* in ACL/FIPA thus far, we did not include the modality in our logic in order to avoid fruitless complication. And we showed its decidability of the logic and implemented a model checker; if it is directly provable or if it could be validated through the chains of communications, the system returns the proof.

Secondly, we mentioned about the temporal aspectual studies as an application of temporal logics. In order to do this analysis, we utilize aspectual information is called heredity for each occurrences by using a concept of a temporal interval, proposed a logic of occurrence. We proposed a logic of occurrence, extending the universe of structure from conventional U to U_o and U_t, where U_t is a set of temporal extents. In addition, giving inclusion relations in these temporal extents, we could distinguish upward-hereditary events from downward-hereditary states. Also, we introduced precedence relations in temporal extents, so as to represent rightward and leftward heredities in occurrences. And, we showed a Horn clause calculus that enabled us the inference on occurrences, and implemented a computer system.

Lastly, based on the results of above researches, we proposed the $K_{T\Box}$ which combined linear tense logic and interval logic. In the $K_{T\Box}$, temporal relations between intervals are reduced to the accessibility of possible worlds, given inclusion relations. We showed that our logic provided a formal apparatus for a precise aspectual classification. Moreover, we introduced a sequent system for our logic and showed its decidability. Additionally we show some axioms with multiple modal operators, and inference rules for these axioms of LK, as an axiomatic system of $K^+_{T\Box}$. As a future subject, we show a decidability of $K^+_{T\Box}$.

Appendix A

A.1 Proof of Theorem 4

In this section, we show a proof of theorem 4 by the following steps; (1) first, we show a definition of the Ξ-partial valuations and observe a relation between a set of sub formulae and the Ξ-partial valuations, (2) for a set of the Ξ-partial valuations, we define a model which implements finitely property, and (3) we show a proof of theorem 4 by the property of a model for the Ξ-partial valuations and a result of (2).

A.1.1 A Relation between Ξ-partial Valuations and Sub Formulae

Definition 26 *(Ξ-partial valuations) Let $\mathcal{L}$ and Ξ be a set of formulae which is closed under subformulae. A sequent $\Sigma \to \Lambda$ is a Ξ-partial valuation in a system $G^-(\mathcal{L})$ if the following conditions are satisfied; (i) $G^-(\mathcal{L}) \not\vdash \Sigma \to \Lambda$; (ii) $\Sigma \cup \Lambda = Sub(\Sigma \cup \Lambda)$; (iii) $Sub(\Sigma \cup \Lambda) \subseteq \Xi$.*

That is, $\Sigma \to \Lambda$ is a Ξ-partial valuation if and only if $\Sigma \cup \Lambda$ is a subset of Ξ which is closed under subformulae such that $\Sigma \to \Lambda$ is not provable in $G^-(\mathcal{L})$. Ξ-partial valuations are denoted by $u, v, w, \cdots$, and $a(u)$ and $s(u)$ denote the antecedent of u and succedent of u, respectively; i.e. $a(\Sigma \to \Lambda) = \Sigma$ and $s(\Sigma \to \Lambda) = \Lambda$. Thus both a and s can be regarded as functions from the set of sequents to the collection of sets of formulae. The following proposition 7 says a key fact on partial valuations.

Proposition 7 *Suppose that a sequent $\Sigma \to \Lambda$ is not provable in $G^-(\mathcal{L})$ and Xi is any set of formulae which includes $Sub(\Sigma \cup \Lambda)$. Then there exists a Ξ-partial valuation u such that $\Sigma \subseteq a(u) \subseteq Sub(\Sigma \cup \Lambda)$ and $\Lambda \subseteq s(u) \subseteq Sub(\Sigma \cup \Lambda)$.*

The above proposition is provable by the inductive method for a formulae included in $Sub(\Sigma \cup \Lambda)$[39].

A.1.2 A Model for the Ξ-partial Valuations

We suppose that $\Sigma \to \Lambda$ is not provable in $G^-(\mathcal{L})$. We define the model $(W, R_T, R_\Box, \Vdash)$ for the tense interval logic as follows:

$W = \{u | u$ is a $Sub(\Sigma \cup \Lambda)$-partial valuation$\}$,

$uR_T v$ *iff* $\forall \varphi$, $G\varphi \in a(u)$ implies $G\varphi \in a(v)$ and $\varphi \in a(v)$, and
$\forall \varphi$, $H\varphi \in a(v)$ implies $H\varphi \in a(u)$ and $\varphi \in a(u)$,

$uR_\Box v$ *iff* $\forall \varphi$, $\Box\varphi \in a(u)$ implies $\Box\varphi \in a(v) and \varphi \in a(v))$, and
$\Box\varphi \in a(v) implies \Box\varphi \in a(u)$,

$u \Vdash p$ *iff* $p \in a(u)$, where p is a propositional variable.

Now, $\Sigma \to \Lambda$ is not provable, so there exists a $Sub(\Sigma \cup \Lambda)$-partial valuation u such that $\Sigma \subseteq a(u)$ and $\Lambda \subseteq s(u)$. That is, W is not empty set. Since $Sub(\Sigma \cup \Lambda)$ is a finite set, W is also a finite set. In fact, if the number of formulae in $Sub(\Sigma \cup \Lambda)$ is k, then the number of elements of W is at most 2^k.

Proposition 8 *The model defined above is a $K_T + K_\Box$-model.*

PROOF. We will give a proof here only (5) in section 5.2.2 i.e. $\forall u \exists v(uR_\Diamond v)$. Take an arbitrary $u \in W$. Now define Σ and Λ as follows:

$$\Sigma := \{\psi | \Box\psi \in a(u)\}$$

$$\Lambda := \{\psi | \Box\psi \in s(u)\}$$

Then the sequent $\Box\Sigma, \Sigma \to \Box\Lambda$ is unprovable in $G^-(K_T + K_\Box)$. Otherwise, by following proof figure, $a(u) \to s(u)$ becomes provable, which is a contradiction.

$$\dfrac{\dfrac{\dfrac{\vdots}{\Box\Sigma, \Sigma \to \Box\Lambda}}{\Box\Sigma \to \Box\Lambda}}{a(u) \to s(u)}$$

Let $\Pi = Sub(\Box\Sigma \cup \Box\Lambda)$. Then $\Pi \subseteq Sub(a(u) \cup s(u)) \subseteq Sub(\Gamma \cup \Delta)$. Hence there exists a $Sub(\Gamma \cup \Delta)$-partial valuation v such that $\Box\Sigma \cup \Sigma \subseteq a(v) \subseteq \Pi$ and $\Box\Lambda \subseteq s(v) \subseteq \Pi$ by proposition 7.

Then, we will show that $uR_\Box v$. If $\Box\psi \in a(u)$, then $\psi \in \Sigma$ and $\Box\psi \in \Box\Sigma$. hence $\psi \in a(v)$ and $\Box\psi \in a(v)$. If $\Box\psi \in a(v)$, then $\Box\psi \in \Pi$, and Hence $\Box\psi \in Sub(a(u) \cup s(u)) = a(u) \cup s(u)$. If $\Box\psi \in s(u)$, then $\Box\psi \in \Box\Lambda$, so $\Box\psi \in s(v)$. This contradicts $\Box\psi \in a(v)$. Therefore $\Box\psi \in a(u)$.

Proposition 9 $\forall u \in W$, *the following hold.*

(1) $\varphi \wedge \psi \in a(u) \longrightarrow \varphi \in a(u)$ *and* $\psi \in a(u)$,
(2) $\varphi \wedge \psi \in s(u) \longrightarrow \varphi \in s(u)$ *or* $\psi \in s(u)$,
(3) $\varphi \vee \psi \in a(u) \longrightarrow \varphi \in a(u)$ *or* $\psi \in a(u)$,
(4) $\varphi \vee \psi \in s(u) \longrightarrow \varphi \in s(u)$ *and* $\psi \in s(u)$,
(5) $\varphi \Rightarrow \psi \in a(u) \longrightarrow \varphi \in s(u)$ *or* $\psi \in a(u)$,
(6) $\varphi \Rightarrow \psi \in s(u) \longrightarrow \varphi \in a(u)$ *and* $\psi \in s(u)$,
(7) $\neg\varphi \in a(u) \longrightarrow \varphi \in s(u)$,
(8) $\neg\varphi \in s(u) \longrightarrow \varphi \in a(u)$,
(9) $\Box\varphi \in a(u) \longrightarrow \forall v \in W$, $uR_{\Box}v$ *implies* $\varphi \in a(v)$,
(10) $\Box\varphi \in s(u) \longrightarrow \exists v \in W, uR_{\Box}v$ *and* $\varphi \in s(v)$,
(11) $G\varphi \in a(u) \longrightarrow \forall v \in W$, $uR_{T}v$ *implies* $\varphi \in a(v)$,
(12) $G\varphi \in s(u) \longrightarrow \exists v \in W, uR_{T}v$ *and* $\varphi \in s(v)$,
(13) $H\varphi \in a(u) \longrightarrow \forall v \in W$, $vR_{T}u$ *implies* $\varphi \in a(v)$,
(12) $H\varphi \in s(u) \longrightarrow \exists v \in W, vR_{T}u$ *and* $\varphi \in s(v)$.

PROOF. (1) suppose that $\varphi \notin a(u)$ or $\psi \notin a(u)$. Since $\varphi\wedge\psi \in a(u) \subseteq Sub(a(u)\cup s(u))$, $\varphi \in Sub(a(u) \cup s(u))$ and $\psi \in Sub(a(u) \cup s(u))$, so $\varphi \in a(u) \cup s(u)$ and $\psi \in a(u) \cup s(u)$. If $\psi \notin a(u)$, then $\psi \in s(u)$, so the following proof figure gives us a contradiction since u is a $Sub(\Gamma \cup \Delta)$-partial valuation.

$$\dfrac{\dfrac{\varphi \to \varphi}{\varphi \wedge \psi \to \varphi}\,(\wedge \to)}{\overline{\overline{a(u) \to s(u)}}}$$

Thus $\varphi \in a(u)$. Similarly, we can derive a contradiction if $\psi \notin a(u)$. Thus $\psi \in a(u)$.

(10) Suppose that $\Box\varphi \in s(u)$. Define Σ and Λ as follows:

$$\Sigma := \{\psi | \Box\psi \in a(u)\}$$

$$\Lambda := \{\psi | \Box\psi \in s(u)\}$$

Then the sequent $\Box\Sigma, \Sigma \to \Box\Lambda, \varphi$ is unprovable in $G^{-}(K_T + K_{\Box})$. For, otherwise we can derive a contradiction by using the following proof figure.

$$\dfrac{\dfrac{\dfrac{\vdots}{\Box\Sigma, \Sigma \to \Box\Lambda, \varphi}}{\Box\Sigma \to \Box\Lambda, \Box\varphi}}{\overline{\overline{a(u) \to s(u)}}}$$

Let $\Pi = Sub(\Box\Sigma \cup \Box\Lambda \cup \{\varphi\})$. Then $\Pi \subseteq Sub(a(u) \cup s(u)) \subseteq Sub(\Gamma \cup \Delta)$. Hence there exists a $Sub(\Gamma \cup \Delta)$-partial valuation v such that $\Box\Sigma \cup \Sigma \subseteq a(v) \subseteq \Pi$ and $\Box\Lambda \cup \{\varphi\} \subseteq s(v) \subseteq \Pi$ by proposition 7. It is clear that $\varphi \in s(v)$.

Now, we show that $uR_{\Box}v$. If $\Box\psi \in a(u)$, then $\psi \in \Sigma$ and hence $\Box\psi \in \Box\Sigma$. Thus $\psi \in a(v)$ and $\Box\psi \in a(v)$. If $\Box\psi \in a(v)$, then $\Box\psi \in \Pi$. So $\Box\psi \in Sub(a(u) \cup s(u)) = a(u) \cup s(u)$. If $\Box\psi \in s(u)$, then $\Box\psi \in \Box\Lambda$, and hence $\Box\psi \in s(v)$. This contradicts $\Box\psi \in a(v)$. Therefore $\Box\psi \in a(u)$.

Recall that $\Vdash$ is defined by the condition $u \Vdash p$ *iff* $p \in a(u)$ for a propositional variable p. We can show the following proposition 10 by this result and proposition 9. The following proposition 10 can be proved by simultaneous induction on the length of χ.

Proposition 10 *Suppose* $u \in W$ *and* $\chi \in Sub(\Sigma \cup \Lambda)$.

(1) $p \in s(u) \longrightarrow u \nVdash p$ *for every propotisional variable* p

(2) $\chi \in a(u) \longrightarrow u \Vdash \chi$

(3) $\chi \in s(u) \longrightarrow u \nVdash \chi$

PROOF. For (1), if $p \in s(u)$, then $p \notin a(u)$, and hence $u \nVdash p$. (2) and (3) can be proved by simultaneous induction on the length of χ, using proposition 9.

A.1.3 Proof

By the definition 26, If $\Gamma \to \Delta$ is not provable in $G^{-}(\mathcal{L})$, then there exists $Sub(\Gamma \cup \Delta)$-partial valuation u such that $\Gamma \subseteq a(u) \subseteq Sub(\Gamma \cup \Delta)$ and $\Delta \subseteq s(u) \subseteq Sub(\Gamma \cup \Delta)$. $\forall\varphi \in \Gamma$, $\varphi \in a(u)$ is satisfied, thus by the proposition 10, $u \Vdash \varphi$. That is, $u \Vdash \Gamma_{*}$. On the contrary, $\forall\varphi \in \Delta$, $\varphi \in s(u)$, thus $u \nVdash \varphi$ and $u \nVdash \Delta^{*}$. Therefore, $u \nVdash \Gamma_{*} \to \Delta^{*}$, for a system $G^{-}(\mathcal{L})$, this completes the proof of Theorem 4.

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